Optimal Design Methods for Robust Internal Loop Compensator and Its Application to Twin-Servo Brushless DC Linear Motors

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***Abstract*— We propose a robust internal loop compensator and its optimal design method based on $\mathcal{H}_\infty$ control. The controller consists of two parts, internal and external loop. Internal loop is used as a compensator for cancelling disturbances including difference between reference model and real system, and external loop is designed to meet the performance criterion. The properties of the internal loop compensator is compared with that of disturbance observer. The proposed controller structure is characterized by compensator design using reference model, relative insensitivity of the compensated system to modeling inaccuracies and unknown external disturbances, and satisfaction of desired performance specifications. The performance of proposed controller is demonstrated by experiments of a twin-servo mechanism using two brushless DC linear motors.**

I. Introduction

Recently, high speed and high accuracy motion control is one of the most interesting research field as the products become smaller and faster such as data storage or semi-conducting devices. To meet these high performance specifications, conventional optimal control methods such as LQ and LQG control have been widely used. However, mathematical description of the system can not represent real plant exactly, and unpredictable disturbances affect the performance if they are not properly attenuated.

This paper focuses on disturbance estimation and attenuation schemes in order to guarantee desired performance specifications. Various methods for this purpose have been proposed. Time delay control[1, 2] based on direct estimation of disturbance using time delay, and disturbance observer[3, 4] which makes behavior of real system as that of given nominal system in low frequency region using low pass filter are good examples. But these methods are not so generic and have not optimality.

In this paper, we design a robust controller based on model following control approach. The objective is to establish robust disturbance attenuation algorithm and to provide optimization concept in the controller design. The controller has two independent control loop, namely, internal and external loop, and their characteristics can be designed independently. The internal loop is used as a compensator rather than a controller, so we call this *internal loop compensator* and external loop is used as a controller, we call this *external loop controller*. Internal loop compensator is for the rejection of uncertain disturbances. This disturbance attenuation property looks similar to that of disturbance observer, and we can integrate various controllers for advanced motion control schemes in the design of external loop controller after the internal loop design.

In the next section, robust internal loop compensator and optimization method based on $\mathcal{H}_\infty$ control are proposed. In section III, we show our twin-servo system using two brushless DC linear motors and deal with the design of external loop controller. Experimental results of twin-servo system are shown in section IV, and conclusion follows.

II. Robust Internal Loop Compensator Design and Optimization

We consider multivariable system with r inputs, r outputs described by the following dynamic equation

$$\text{Plant } P: \quad \begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}\boldsymbol{u} \\ \boldsymbol{y} &= \boldsymbol{C}\boldsymbol{x} \end{aligned} \tag{1}$$

$$\text{Model } P_m: \quad \begin{aligned} \dot{\boldsymbol{x}}_m &= \boldsymbol{A}_m\boldsymbol{x}_m + \boldsymbol{b}_m\boldsymbol{u}_m \\ \boldsymbol{y}_m &= \boldsymbol{C}_m\boldsymbol{x}_m \end{aligned} \tag{2}$$

where $\boldsymbol{x} \in \mathbb{R}^n$ is plant state, $\boldsymbol{x}_m \in \mathbb{R}^n$ is model state, $\boldsymbol{u} \in \mathbb{R}^r$ is plant input, $\boldsymbol{u}_m \in \mathbb{R}^r$ is model input, $\boldsymbol{y} \in \mathbb{R}^r$ is plant output, $\boldsymbol{y}_m \in \mathbb{R}^r$ is model output, and $\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{C}\boldsymbol{A}_m, \boldsymbol{b}_m, \boldsymbol{C}_m$ are matrices of appropriate dimension.

The error vector, $\boldsymbol{e}_m$ is defined as the difference between the reference model state vector and the plant state vector,

$$\boldsymbol{e}_m = \boldsymbol{x}_m - \boldsymbol{x}. \tag{3}$$

Using Eq.(1),(2), the error dynamic equation becomes

$$\dot{\boldsymbol{e}}_m = \boldsymbol{A}_m\boldsymbol{x}_m + \boldsymbol{B}_m\boldsymbol{u}_m - (\boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}). \tag{4}$$

Now, we use model following controller in Fig. 1[5], and control action $\boldsymbol{u}$ is defined as

$$\boldsymbol{u} = \boldsymbol{K}_u\boldsymbol{u}_m + \boldsymbol{K}_m\boldsymbol{x}_m - \boldsymbol{K}_p\boldsymbol{x}. \tag{5}$$

Then, the error dynamics takes the following form:

$$\begin{aligned} \dot{\boldsymbol{e}}_m = (\boldsymbol{A}_m - \boldsymbol{B}\boldsymbol{K}_m)\boldsymbol{e}_m + (\boldsymbol{B}_m - \boldsymbol{B}\boldsymbol{K}_u)\boldsymbol{u}_m \\ + \{\boldsymbol{A}_m - \boldsymbol{A} + \boldsymbol{B}(\boldsymbol{K}_p - \boldsymbol{K}_m)\}\,\boldsymbol{x}. \end{aligned} \tag{6}$$

In order to achieve perfect model following, from Eq.(6) following equations have to be satisfied:

$$\begin{aligned} \boldsymbol{K}_m - \boldsymbol{K}_p &= \boldsymbol{B}^+(\boldsymbol{A}_m - \boldsymbol{A}), \\ \boldsymbol{K}_u &= \boldsymbol{B}^+\boldsymbol{B}_m. \end{aligned} \tag{7}$$

Therefore sufficient conditions for perfect model following are obtained as

$$\begin{aligned} (\boldsymbol{I} - \boldsymbol{B}\boldsymbol{B}^+)(\boldsymbol{A}_m - \boldsymbol{A}) &= 0, \\ (\boldsymbol{I} - \boldsymbol{B}\boldsymbol{B}^+)\boldsymbol{B}_m &= 0. \end{aligned} \tag{8}$$

Eq.(8) holds if the matrix $(\boldsymbol{I} - \boldsymbol{B}\boldsymbol{B}^+)$ is orthogonal to $(\boldsymbol{A}_m - \boldsymbol{A})$ and to $\boldsymbol{B}_m$. If reference model and the plant have a Luenberger controllable canonical form, then this property is always satisfied[5, 6] and perfect model following is achieved.

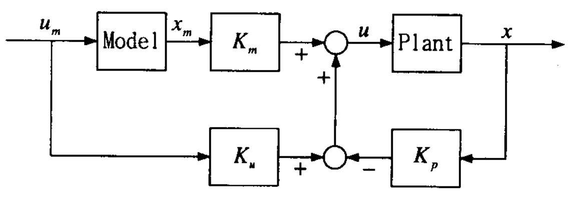


Fig. 1. Model Following Control System

A. Robust Internal Loop Compensator Design

Uncertainties and parameter variations always exist in real environments, and we cannot accurately describe these quantities. Therefore Eq.(8) can not be satisfied in most control problems. To overcome this difficulty, we focus on robust compensator design. Firstly, we design the reference model as following form;

$$\begin{aligned} \boldsymbol{A}_m &= \mathrm{diag}\{\boldsymbol{A}_{m,1}, \boldsymbol{A}_{m,2}, \cdots, \boldsymbol{A}_{m,r}\}, \\ \boldsymbol{B}_m &= \mathrm{diag}\{\boldsymbol{b}_{m,1}, \boldsymbol{b}_{m,2}, \cdots, \boldsymbol{b}_{m,r}\} \end{aligned} \tag{9}$$

where $\boldsymbol{A}_{m,i} \in \mathbb{R}^{\mu_i \times \mu_i} (i = 1, \dots, r)$ is in controllable canonical form and $\boldsymbol{b}_{m,i} \in \mathbb{R}^{\mu_i}$ is written as

$$\boldsymbol{b}_{m,i} = [\, 0 \quad 0 \, \cdots \, \underset{\mu_i \mathrm{th}}{1} \,]^T. \tag{10}$$

In order to design robust internal loop compensator(RIC), we select following gains of Eq.(5) as

$$\boldsymbol{K} = \boldsymbol{K}_m = \boldsymbol{K}_p, \qquad \boldsymbol{K}_u = \boldsymbol{I}. \tag{11}$$

From Eq.(1) and Eq.(2), Luenberger controllable canonical form is divided into rth block terms with μ_ith order differential equation and reference model is divided into canonical form as follows;

$$\dot{\boldsymbol{x}}_i = \boldsymbol{A}_i \boldsymbol{x}_i + \boldsymbol{b}_i u_i, \qquad i = 1, 2, \dots, r, \tag{12}$$

$$\dot{\boldsymbol{x}}_{m,i} = \boldsymbol{A}_{m,i} \boldsymbol{x}_{m,i} + \boldsymbol{b}_{m,i} u_{m,i}, \qquad i = 1, 2, \dots, r. \tag{13}$$

If we assume that the plant and model are stabilizable and reference model has a piecewise continuous and uniformly bounded reference input u_m and an output y_m, then the closed loop system of Eq.(1),Eq.(2), and Eq.(5) approaches to reference model $\boldsymbol{P}_m$ as $\boldsymbol{K} \to \infty$ [7].

B. Optimal RIC Design in the $\mathcal{H}_\infty$ Framework

Until previous section, we developed internal loop compensator based on full state feedback, but it is sometimes impractical to use full state of plant in real environments, and hence we need to derive compensator equation using only plant output signal. We will design RIC considering the frequency characteristics of transfer function from $\boldsymbol{u}_m$ to $\boldsymbol{y}$ using $\boldsymbol{K}$ and $\boldsymbol{P}_m$. In Fig. 2, if there exist external disturbance and measurement noise, the control input can be written as follows;

$$\begin{aligned} u_i(s) = \{1 + K_i(s) P_{m,i}(s)\}\, u_{m,i}(s) \\ - K_i(s)\{y_i(s) + \xi_i(s)\} + d_i(s) \end{aligned} \tag{14}$$

where $d_i(s)$ is a disturbance which comes through input channel and $\xi_i(s)$ is a measurement noise.

Now, let's define the transfer function Q_i which controls $P_{m,i}$ using feedback controller K_i as shown in Fig. 3(a). Q_i is described as

$$Q_i = \frac{P_{m,i} K_i}{1 + P_{m,i} K_i}. \tag{15}$$

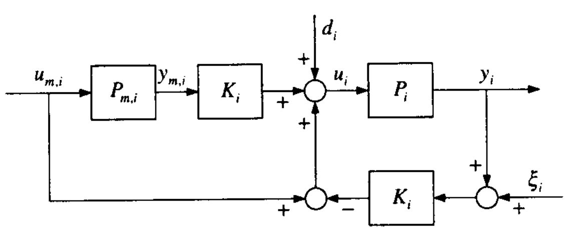
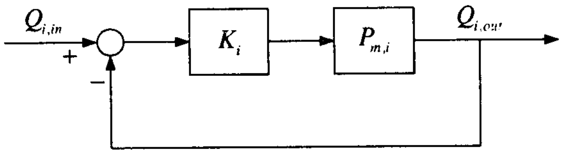
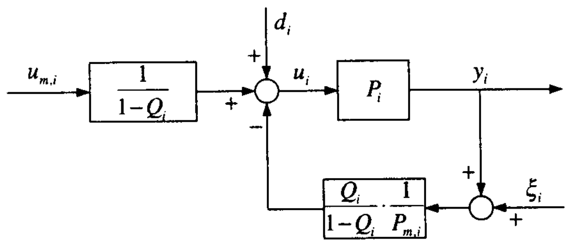


Fig. 2. RIC Structure Based on Model Following Controller



(a) Transfer Function Q



(b) Equivalent Structure of RIC Using Q

Fig. 3. Function Q and Equivalent Structure of RIC

If we recalculate this equation to K_i, and plug this K_i into Fig. 2, we obtain Fig. 3(b) which is a well known structure of disturbance observer[4]. Therefore optimal RIC design problem becomes the optimization problem of $\boldsymbol{K}$ for reference model $\boldsymbol{P}_m$ because the performance is characterized by $\boldsymbol{K}$ and $\boldsymbol{P}_m$.

Note also that the above structure of internal loop compensator can be expressed as a disturbance observer and our proposed internal compensator structure has very general characteristics although it was derived from model following concept. It is well known that the disturbance observer makes the system robust by using low pass filter $\boldsymbol{Q}$ which cuts off the external disturbances in low frequency region. Needless to say, the optimal shape of $\boldsymbol{Q}$ is obtained based on how $\boldsymbol{P}_m$ and $\boldsymbol{K}$ are designed. Also, this design of $\boldsymbol{Q}$ in conventional disturbance observer structure is very heuristic because in disturbance observer, it is very difficult to apply optimality concept in the design of $\boldsymbol{Q}$. However, in RIC structure, it is very simple.

As mentioned previously, the RIC is determined by reference model $\boldsymbol{P}_m$ and feedback controller $\boldsymbol{K}$ and also can be described by low-pass filter $\boldsymbol{Q}$ through Eq.(15) simultaneously. From Fig. 3(a), the sensitivity S_{Q_i} and complementary sensitivity T_{Q_i} of Q_i is obtained by

$$S_{Q_i} = \frac{1}{1 + P_{m,i} K_i}, \qquad T_{Q_i} = \frac{P_{m,i} K_i}{1 + P_{m,i} K_i}. \tag{16}$$

We use H_∞ mixed sensitivity method to determine optimal compensator gain $\boldsymbol{K}$ as shown in Fig. 4. This is another advantage of RIC structure in that the structure accepts the optimality concept in designing $\boldsymbol{Q}$ or $\boldsymbol{K}$. In the mixed sensitivity problem formulation, nominal disturbance attenuation specifications and stability margin specifications equations are combined into a single infinity norm specification. Now, the mixed $\mathcal{H}_\infty$ sensitivity problem is formulated as follows;

$$\min_{K_i} \left\| \begin{bmatrix} W_{1,i}(1 + P_{m,i} K_i)^{-1} \\ W_{2,i} P_{m,i} K_i (1 + P_{m,i} K_i)^{-1} \end{bmatrix} \right\|_\infty < 1. \tag{17}$$

Since Eq.(17) uses reference model parameter, this equation can be solved easily and we can assign frequency characteristics of $\boldsymbol{Q}$ with optimality.

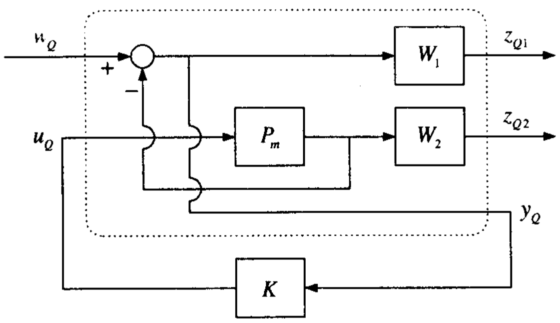


Fig. 4. Optimization of RIC by H_∞ Mixed Sensitivity Method

III. Motion Controller Design for Twin-Servo Precision Linear Motor System

A. Twin-Servo Linear Motor System

Although the linear motor is suitable for high speed, high accuracy positioning control systems, direct drive mechanism is week to the external load variation and parameter uncertainties. Preview action to suppress the effect of disturbances without increasing the input signal magnitude[8] and feedforward learning controller to reduce the effects by force ripple of a linear motor[9] were proposed to overcome these difficulties.

The system we are dealing with in this paper is the twin-servo precision linear motor system to increase the payload capacity and speed as shown in Fig. 5.

Friction identification is the first important procedure of controller synthesis[10] to achieve precision positioning performance. However the structure itself has strong nonlinear friction characteristics, and irregular gap between core and permanent magnet brings about parameter variation. Moreover force ripple caused by variation of magnetic force and inaccurate placement of the permanent magnet make control problem more difficult.

Dynamic equation of this system is written as second order differential equation

$$M\ddot{x} + u_{fric}(x,\dot{x}) + u_{ripple}(x) + d = u \qquad (18)$$

with inertia M, friction force u_{fric}, force ripple term u_{ripple}, position variable x, the control force u, and disturbance d. If this equation includes motor actuator dynamics, then u is an input voltage. Friction can be described by linear viscous friction term and nonlinear term as

$$u_{fric}(x,\dot{x}) = B\dot{x} + \bar{u}_{fric}(x,\dot{x}) \qquad (19)$$

where B is viscous damping coefficient and $\bar{u}_{fric}$ is nonlinear friction force term affected as a function of position

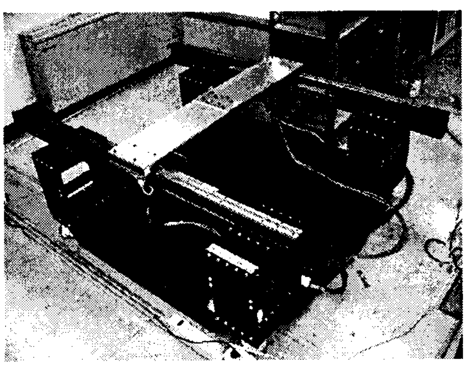


Fig. 5. Twin-Servo Precision Linear Motor System

TABLE I
Parameters of Twin-Servo System

Parameter	M	B
X_1	0.55	0.45
X_2	0.4	0.3

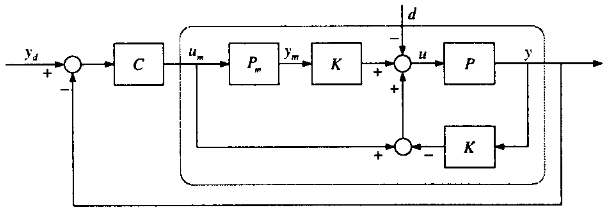


Fig. 6. Robust Motion Control Structure Based on RIC

and velocity. Table 1 shows the parameter value of inertia and damping coefficients of twin-servo motor system.

B. Precision Motion Control

RIC is used to compensate different dynamic characteristics of two motors as shown in Fig. 6. In this figure, transfer function from desired input to position output is given by

$$G = \frac{C(1+KP_m)P}{1+PK+C(1+KP_m)P} \qquad (20)$$

where C is an external loop feedback controller. When we express the plant having uncertainties as

$$P = P_m(1+\Delta_P), \qquad (21)$$

then the robust stability of the above multiplicative uncertainties can be stated in the following theorem.

Theorem 1: If the plant is modeled with multiplicative uncertainties as shown in Eq.(21), the necessary and sufficient condition of robust stability is

$$\|\Delta_P\| < \left\| \frac{1+KP_m+C(1+KP_m)P_m}{KP_m+C(1+KP_m)P_m} \right\|. \qquad (22)$$

Proof. Refer to [11]. ■

From Theorem 1, we can design external loop controller C to stabilize two linear motor systems. To satisfy performance specifications, C is designed based on reference model after the system is compensated by RIC.

IV. Experimental Results

The proposed servo algorithm is experimented by using the twin-servo precision linear motors of Fig. 5. Firstly, we design optimal RIC in the $\mathcal{H}_\infty$ framework. Reference model is selected to minimize the difference between the dynamics of two liner motors as;

$$P_m(s) = \frac{1}{0.5s^2}. \qquad (23)$$

The design goal is to find a feedback controller K of internal loop compensator that has a control loop bandwidth 100 rad/s.

A mixed sensitivity problem is formulated as follows;

$$\min_K \left\| \begin{bmatrix} W_1(1+P_mK)^{-1} \\ W_2P_mK(1+P_mK)^{-1} \end{bmatrix} \right\|_\infty < 1. \qquad (24)$$

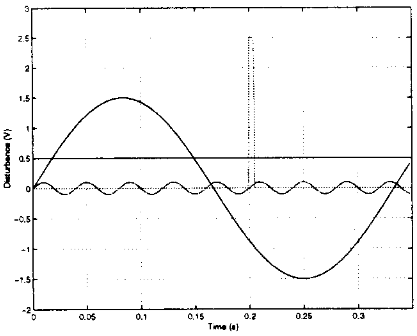
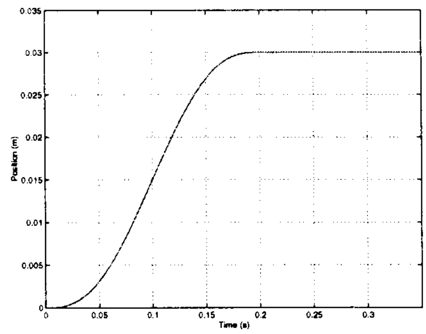
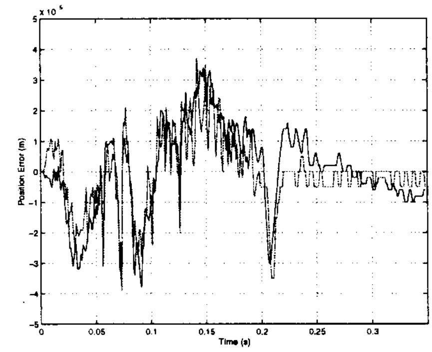


Fig. 7. External Disturbance applied to the Twin-Servo System



(a) Position



(b) Position Error

Fig. 8. Experimental Results with Disturbance

and the problem of imaginary model poles can be solved via axis shifting technique. But there are still two model zeros at infinity, which are also on the imaginary axis. This can be treated via a W_2 weighting selection:

$$W_2 = \frac{s^2}{7 \times 10^4} \tag{25}$$

where the double-differentiator makes the plant full rank at infinity. Inverse of W_1 is the desired shape of the sensitivity function. W_1 is selected as

$$W_1 = \frac{\beta(\alpha s^2 + 2\zeta_1 \omega_c \sqrt{\alpha} s + \omega_c^2)}{(\beta s^2 + 2\zeta_2 \omega_c \sqrt{\beta} s + \omega_c^2)} \tag{26}$$

where $\beta = 250$ is DC gain which controls the disturbance rejection, $\alpha = 0.5$ is high frequency gain which controls the response peak overshoot, $\omega_c = 200$ is cross-over frequency, and $\zeta_1 = \zeta_2 = 0.8$ is damping ratios of the corner frequencies.

From Eq.(23)-(26), we can obtain optimal controller K using MATLAB[12].

$$K(z) = \frac{(3.1088 \times 10^4)z^2 + (9.7835 \times 10^2)z - (3.0109 \times 10^4)}{z^2 + (1.8390 \times 10^{-1})z - (7.9021 \times 10^{-1})} \tag{27}$$

This controller was reduced to 2nd order through optimal Hankel minimum degree approximation.

For the trajectory tracking problems, conventional PID controller is used to stabilize whole system and track the desired position accurately. The fifth order polynomial function is used to specify the position, velocity, and acceleration at the beginning and end of path. The target position is 30mm. Control sampling frequency is 1000Hz and all controllers are discretized by using the bilinear transformation. Disturbance shown in Fig. 7 comes through control input channel.

Fig. 8 shows the experimental results with disturbance of Fig. 7. As can be seen here, the tracking error shows good performance within $\pm 40\mu m$ through all trajectory showing the robustness of the proposed controller.

V. Conclusions

We proposed a robust motion control scheme which consists of internal loop compensator and external loop controller. Internal loop compensator can be designed by many methods due to the generality of the controller structure and H_∞ mixed sensitivity algorithm is used to optimize compensator gain in this paper. It makes the system stable under uncertainties and nonlinearities. External loop controller can also be arbitrarily designed to meet the specification of the system using the neat result of internal loop compensator. The effectiveness of the proposed algorithm is verified through trajectory tracking control algorithm and the results show excellent performance under various nonlinear friction characteristics and disturbances using twin-servo brushless DC linear motor system.

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