Finite Motion Analysis for Multifingered Robotic. Hand Considering Sliding Effects

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ABSTRACT: An algorithm for the motion planning of the robotic hand is proposed to generate finite displacements and changes in orientation of objects by considering sliding effects between the fingertips and the object at contact points. Specifically, an optimization problem is firstly solved to find minimum contact forces and minimum joint velocities to impart a desired motion to the object at each time step. Then the instantaneous relative velocity at the contact point is found by determining velocities of the fingertip and the velocity of the object at the contact point. Finally, time derivatives of the surface variables and contact angle of the fingertip and the object at the present time step is computed using the Montana's contact equation to find the contact parameters of the fingertip and the object at the next time step. To show the validity of the proposed algorithm, a numerical example is illustrated by employing the robotic hand manipulating a sphere with three fingers each of which has four joints.

1.INTRODUCTION

In recent years, dextrous multifingered robotic hands have become of interest as fine manipulations are required for more sophisticated tasks in robot applications. Various multifingered robotic hands have been designed and manufactured and many research works including basic analysis of kinematics and force control for stable grasping have also been performed. Another important problem arising from the study of multifingered hands is how to impart finite displacements and/or changes of orientation to a grasped object.

Several research works on such issues have been proposed[1]-[8], where most of them only consider rolling contacts between the fingertips and the object due to the difficulties in finding the evolution of contact points, even though the object could be manipulated more efficiently by allowing sliding contacts at the contact points. Kerr[1] discussed how to move each finger in order to execute a finite displacement of the object. Kinematic equations are derived from the rolling constraint that the fingertip and object velocities are equal at the contact point. Montana[3] and Cai and Roth[4] independently studied the kinematic relations of rigid bodies that maintain contact while in relative motion. The kinematic equations for the contact point evolution was derived. They did not, however, consider the effects of the kinematics of a finger attached to the fingertip. Cole et al.[5] derived the kinematics of rolling contact for two arbitrary shaped surfaces rolling on each other and presented a scheme for the control of these hands. Cole et al.[6] also considered the problem of dynamic control of a multifingered hand and presented a new control law that applies specifically to the situation of a hand manipulating a grasped object while certain prespecified fingers slide along the object surface. Brock[8] derived a kinematic relation between the object motion, the constraints of motion, and the grasp forces. Based on this relation, a method of reorienting a grasped object is proposed. Fearing[7] considered slip from a quasi-static viewpoint to achieve grasp stability. To the authors' knowledge, no previous work has been reported to positively utilize sliding contacts in the manipulation of the object by multifingered hands.

In this paper, we propose a finite motion pluming algorithm for multifingered hands manipulating an object of arbitrary shape considering general relative motions between the lingertip and the object, at thr contact point. The minimum contact forces and minimum joint velocities are obtained by solving a nonlinear optimization problem given initial contact parameters which is defined as the position and rotation matrices of the coordinate frame attached to the contact point with respect to the body coordinate frame. The relative velocities then can be determined by calculating the object velocity and fingertip velocities at the contact point. The contact point evolution at the next time step is also determined by utilizing the Montana's contact equation[3] and obtained relative velocities to update the contact parameters. A simulation is finally illustrated by employing three fingered robotic hand manipulating a sphere to evaluate the validity of the proposed algorithm

In the following section, finite motion planning problems for multifingrred robotic hands are formulated. In Section 3, kinematics of multifingered hands grasping an object is described and the kinematrcs of contact is also described in Section 4. In Section 5, the finite motion planning is shown to be equivalent to finding minimum contact forces and minimum joint velocities for each finger Simulation results arc summarized in Section 6 and conclusions are drawn in the final section.

2. PROBLEM STATEMENT

The finite motion planning problem for multifingered hands manipulating an object considered in this study can be divided into two stages. It is remarked that large contact forces might result in low grasping stability because even a small position error may cause a large disturbing moment at the mass center of the object and the excessive contact forces are not proper for grasping fragile objects. It is also remarked that the manipulability of fingertip may be implicitly obtained as well as energy consumption may be minimized by minimizing the joint velocities. Thus, PROBLEM I can be proposed.

(PROBLEM I) Find the minimum contact forces and minimum joint velocities of the fingers to generate the <desired motions of the object satisfying the dynamic force/moment equilibrium equation, the compatibility equation of thr relative motions, and Coulomb's law of friction as well as some physical constraints given the initial contact parameters.

Let the contact angle be defined as the angle between the corresponding axis of two coordinate frames in the common tangent plane attached to respective contact points of two contacting bodies. Then, the contact parameters evolve by updating the surface variables and contact angle in response to a relative motion of the fingertip and the object assuming that the surfaces of fingertip and the object are parameterized by the longitudinal and latitudinal variables. Thus, PROBLEM II can be proposed.

(PROBLEM II) Find the time derivatives of the surface variables and contact angle of the object and the fingertips at. present time step to predict the contact parameters at the next time step by determining relative velocities obtained from PROBLEM I.

3. KINEMATICS OF MULTIFINGERED ROBOTIC HANDS

In this section, kinematics of multifingered robotic hands is described. A k-fingered hand grasping an object is shown in Fig. 1. Let the number of joints and the joint variables of finger i, i=1,··,k, be denoted as m_i and $q_i \in R^{m_i}$, respectively. To describe the relative motions between a fingertip and an object, a set of coordinate frames are defined as follows. The reference frame. $\{C_p\}$, is fixed to the palm of the hand; the body coordinate frame, $\{C_b\}$, is fixed to the mass center of the object; the finger frame, $\{C_{fi}\}$, is fixed to the last link of finger i; at the i-tb point of contact between the finger i and the object, the local frame of the object,, $\{C_{bi}\}$, is fixed relative to $\{C_{bi}\}$ and the local frame of the finger i, $\{C_{li}\}$, is fixed relative to $\{C_{fi}\}$, where their z-axes coincide with the outward pointing normal to the object surface and the fingertip surface, respectively and their x-and y-axes lie in the common tangent plane as well as they share a common origin at the contact point.

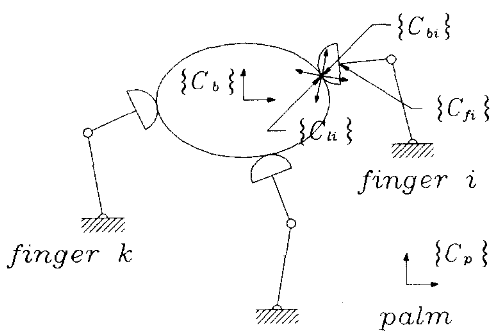

Fig. 1 A k-fingered robotic hands grasping an object

Let $r_{\beta,\alpha} \in R^3 and A_{\beta,\alpha} \in SO(3)$ denote the position vector and the rotation matrix of a coordinate frame $\{C_\beta\}$ with respect to a coordinate frame $\{C_\alpha\}$, respectively. If $(r_{\beta,\alpha(t)}, A_{\beta,\alpha(t)})$ is any curve in $SE(3) \equiv R^3 \times SO(3)$ representing the trajectory of $\{C_\beta\}$ relative to $\{C_\alpha\}$, the translational and rotational velocity of $\{C_\beta\}$ relative to {C,} can be described by

$$v_{\beta,\alpha} = A^t_{\beta,\alpha}\dot{r}_{\beta,\alpha} \quad and \quad \omega_{\beta,\alpha} = S^{-1}(A^t_{\beta,\alpha}\dot{A}_{\beta,\alpha}), \tag{1}$$

where S is an operator defined by

$$\omega = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}, \quad S(\omega) = \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix} \tag{2}$$

and superscript t implies the transpose

The vector $(v^t_{\beta,\alpha}, \omega^t_{\beta,\alpha})^t \in R^6$ *is* called the generalized velocity of $\{C_\beta\}$ with respect to $\{C_\alpha\}$ For any three coordinate frame {C,), $\{C_\beta\}$, $\{C_\gamma\}$, the following relation exists between their relative velocities

$$v_{\gamma,\alpha} = A^t_{\gamma,\beta}(v_{\beta,\alpha} + \omega_{\beta,\alpha} \times r_{\gamma,\beta}) + v_{\gamma,\beta} \tag{3-1}$$

$$\omega_{\gamma,\alpha} = A^t_{\gamma,\beta}\omega_{\beta,\alpha} + \omega_{\gamma,\beta} \tag{3-2}$$

In particular, when $\{C_\gamma\}$ is fixed with respect to $\{C_\beta\}$, the velocity $\{C_\gamma\}$ is related to that of $\{C_\beta\}$ by a constant tranformation, given by,

$$\begin{pmatrix} v_{\gamma,\alpha} \\ \omega_{\gamma,\alpha} \end{pmatrix} = \begin{pmatrix} A^t_{\gamma,\beta} & -A^t_{\gamma,\beta}S(r_{\gamma,\beta}) \\ 0 & A^t_{\gamma,\beta} \end{pmatrix} \begin{pmatrix} v_{\beta,\alpha} \\ \omega_{\beta,\alpha} \end{pmatrix} \equiv T_{\gamma,\beta} \begin{pmatrix} v_{\beta,\alpha} \\ \omega_{\beta,\alpha} \end{pmatrix} \tag{4}$$

In other words, if we let $\{C_\alpha\}$ be the reference frame, $\{C_\beta\}$ the body coordinate frame, and $\{C_\gamma\}$ the object local frame, then the velocity of the object represented by the body frame and by the local frame are related by a constant transformation, which in turn is a function of the contact parameters of the object. A similar relation holds for the finger.

Let (v^i_x, v^i_y, v^i_z) and $(\omega^i_x, \omega^i_y, \omega^i_z)$ denote the translational and rotational velocities of $\{C_{bi}\}$ with respect to $\{C_{li}\}$, respectively. These are velocities of the object with respect to finger i expressed in local frames. Using (3), tbe velocity of $\{C_{bi}\}$ can be expressed as

$$\begin{pmatrix} v_{bi,p} \\ \omega_{bi,p} \end{pmatrix} = \begin{pmatrix} A_{\psi i} & 0 \\ 0 & A_{\psi i} \end{pmatrix} \begin{pmatrix} v_{li,p} \\ \omega_{li,p} \end{pmatrix} + \begin{pmatrix} v^i_x \\ v^i_y \\ v^i_z \\ \omega^i_x \\ \omega^i_y \\ \omega^i_z \end{pmatrix}, \tag{5}$$

where

$$A_{\psi i} = \begin{pmatrix} \cos\psi i & -\sin\psi i & 0 \\ -\sin\psi i & -\cos\psi i & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{6}$$

is the orientation matrix of $\{C_{bi}\}$ with respect to $\{C_{li}\}$.

On the other hand, the velocity of $\{C_{bi}\}$ is related to the velocity of $\{C_b\}$ by

$$\begin{pmatrix} v_{bi,p} \\ \omega_{bi,p} \end{pmatrix} = \begin{pmatrix} A^t_{bi,b} & -A^t_{bi,b}S(r_{bi,b}) \\ 0 & A^t_{bi,b} \end{pmatrix} \begin{pmatrix} v_{b,p} \\ \omega_{b,p} \end{pmatrix} \equiv T_{bi,b} \begin{pmatrix} v_{b,p} \\ \omega_{b,p} \end{pmatrix} \tag{7}$$

and similarly one has for finger i that

$$\begin{pmatrix} v_{li,b} \\ \omega_{li,p} \end{pmatrix} = \begin{pmatrix} A^t_{li,fi} & -A^t_{li,fi}S(r_{li,fi}) \\ 0 & A^t_{bi,b} \end{pmatrix} \begin{pmatrix} v_{fi,p} \\ \omega_{fi,p} \end{pmatrix} \equiv T_{li,fi} \begin{pmatrix} v_{fi,p} \\ \omega_{fi,p} \end{pmatrix} \tag{8}$$

Moreover, the velocity of the linger frame. $\{C_{fi}\}$, is related to the velocity of the finger joints, $\dot{q}_i$, by the finger Jacobian,

$$\begin{pmatrix} v_{fi,p} \\ \omega_{fi,p} \end{pmatrix} = J_i(q_i)\dot{q}_i \tag{9}$$

Finally, the translational and rotational relative velocities of the object with respect to fingertip at the i-th contact pant can be expressed in terms of the velocity of the finger joints as well as the contact parameters.

$$\begin{pmatrix} v^i_x \\ v^i_y \\ v^i_z \end{pmatrix} = v_{bip} - [A_{\psi i}A^t_{li,fi}J_u\dot{q}_i - A_{\psi i}A^t_{li,fi}S(r_{li,fi})J_L\dot{q}_i] \tag{10}$$

$$\begin{pmatrix} \omega^i_x \\ \omega^i_y \\ \omega^i_z \end{pmatrix} = \omega_{bi,p} - [A_{\psi i}A^t_{li,fi}J_L\dot{q}_i], \tag{11}$$

where v^i_x and v^i_y represent sliding, ω^i_x and ω^i_y rolling, and ω^i_z spin motion.

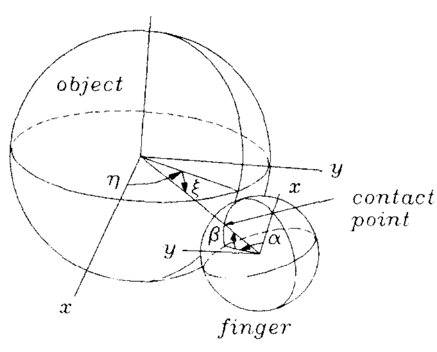

Fig. 2 The surface variables of the fingertip and the object

4. THE KINEMATICS OF CONTACT

This section describe the motion of a point of contact over the surfaces of two contacting object in response to a relative motion of these objects. When the fingertips roll or slide over the object, the contact parameters $(r_{bi,b}, A_{bi,b})$ of the object and $(r_{li,fi}, A_{li,fi})$ of the finger evolve according to the kinematic equations of contact. If the fingertip and the object surfaces are parameterized by the longitude variable u(α and η)and latitude variable v(β and ξ), we can describe the contact parameters of fingertip aud object by these variables (Fig. 2).

Let the symbols K, T, and M represent. respectively, the curvature form, torsion form, and metric at time t at the point of contact with respect to its coordinate system [11]. Let R_ψ represent the orientation matrix of the x- and y- axes of $\{C_{li}\}$ with respect to the x- and y- axes of $\{C_{fi}\}$ and the subscripts o and f denote the object and fingertip, respectively Also let $\tilde{K_f}$ **be defined as** $R_\psi K_f R_\psi$ and Ii, + $\tilde{K_f}$ be the relative curvature form. At a point of contact, if the relative curvature form is invertible, then the point of contact and angle of contact evolve according to

$$\dot{u_o} = M_o^{-1}(K_o + \tilde{K_f})^{-1}([-\omega_y \ \omega_x]^t - \tilde{K_f}[v_x \quad v_y]^t), \tag{12}$$

$$\dot{u_f} = M_f^{-1}R_\psi(K_0 + \tilde{K_f})^{-1}([-\omega_y \ \omega_x]^t + K_o[v_x \quad v_y]^t), \tag{13}$$

$$\dot{\psi} = \omega_z + T_o M_o \dot{u_o} + T_f M_f \dot{u_f}, \tag{14}$$

$$0 = v_z \tag{15}$$

Thus, the contact equation gives the time derivatives of the surface variables and contact angle by receiving the relative velocities of two contacting object (Fig. 3). Eqs. (12).(14) are called the first, second, and third contact equations, respectively. Eq. (15) is the kinematic constraint of contact imposing the constraint on the relative motion necessary to maintain contact.

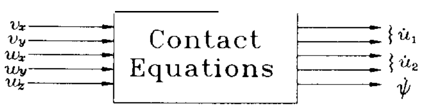

Fig. 3 The relative velocities and contact equations

5. PROBLEM FORMULATION AND NONLINEAR PROGRAMMING APPROACHES

5.1 Force/Moment Equilibrium Equation

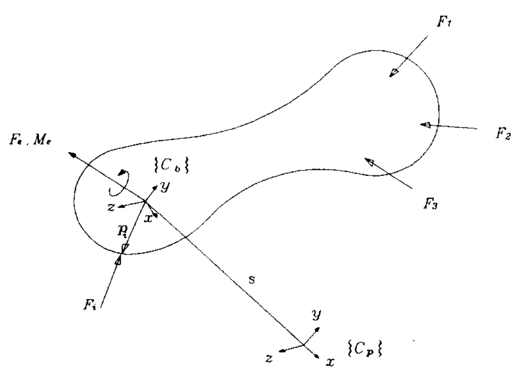

Fig. 4 Modelling of a manipulation of an object by multifingered robotic **hands**

In Fig. 4, all vectors are represented with respect to the object coordinate system $\{C_b\}$ and a frictional point contact model is assumed at the contact point. Let F_i be the force vectors applied to the object at i-th contact point by each finger and T $=[F_e^t M_e^t]^t \in R^{6\times 1}$, denote the resultant force and moment vectors, respectively Let p_i and n be the position vector from the origin of the object coordinate system to i-th contact point c_i and the number of contact points, respectively. Then, the force/moment equilibrium equation can be written as follows.

$$F_e = \sum_{i=1}^{n} F_i \tag{16 - 1}$$

and

$$M_e = \sum_{i=1}^{n} r_i \times F_i \tag{16 - 2}$$

Eq. (16) can be written in the matrix form as

$$T = GF, \tag{17}$$

where $G \in R^{6\times 3n}$ is defined by

$$G \equiv \begin{pmatrix} I_3 & I_3 & \cdots & I_3 \\ P_1 & P_2 & \cdots & I_n \end{pmatrix} \tag{18}$$

and is time dependent as the contact parameters evolve. Here I_3's are 3 × 3 unit matrices and P_i ***are*** the 3 × 3 skew symmetric matrix with zero diagonal elements equivalent to the vector product of position vectors $p_i = (p_{ix}, p_{iy}, p_{iz})^t \in 3 \times 1$ shown as

$$P_i \equiv \begin{pmatrix} 0 & -p_{iz} & p_{iy} \\ p_{iz} & 0 & -p_{ix} \\ -p_{iy} & p_{ix} & 0 \end{pmatrix} \tag{19}$$

It is noted that the dynamic equilibrium is also maintained by using above static force/moment equilibrium equation if the inertia force equal to the product of the mass of the object and its acceleration and directed oppositely to the acceleration is added to T. It is remarked that given T and G from a task and contact points, F can be determined by solving Eq (17). However, if the number of contact points are greater than 2, F may have infinite number of solutions, since matrix A has $(3n-6)$ of redundancy. Thus, among the solutions should we determine the optimal solution which **minimizes the** contact forces under some physical conatraints.

5.2 Forces Transmitted at a point of Contact

The resultant force transmitted from one surface to another through a pomt of contact is resolved into a normal force F_n acting along the common normal, which generally must be compressive, and a tangential force F_t in the tangent plane sustained by friction The magnitude of F_t must be less than or, in the limit, equal to the force of limiting friction, i.e.

$$F_t \leq \mu F_n, \tag{20}$$

where μ is the coefficient of limiting friction

5.3 Contact Maintenance Condition

If the contact between the surfaces of the fingertip and those of the object is continuous, their velocity components along the common normal must be equal such that the surfaces are neither separating nor overlapping Thus, v_z^t always equals zero and can be represented in terms of joint velocity vectors and contact parameters as follows: $[A_{bi,b}^t v_{b,p} - A_{bi,b}^t S(r_{bi,b})\omega_{b,p}]_z$

$$-[A_{\psi i}A_{li,fi}^t J_u \dot{q_i} - A_{\psi i}A_{li,fi}^t S(r_{li,fi}) J_L \dot{q_i}]_z = 0 \tag{21}$$

where subscript z implies z-compont. Any motion of contacting surfaces must satisfy the contact maintenance condition and can be regarded as the combination of sliding, rolling and spin.

5.4 Consistency of Roll/Slide Mode between Force and Motion

When the object is manipulated by the multifingered hand. either rolling or sliding at the contact points may be resulted from the contact forces. To generate the corresponding relative motions for the contact forces applied at the contact point.. following mode parameter is defined.

$$\delta = \mu F_n - F_t \tag{22}$$

While the contact forces result in rolling motions if δ is greater than zero, the contact forces sliding motions if 6 is equals to zero. Let v_t denote the translational relative velocity vector. Then, the consistency of the contact forces and relative motions at the contact points are accomplished by satisfying the following compatibility equation

$$\delta \cdot v_t = 0 \tag{23}$$

Thus, while v_t should the zero implying rolling motions if δ is greater than zero, v_t have any magnititude implying sliding motions if δ is zero.

5.5 The Direction of Tangential Forces and Sliding Velocities

The tangential force of friction is constrained to he no greater than the product, of the normal force with the coefficient of static friction. In a purely sliding contact the tangential force reaches its lirniting value in a direction opposed to the sliding velocity. In this paper, the sliding velocity is defined the translational relative velocities of the object with respect to the fingertip at the contact point. Thus, the direction of the tangential force and sliding velocity should he coincident. Let $||*||$ denote the Euclidean norm. Then, the directional condition oft angential force and sliding velocities is described as follows:

$$\frac{F_t}{||F_t||} = \frac{v}{||v_t||} \tag{24}$$

5.6 Nonlinear Optimization Problem Formulation

Now the finite motion planning problems can be formulated into an optimization problem to find the minimum contact forces and joint velocities at each time step given contact parameters satisfying above constraints as well as some physical constraints.

Minimize

$$||F_i|| + ||\dot{q}_i|| \tag{25}$$

Subject to

$$GF = T \tag{26}$$

$$F_t \leq \mu F_n \tag{27}$$

$$v_z^i = 0 \tag{28}$$

$$\dot{q}_{imin} \leq \dot{q}_i \leq \dot{q}_{imax} \tag{29}$$

$$\ddot{q}_{imin} \leq \ddot{q}_i \leq \ddot{q}_{imax} \tag{30}$$

$$F_{imin} \leq F_i \leq F_{imax} \tag{31}$$

$$\delta \cdot v_t = 0 \tag{32}$$

$$\frac{F_t}{||F_t||} = \frac{v}{||v_t||} \tag{33}$$

The summation of the Euclidean norm of contact forces and that of joint velocities are chosen as an object function in Eq (25) Eq (26) is the dynamic force/moment equlibrium equation. Eq. (27) and (33) are the Coulomb friction constraints and Eq. (28) is contact maintenance condition Eqs. (29) and (30) are joint velocity and acceleration constraints, respectively. Eq. (31) is the constraint of lhe magnitude of the contact force. Eq. (32) is the mode compatibility condition of roll or slide between contact force and relative velocities. Thus, give" the contact parameters, the contact forces and joint velocities of the fingers are obtained by solving the above nonlinear optimization problem

The procedure to find the contact forces and joint velocities at each time step can be summarized as follows.

- step 0 Read the initial configurations, contact parameters at the present. time step
- step 1 Calculate thr object velocity at the contact point.
- step 2 Calculate the contact forces and joint velocities of the fingers to find the fingertip velocities.
- Step 3 Calculate the relative velocities at the contact point
- step 4 Determine the time derivatives of the surface variables and cont act angle at the present time step by the contact equation.
- step 5 Update the contact parameters and the joint configurations of thr fingers.
- Step 6 Go to step 1.

6. SIMULATION RESULTS

The re-orienting task of a sphere (Fig. 5) is considered to show the validities of our proposed method for a robotic hand with three fingers each of which has four joints. The specification of the hand is given in Table 1. In Table 2. the specification of the object is summarized. In Table 3, the initial joint configurations of the fingers are give" and the posture of the hand is also show" in Fig 6 Initial contact parameters of the fingertip surface and object surface are given in Table 4 The orientation of reference frame and body coordinate frame are chose" to be coincident The rotative motion of 0.524 rad/sec about x-axis of reference frame is given to the object The solution of problem is obtained by utilizing the Augmented Lagrange Multiplier Method[10].

Figs. G-X show the contact forces of each finger and the joint velocities of fingers are shown in Figs 9-1 I. The contact point evolutions on the object surface are illustrated in Figs 12-14 The x-coordinates of contact points for all fingers are located around zero, because the object motion is executed in yz-plane. The y-coordinates of contact points for Finger I & II move to the positive y-direction, which agrees to the intuition of human. The z-coordinates of contact point of Finger I move downward, while that of Finger II upward, which also agrees to the intuition of human From this point of view, thr result that the contact point of Finger III remained stationary shown in Fig 14 can be explained

Table 1 Specification of Robotic Hand

No. of Fingers		3	
No. of Joints/Finger		4	
Link	Length of Each Finger		[cm]
link 1: 2.8	link 2: 6.2	link 3: 3.6	link 4: 1.4
Geometry of Fingertip Surface			Hemispher
Radius of Fingertip [cm]		1	

Table 2. Specification of the Object

Geometry of Surface	Sphere
Radius [cm]	3
Mass [g]	100
Friction Coefficient	0.45
Contact Type	Frictional Point Contact

Table 3 Initial Joint Configurations of Fingers [Rad]

	Joint 1	Joint 2	Joint 3	Joint 4
Finger 1	0	0.5233599	4 974 188	5.497787
Finger 2	0.	0.523399	4 974188	5 497787
Finger 3	0	5 7595871	3 0 8 9 9 ' 7	0.785398

Table 4. Initial Surface Variable and the Contact Angle of Fingertip and Object [Rad]

	Fingertip Surface		Object Surface		Contact
	longit. u	latit. v	longit. u	latit. v	Angle ψ
Contact Pt.1	1.5708	1.0472	1.5708	1.0472	0.
Contact Pt.2	-1.5708	1.0472	-1.5708	1.0472	0.
Contact Pt.3	0	1.5708	0	-1.5708	3.14159

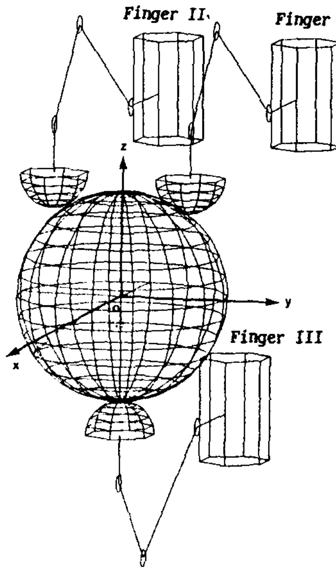

Fig. 5 The reorienting task of a sphere by a multifingered hand

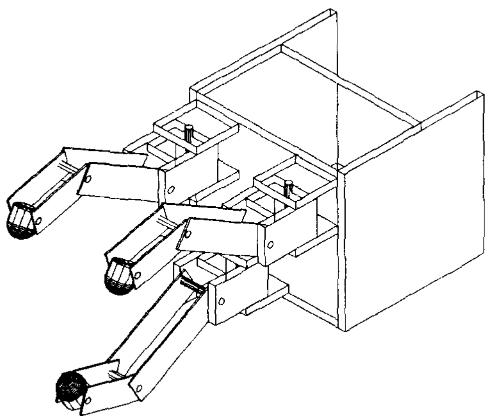

Fig. 6 The joint posture of the robotic hand

7. CONCLUSION

A finite motion planning algorithm for multifingered robotic hands manipulating an object of arbitrary shape was presented. In this study, the general relative motions Including sliding motion at the contact point was considered to find the trajectory of fingertips' contact points over the object surface. The minimum contact forces and joint velocities to generate an desired object motion was found by utilizing an nonlinear optimization technique A simulation was presented by employing three fingered robotic hand re-orienting a sphere.

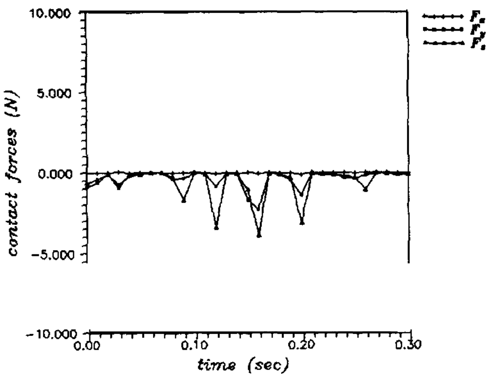

Fig. 7 The contact forces of Finger I

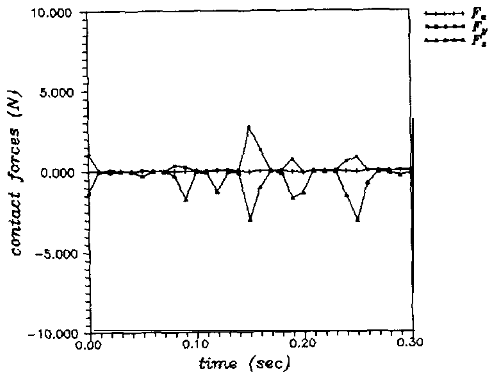

Fig. 8 The contact forces of Finger II

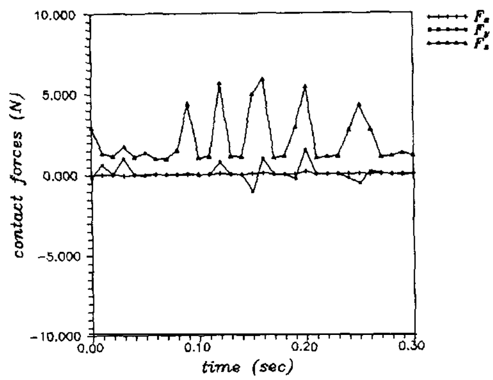

Fig. 9 The contact forces of Finger III

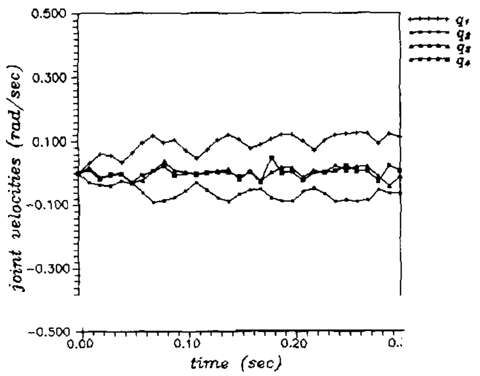

Fig. 10 The joint velocities of Finger I

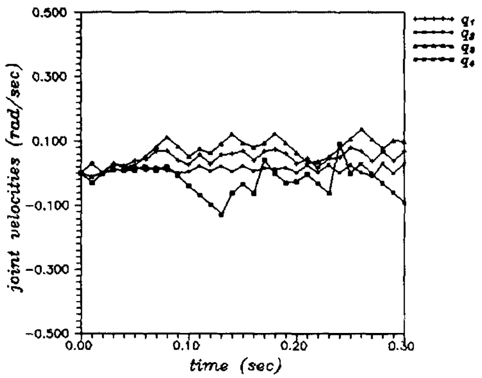

Fig. 11 The joint velocities of Finger II

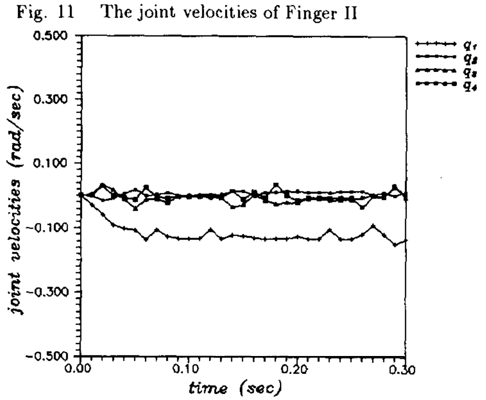

Fig. 12 The joint velocities of Finger III

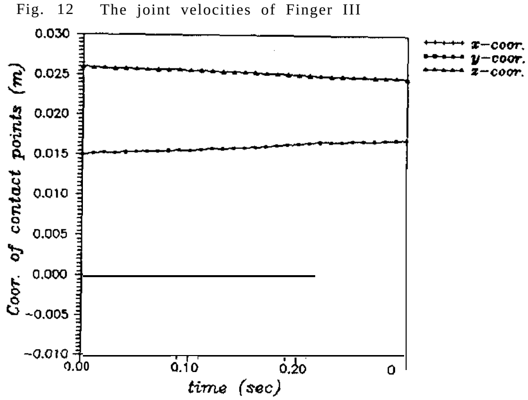

Fig. 13 The evolution of contact points of Finger I

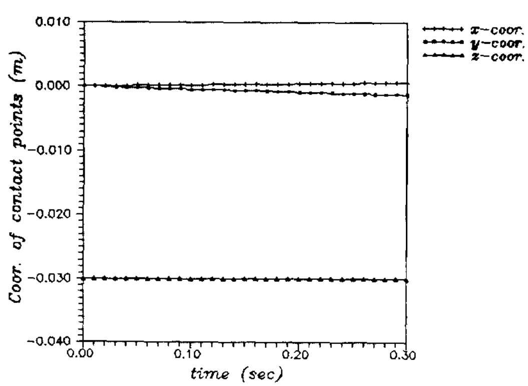

Fig. 15 The evolution of contact points of Finger III

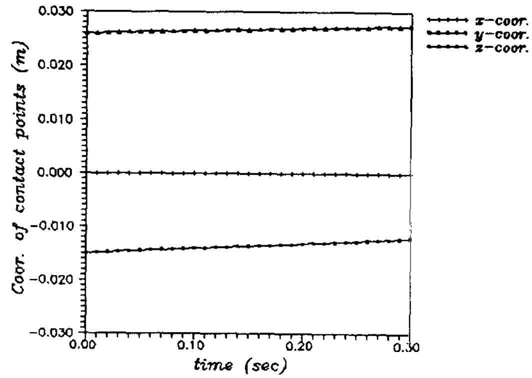

Fig 14 The evolution of contact points of Finger II

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