Tracking Line Analysis of a Robot Manipulator for Conveyor Systems

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Abstract : The concept of *dynamic tracking line* is proposed as the feasible tracking region for a robot in a robot-conveyor system, which takes the variable conveyor speed into consideration. This paper presents an effective method to find the dynamic tracking line in a robotic workcell. The *maximum permissible line-speed* which is a quantitative measure of the robot capability for conveyor tracking, is defined on the basis of the relation between the end-effector speed and the bounds on the joint velocities, accelerations, and torques. This measure is derived in an analytic form, and some of its properties are established mathematically. The problem of finding the dynamic tracking line is then formulated as a root-solving problem for a single-variable equation, and solved by the use of a simple numerical technique.

I. Introduction

Most of industrial robot operations in manufacturing are carried out on continuously moving product lines. Conveyor tracking, or line tracking requires a robot to perform designated operations while the workpiece is being moved continuously by the conveyor. The primary advantage of conveyor-tracking applications is that the continuous operations may maximize the robot utilization and minimize the production cycle time, contributing to economic performance and system productivity [1]. Most applications of line tracking include spray painting, spot welding, material handling and inspection.

In general, the working region of a robot workcell is represented by the robot workspace. The knowledge of robot workspace is important in arranging the associated workcell and assessing the efficiency of a production line [4]. However, the tracking line, or tracking window [2],[3] has been regarded as the working region of a robot manipulator in the conveyor tracking system. It has been defined as the intersection of the robot workspace with the line of travel of the workpiece along the conveyor, which can be determined from the geometric charateristics of the workcell. The purpose of this paper is to analyze the dynamic charateristics of the tracking line for the working-region specification of the robot-conveyor system.

In this paper, we propose a new concept of dynamic tracking line as the feasible working region for the robot in the robot-conveyor system, which considers the conveyor speed as well as the constraints on the joint velocities, accelerations and torques. The dynamic tracking line can be used as an aid to optimal operating and designing the robot-conveyor system. Also, an effective method to find the dynamic tracking line is presented through the mathematical formulation. The robot capability for uniform straight-line motion is derived in an analytic function using the parameterized dynamics and kinematics of the robot. The problem of finding the dynamic tracking line is then formulated as a single-variable root-solving problem, and solved by use of a simple numerical method. By applying the method to an example system, we demonstrate that the presented method is useful to specify the working region of the robotic workcell.

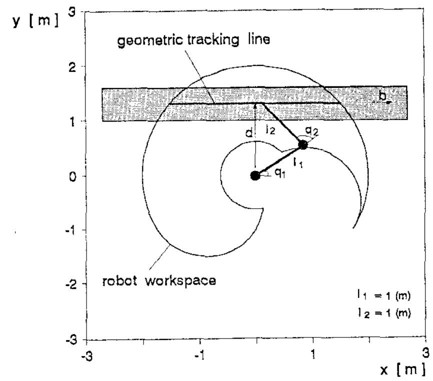

Fig.1. A robotic workcell for conveyor tracking.

II. Problem Statement

We deal with a workcell which consists of a non-redundant manipulator and a line-conveyor system. Fig.1 shows an example of the robot-conveyor system. The conveyor transfers workpieces into robot workspace, and the robot performs designated operations while the conveyor is being moved continuously. The system is assumed to have the following characteristics :

(A1) The conveyor runs at a constant speed.

(A2) The workpiece is stationary with respect to the conveyor, so the robot tracks the workpiece along a straight-line path over the conveyor.

(A3) The orientation of the end-effector is kept fixed while the robot tracks the workpiece.

The robot workspace is defined as the set of all three-dimensional points that can be reached by a reference point located on the end-effector. The workspace can be determined from joint parameters such as link lengths and joint limits. However, the working range of the robot manipulator in the conveyor tracking system is restricted within the path of the workpiece along the conveyor, which is assumed in (A2). For the working-range specification of the robot-conveyor system, the concept of geometric tracking line is defined as follows.

Definition 1 : The intersection of the robot workspace with the straight-line path of a workpiece along the conveyor, is defined as *geometric tracking line* and denoted by GTL, where there should be no singular points of the robot manipulator in the GTL.

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Fig.1 illustrates the robot workspace and the geometric tracking line of the example system. The direction of the conveyor, the distance of the conveyor from the robot base, and the joint parameters of the robot all play a part in determining the geometric tracking line. The set of the geometric tracking lines over a conveyor has been known as tracking window [2], [3]. For the robotic workcell, the total motion cycle in a particular application must be consistent with the tracking window for that application.

In the conveyor tracking system, the workpiece is not at a standstill but moves continuously while at work. Since the conveyor transfers the workpiece at a constant speed, the conveyor tracking requires the robot capability for uniform straight-line motion of the end-effector. But the magnitudes of joint velocities, accelerations and torques of the robot are limited by their maximum values. We assume that the joint velocities, accelerations and torques are bounded as

$$|\dot{q}_i| \leq \dot{q}_{i,max} \quad i = 1, \cdots, n \tag{1}$$
$$|\ddot{q}_i| \leq \ddot{q}_{i,max} \quad i = 1, \cdots, n \tag{2}$$
$$|\tau_i| \leq \tau_{i,max} \quad i = 1, \cdots, n \tag{3}$$

where $\boldsymbol{q} = [q_1, \cdots, q_n]^T \in R^{n\times 1}$ and $\boldsymbol{\tau} = [\tau_1, \cdots, \tau_n]^T \in R^{n\times 1}$ are the joint position vector and the joint torque vector of a n-joint manipulator, respectively. These constraints restrict the end-effector speed because the end-effector speed is a function of the joint velocities, accelerations and torques [6] [7] [8]. Consequently, the robot capability for uniform straight-line motion is limited by the constraints in (1), (2) and (3). This paper proposes a new concept of maximum permissible line-speed, which quantifies the motion capability of robot for conveyor tracking.

Definition 2 : Suppose that the end-effector moves along a straight-line path. At a location on the path, the maximum permissible speed of the end-effector under the constraints of the joint velocities, accelerations and torques ((1)-(3)), is said to be *maximum permissible line-speed* and denoted by $\overline{v}$.

From the kinematics of the robot manipulator, we know that the joint velocities, accelerations and torques are functions of the end-effector location. So the maximum permissible line-speed of definition 2 varies with the end-effector location along the geometric tracking line. In case that the maximum permissible line-speed is lower than the conveyor speed, the end-effector cannot be synchronized with the workpiece at the location. Hence the feasible working region of the robot-conveyor system should be determined from the additional consideration of the conveyor speed and the maximum permissible line-speed. Now we propose a new concept of dynamic tracking line to represent the feasible tracking region of the robot-conveyor system.

Definition 3 : The subregion of the geometric tracking line, in which the maximum permissible line-speed is higher than the conveyor speed v, is said to be *dynamic tracking line* and denoted by $DTL(v)$.

According to the conveyor speeds, different dynamic tracking lines can be obtained in the same geometric tracking line. The problem of finding the $DTL(v)$ is called as the *dynamic tracking-line problem* (DTLP), which is under the category of the workspace specification. This paper presents an effective method to solve the DTLP through the following mathematical formulation.

III. Problem Formulation

A. Equations of Motion of a Robot Manipulator

The overall motion of the robot for conveyor tracking is divided into two states : steady state and transient state [5]. In the steady state, the end-effector moves at a constant speed and some desired operations are achieved with the moving workpiece. Because the robot must be at a standstill at the initial and final time, the end-effector has to accelerate and decelerate for the start/stop transition. However, for a typical job, the time spent in this transient state is considerably small as compared with the time spent in the steady state. Therefore, we consider only the case of uniform straight-line motion of the end-effector.

Let $\boldsymbol{b} \in R^{3\times 1}$ be a directional unit vector of the straight-line path, and $\boldsymbol{d} \in \mathrm{R}^{3\times 1}$ be a distance vector from the robot base to the straight-line path. Then these vectors specify the geometric tracking line in the three dimensional Cartesian space, which is illustrated in Fig.1. Let $\boldsymbol{p} \in \mathrm{R}^{3\times 1}$ and $\boldsymbol{\omega} \in \mathrm{R}^{n-3\times 1}$ be the position and the orientation vector of the end-effector in the Cartesian space, then we have

$$\begin{bmatrix} \boldsymbol{p} \\ \boldsymbol{\omega} \end{bmatrix} = \begin{bmatrix} \boldsymbol{d} \\ \boldsymbol{\omega}_o \end{bmatrix} + \lambda \begin{bmatrix} \boldsymbol{b} \\ \mathbf{o} \end{bmatrix} \tag{4}$$
$$\begin{bmatrix} \dot{\boldsymbol{p}} \\ \dot{\boldsymbol{\omega}} \end{bmatrix} = v \begin{bmatrix} \boldsymbol{b} \\ \mathbf{o} \end{bmatrix} \tag{5}$$

where $\boldsymbol{\omega}_o$ is an initial orientation of the end-effector, and $\mathbf{o}$ is an $n-3\times 1$ null vector. The parameter $\lambda \in R$ specifies the location of the end-effector along the straight-line path, and v $(=\dot{\lambda})$ denotes the end-effector speed.

For an n-joint non-redundant manipulator, there exists a Jacobian matrix $J \in R^{n\times n}$ [12] such that

$$J\dot{\boldsymbol{q}} = \begin{bmatrix} \dot{\boldsymbol{p}} \\ \dot{\boldsymbol{\omega}} \end{bmatrix} \tag{6}$$

Substitution (5) into (6) results in

$$J\dot{\boldsymbol{q}} = v \begin{bmatrix} \boldsymbol{b} \\ \mathbf{o} \end{bmatrix} \tag{7}$$

Differentiating (7) with respect to time yields

$$J\ddot{\boldsymbol{q}} + B\dot{\boldsymbol{q}} = \mathbf{o} \tag{8}$$

Here $B \in R^{n\times n}$ is a matrix whose elements B_{ij} are given by

$$B_{ij} = (\nabla J_{ij})^T \dot{\boldsymbol{q}}$$

where $\nabla = (\frac{\partial}{\partial q_1}\ \frac{\partial}{\partial q_2}\ \cdots\ \frac{\partial}{\partial q_n})^T$ is an operation vector and J_{ij} is the ijth element of J. Solving (7) and (8) for $\dot{\boldsymbol{q}}$ and $\ddot{\boldsymbol{q}}$, respectively, yields

$$\dot{\boldsymbol{q}} = vJ^{-1} \begin{bmatrix} \boldsymbol{b} \\ \mathbf{o} \end{bmatrix}, \quad \ddot{\boldsymbol{q}} = -v^2 J^{-1} \tilde{B} J^{-1} \begin{bmatrix} \boldsymbol{b} \\ \mathbf{o} \end{bmatrix} \tag{9}$$

where the elements of $\tilde{B}$ are given by

$$\tilde{B}_{ij} = (\nabla J_{ij})^T J^{-1} \begin{bmatrix} \boldsymbol{b} \\ \mathbf{o} \end{bmatrix}$$

As a result, the joint velocity $\dot{\boldsymbol{q}}$ is directly proportional to the end-effector speed, and the joint acceleration $\ddot{\boldsymbol{q}}$ is directly proportional to square of the end-effector speed.

Next we represent the equations of joint motion as the parametric form. From the kinematic relations of the manipulator, the joint position $(\boldsymbol{q})$ can be determined from the end-effector location ($\boldsymbol{p}$ and $\boldsymbol{\omega}$). Since the $\boldsymbol{p}$ and $\boldsymbol{\omega}$ are specified by the parameter λ as (4), the $\boldsymbol{q}$ can be represented as the parametric form such that

$$\boldsymbol{q} = \boldsymbol{f}(\lambda) \tag{10}$$

where $\boldsymbol{f}(\cdot) : \mathrm{R} \rightarrow \mathrm{R}^{n\times 1}$ is a function which depends on the configuration of the manipulator. Differentiating (10) twice with respect to time yields the following equations.

$$\dot{\boldsymbol{q}} = v\boldsymbol{f}_{\lambda}(\lambda) \quad , \quad \ddot{\boldsymbol{q}} = \mathrm{v}^2\boldsymbol{f}_{\lambda\lambda}(\lambda) \tag{11}$$

where $\boldsymbol{f}_{\lambda} \triangleq \frac{\partial \boldsymbol{f}}{\partial \lambda}$ and $\boldsymbol{f}_{\lambda\lambda} \triangleq \frac{\partial^2 \boldsymbol{f}}{\partial \lambda^2}$. From (9), we now obtain the relations as

$$\boldsymbol{f}_{\lambda}(\lambda) = \mathrm{J}^{-1}\begin{bmatrix} \boldsymbol{b} \\ \mathbf{o} \end{bmatrix} \quad , \quad \boldsymbol{f}_{\lambda\lambda}(\lambda) = -\mathrm{J}^{-1}\tilde{\mathrm{B}}\mathrm{J}^{-1}\begin{bmatrix} \boldsymbol{b} \\ \mathbf{o} \end{bmatrix} \tag{12}$$

Finally, we describe the dynamical behavior of a robot manipulator. The dynamic equation of motion can be compactly written as [7]

$$\boldsymbol{\tau} = \mathrm{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^{\mathrm{T}}\mathrm{C}(\boldsymbol{q})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) \tag{13}$$

where $M(\boldsymbol{q}) : \mathrm{R}^{n\times 1} \rightarrow \mathrm{R}^{n\times n}$ is the inertia matrix, $C(\boldsymbol{q}) : \mathrm{R}^{n\times 1} \rightarrow \mathrm{R}^{n\times n\times n}$ is the tensor in the formulation of the Coriolis and centripetal torques, and $\boldsymbol{g}(\boldsymbol{q}) : \mathrm{R}^{n\times 1} \rightarrow \mathrm{R}^{n\times 1}$ is the vector of gravity torques. With substitution from (10) and (11), (13) now becomes

$$\boldsymbol{\tau} = \mathrm{v}^2\boldsymbol{h}(\lambda) + \boldsymbol{g}(\lambda) \tag{14}$$

where

$$\begin{aligned} \boldsymbol{h}(\lambda) &= M(\boldsymbol{f}(\lambda))\boldsymbol{f}_{\lambda\lambda}(\lambda) + \boldsymbol{f}_{\lambda}(\lambda)^{\mathrm{T}}\mathrm{C}(\boldsymbol{f}(\lambda))\boldsymbol{f}_{\lambda}(\lambda) && (15) \\ \boldsymbol{g}(\lambda) &= \boldsymbol{g}(\boldsymbol{f}(\lambda)) && (16) \end{aligned}$$

which is the parametric form of the robot dynamics.

B. Formulation of the Maximum Permissible Line-Speed

Some researches on the maximum permissible speed of the end-effector have been announced so far. Hollerbach [7] and Wu [8] proposed specific methods to find the maximum permissible speed in case of the translational contouring. But algorithmic formulations of these methods make it difficult to examine the mathematical properties of the maximum permissible speed. Also constraints of the joint velocities and accelerations were not considered in [7], and constraints of the joint torques were not considered in [8]. So these methods cannot be directly applied to the dynamic tracking-line problem of this paper. In this paper, we obtain the maximum permissible speed of the end-effector by analytic formulation. For the dynamic tracking-line problem, we consider only the case of straight-line motion, but take all constraints of joint velocities, accelerations and torques into account. The maximum permissible line-speed of definition 2 is derived in an analytic function through the following formulation.

Lemma 1 : Suppose that $\boldsymbol{t} = [\mathrm{t}_1, \cdots, \mathrm{t}_n]^{\mathrm{T}}$, $\boldsymbol{k} = [\mathrm{k}_1, \cdots, \mathrm{k}_n]^{\mathrm{T}}$ and a positive scalar α are related as

$$\boldsymbol{t} = \alpha\boldsymbol{k} \tag{17}$$

and each element of the vector $\boldsymbol{t}$ is bounded by

$$|t_i| \leq 1 \quad i = 1, \cdots, n \tag{18}$$

where both $\boldsymbol{t}$ and $\boldsymbol{k}$ are not null-vectors. Let $\overline{\alpha}$ denote the maximum permissible value of α, then

$$\overline{\alpha} = \frac{1}{\|\boldsymbol{k}\|_{\infty}} \tag{19}$$

where $\|\boldsymbol{k}\|_{\infty} \triangleq \max_i |\mathrm{k}_i|$.

Proof : By (17), we have

$$|t_i| = |\alpha k_i| = \alpha|k_i| \quad i = 1, \cdots, n$$

Then the bounds on t_i in (18) become the n simultaneous inequalities of α such that

$$\alpha|k_i| \leq 1 \quad i = 1, \cdots, n$$

Hence $\overline{\alpha}$ (the maximum permissible value of α) must satisfy the following equation as

$$\overline{\alpha} \max_i |k_i| = 1$$

, which completes the proof. ■

Theorem 1 : Suppose that the end-effector moves along a geometric tracking line at uniform speed. Let $\overline{v}_v$ be the maximum permissible speed of the end-effector at a location under the joint velocity constraints of (1). Also, let $\overline{v}_a$ be the maximum permissible speed of the end-effector at a location under the joint acceleration constraints of (2). Then $\overline{v}_v$ and $\overline{v}_a$ are functions of λ as

$$\begin{aligned} \overline{v}_v(\lambda) &= \frac{1}{\|V^{-1}\boldsymbol{f}_{\lambda}(\lambda)\|_{\infty}} \triangleq \frac{1}{\varepsilon_v(\lambda)} && (20) \\ \overline{v}_a(\lambda) &= \frac{1}{\|A^{-1}\boldsymbol{f}_{\lambda\lambda}(\lambda)\|_{\infty}^{\frac{1}{2}}} \triangleq \frac{1}{\varepsilon_a(\lambda)} && (21) \end{aligned}$$

where $V \in R^{n\times n}$ and $A \in R^{n\times n}$ are diagonal matrices as

$$\begin{aligned} V &\triangleq \mathrm{diag}\{\dot{q}_{1,max}, \cdots, \dot{q}_{n,max}\} \\ A &\triangleq \mathrm{diag}\{\ddot{q}_{1,max}, \cdots, \ddot{q}_{n,max}\} \end{aligned}$$

Proof : First, we prove the $\overline{v}_v$. From the parameterized kinematic relation of (11), $\overline{v}_v$ is the maximum permissible value of v under the joint velocity constraints of (1). Define the vector $\dot{\tilde{\boldsymbol{q}}}$ as

$$\dot{\tilde{\boldsymbol{q}}} \triangleq V^{-1}\dot{\boldsymbol{q}} \tag{22}$$

then the limits on $\dot{q}_i$ are translated into those on the elements of $\dot{\tilde{\boldsymbol{q}}}$ such that

$$|\dot{\tilde{q}}_i| \leq 1 \ \ \forall_i \tag{23}$$

where $\dot{\tilde{q}}_i$ is the ith element of the vector $\dot{\tilde{\boldsymbol{q}}}$. By substituting $\dot{\boldsymbol{q}}$ from (11), (22) new becomes

$$\dot{\tilde{\boldsymbol{q}}} = vV^{-1}\boldsymbol{f}_{\lambda}(\lambda) \tag{24}$$

where v is a positive scalar. Applying lemma 1 to (24) and (23), results in the maximum permissible value of v as (20), which completes the proof of $\overline{v}_v$. The proof of $\overline{v}_a$ is similar to that of $\overline{v}_v$. ■

Theorem 2 : Suppose that the end-effector moves along a geometric tracking line at uniform speed. Let $\overline{v}_{\tau}$ be the maximum permissible speed of the end-effector at a location under the joint torque constraints of (3). Then $\overline{v}_{\tau}$ is a function of λ as

$$\overline{v}_{\tau}(\lambda) = \frac{1}{\|T^{-1}(\lambda)\boldsymbol{h}(\lambda)\|_{\infty}^{\frac{1}{2}}} \triangleq \frac{1}{\varepsilon_{\tau}(\lambda)} \tag{25}$$

$T(\lambda)$ is an $n \times n$ diagonal matrix as

$$T(\lambda) \triangleq \mathrm{diag}\{\tilde{\tau}_{1,max}(\lambda), \cdots, \tilde{\tau}_{n,max}(\lambda)\}$$

where

$$\tilde{\tau}_{i,max}(\lambda) = \begin{cases} |\tau_{i,max} - g_i(\lambda)| & , \quad h_i(\lambda) > 0 \\ |\tau_{i,max} + g_i(\lambda)| & , \quad h_i(\lambda) < 0 \end{cases}$$

, and $g_i(\lambda)$ and $h_i(\lambda)$ are the ith elements of $\boldsymbol{g}(\lambda)$ and $\boldsymbol{h}(\lambda)$, respectively.

Proof : From the parameterized dynamic equation of (14), $\bar{v}_\tau$ is the maximum permissible value of v under the constraints of (3). Define $\hat{\boldsymbol{\tau}}$ as

$$\hat{\boldsymbol{\tau}} \triangleq \boldsymbol{\tau} - \boldsymbol{g}$$

then $\hat{\tau}_i$ which is the ith element of $\hat{\boldsymbol{\tau}}$, is bounded from the bounds on τ_i of such that

$$-\tau_{i,max} - g_i \le \hat{\tau}_i \le \tau_{i,max} - g_i \quad \forall_i \tag{26}$$

Also, the dynamic equation of (14) can be rewritten as

$$\hat{\boldsymbol{\tau}} = v^2 \boldsymbol{h}(\lambda) \tag{27}$$

From this equation, we know the fact that $\hat{\tau}_i$ is bounded by the upper limit of (26) in case that $h_i > 0$, and bounded by the lower limit in case that $h_i < 0$. Hence (26) can be reformulated as

$$\begin{cases} |\hat{\tau}_i| \le |\tau_{i,max} - g_i| & , \quad h_i > 0 \\ |\hat{\tau}_i| \le |\tau_{i,max} + g_i| & , \quad h_i < 0 \end{cases}$$

Multiplying (27) by the matrix $T(\lambda)$ yields the normalized joint torque $\tilde{\boldsymbol{\tau}}$ as

$$\tilde{\boldsymbol{\tau}} = v^2 T^{-1}(\lambda) \boldsymbol{h}(\lambda) \tag{28}$$

$$|\tilde{\tau}_i| \le 1 \quad \forall_i \tag{29}$$

According to lemma 1, the maximum permissible value of v subject to (28) and (29) is obtained as (25), which completes the proof. ■

Now we turn to main problem about the maximum permissible line-speed of definition 2, which takes the all constraints of joint velocities, accelerations and torques. For the simple formulation, we define $\boldsymbol{e}(\lambda) : \mathrm{R} \to \mathrm{R}^{3n \times 1}$ as

$$\boldsymbol{e}(\lambda) \triangleq \begin{bmatrix} |V^{-1} \boldsymbol{f}_\lambda(\lambda)| \\ \sqrt{A^{-1} \boldsymbol{f}_{\lambda\lambda}(\lambda)} \\ \sqrt{T^{-1}(\lambda) \boldsymbol{h}(\lambda)} \end{bmatrix} \tag{30}$$

where $|\boldsymbol{a}| = [|a_1|, \cdots, |a_n|]^T$ and $\sqrt{\boldsymbol{a}} = [\sqrt{|a_1|}, \cdots, \sqrt{|a_n|}]^T$ for a vector $\boldsymbol{a} = [a_1, \cdots, a_n]^T$.

Corollary 1 : Let $\varepsilon(\lambda) : R \to R$ be a scalar function such that

$$\varepsilon(\lambda) \triangleq \|\boldsymbol{e}(\lambda)\|_\infty \tag{31}$$

then the maximum permissible line-speed of the end-effector is reciprocal of $\varepsilon(\lambda)$, that is

$$\bar{v}(\lambda) = \frac{1}{\varepsilon(\lambda)} \tag{32}$$

Proof : From theorem 1 and theorem 2, $\bar{v}_v$, $\bar{v}_a$ and $\bar{v}_\tau$ are obtained by considering only the constraints of joint velocities, accelerations and torques, respectively. However, the maximum permissible line-speed $\bar{v}$ of definition 2 should be determined from the consideration of all of these constraints. It is clear that $\bar{v}$ is the minimum value of $\bar{v}_v$, $\bar{v}_a$ and $\bar{v}_\tau$ at each end-effector location. At each λ, we have

$$\begin{aligned} \bar{v} &= \min\{\bar{v}_v \, , \, \bar{v}_a \, , \, \bar{v}_\tau\} = \min\{\frac{1}{\varepsilon_v} \, , \, \frac{1}{\varepsilon_a} \, , \, \frac{1}{\varepsilon_\tau}\} \\ &= \frac{1}{\max\{\varepsilon_v \, , \, \varepsilon_a \, , \, \varepsilon_\tau\}} \qquad (\varepsilon_v, \varepsilon_a, \varepsilon_\tau \ge 0) \end{aligned}$$

According to theorem 1 and theorem 2,

$$\begin{aligned} \frac{1}{\bar{v}} &= \max\{\varepsilon_v \, , \, \varepsilon_a \, , \, \varepsilon_\tau\} \qquad (33) \\ &= \max\{\|V^{-1} \boldsymbol{f}_\lambda\|_\infty \, , \, \|\sqrt{\mathrm{A}^{-1} \boldsymbol{f}_{\lambda\lambda}}\|_\infty \, , \, \|\sqrt{\mathrm{T}^{-1} \boldsymbol{h}}\|_\infty\} \\ &= \| \begin{bmatrix} |V^{-1} \boldsymbol{f}_\lambda| \\ \sqrt{A^{-1} \boldsymbol{f}_{\lambda\lambda}} \\ \sqrt{T^{-1} \boldsymbol{h}} \end{bmatrix} \|_\infty = \varepsilon \end{aligned}$$

, which completes the proof. ■

Now the robot capability for conveyor tracking is quantified as an analytic function of (32). For a given geometric tracking line specified by a directional vector $\boldsymbol{b}$ and a distance vector $\boldsymbol{d}$, the $\boldsymbol{f}_\lambda(\lambda)$, $\boldsymbol{f}_{\lambda\lambda}(\lambda)$ and $\boldsymbol{h}(\lambda)$ can be computed from the kinematic and dynamic equations of robot. Also, the matrices of V, A and $T(\lambda)$ can be obtained directly from the boundary values of the joint velocities, accelerations and torques. Then the maximum permissible line-speed $\bar{v}(\lambda)$ can be computed from corollary 1. Due to the analytic formulation of $\bar{v}$, its properties can be established mathematically.

C. Formulation of Dynamic Tracking-Line Problem

As discussed earlier, a straight-line path is specified by a directional vector $\boldsymbol{b}$ and a distance vector $\boldsymbol{d}$. Also the location on the straight-line path can be described by the parameter λ. Hence, for a straight-line path with the vectors of $\boldsymbol{b}$ and $\boldsymbol{d}$, geometric tracking line of definition 1 is described by the set of parameter λ as

$$GTL = \{\lambda | \lambda \in (\lambda_o \, , \, \lambda_f)\} \subset R \tag{34}$$

where λ_o and λ_f are the values of the parameter λ at the intersection points of the robot workspace with the straight-line path. Because of the workspace boundary singularities [12], λ_o and λ_f are excluded from the geometric tracking line.

The dynamic tracking line of definition 3 is the feasible range of the straight-line path, in which the maximum permissible line-speed $\bar{v}(\lambda)$ is higher than the conveyor speed v. From corollary 1, the dynamic tracking line can be described as the set of λ which satisfy the following inequality.

$$\varepsilon(\lambda) \le \frac{1}{v} \tag{35}$$

where $\lambda \in (\lambda_o, \lambda_f)$. For the simple and effective solution of the inequality, it is necessary to obtain the rough plot of $\varepsilon(\lambda)$. Hence we investigate the analytic properties of $\varepsilon(\lambda)$.

Corollary 2 : The function $\varepsilon(\lambda)$, defined for $\lambda \in (\lambda_o, \lambda_f)$, has mathematical characteristics as follows :

(i) $\varepsilon(\lambda) \ge 0$

(ii) $\lim_{\lambda \to \lambda_o} \varepsilon(\lambda) = \lim_{\lambda \to \lambda_f} \varepsilon(\lambda) = \infty$

Proof : (i) From corollary 1, $\varepsilon(\lambda)$ is defined as an infinite vector norm. By the property of the vector norm [9], it is clear that $\varepsilon(\lambda) \ge 0$ for all λ.

(ii) Let $\boldsymbol{q}_\mathrm{o}$ and $\boldsymbol{q}_\mathrm{f}$ be the joint positions at the boundary points of the geometric tracking line, that is, $\boldsymbol{q}_\mathrm{o} = \boldsymbol{f}(\lambda_\mathrm{o})$ and $\boldsymbol{q}_\mathrm{f} = \boldsymbol{f}(\lambda_\mathrm{f})$, then both $\boldsymbol{q}_\mathrm{o}$ and $\boldsymbol{q}_\mathrm{f}$ are workspace-boundary singular points. The determinant of the Jacobian matrix at the singular point is known to be zero [12], that is $|J(\boldsymbol{q}_\mathrm{o})| = |\mathrm{J}(\boldsymbol{q}_\mathrm{f})| = 0$.

By (12) and (20), we have

$$\lim_{\lambda\to\lambda_o} \varepsilon_v(\lambda) = \lim_{\boldsymbol{q}\to\boldsymbol{q}_o} \frac{1}{|J(\boldsymbol{q})|}\|V^{-1}\Delta(\boldsymbol{q})\begin{bmatrix} \boldsymbol{b} \\ \mathbf{o} \end{bmatrix}\|_\infty = \infty$$

where $\Delta(\boldsymbol{q})$ is the adjoint matrix of J, which is same to the case of $\lambda \to \lambda_o$. From corollary 1, $\varepsilon(\lambda) = \max\{\varepsilon_v(\lambda), \varepsilon_a(\lambda), \varepsilon_\tau(\lambda)\}$, so $\varepsilon(\lambda) \to \infty$ as $\varepsilon_v(\lambda) \to \infty$, which completes the proof. ∎

In this paper, we assume that the $\varepsilon(\lambda)$ is a continuous over (λ_o, λ_f). The norm of a vector is continuous if all of the vector elements are continuous [9]. Hence the $\varepsilon(\lambda)$ is continuous if the elements of the vector $\boldsymbol{f}_\lambda(\lambda)$, $\boldsymbol{f}_{\lambda\lambda}(\lambda)$, $\boldsymbol{h}(\lambda)$ and $\boldsymbol{g}(\lambda)$ are continuous function. Roughly speaking, these vector functions are continuous in case that the kinematic and the dynamic equations of the robot consist of the analytic functions. It is true for many cases of industrial robot manipulators.

By the property (ii) of corollary 2, the $\varepsilon(\lambda)$ decreases, (increases, $\cdots$, decreases) and increases as the λ increases from λ_o to λ_f. Also, by the property (i) of corollary 2, the plot of $\varepsilon(\lambda)$ exists on the upper side of the λ-ε plane. From the rough plot of $\varepsilon(\lambda)$, we know the fact that the number of real roots of the equation $\varepsilon(\lambda) = \frac{1}{v}$ $(v > 0)$ is always even if the real roots exit, where the number of double root and critical root is considered as two. From these results, the inequality of (35) can be solved easily from the roots of the equation.

Now the problem for finding the dynamic tracking line is formulated as follows.

$$\begin{array}{lrl} \text{DTLP}: & \text{given} & v, \boldsymbol{b}, \boldsymbol{d}, \dot{\boldsymbol{q}}_{\max}, \ddot{\boldsymbol{q}}_{\max}, \boldsymbol{\tau}_{\max} \\ & \text{find} & DTL(v) = [\lambda_1, \lambda_2] \cup \cdots \cup [\lambda_{2k-1}, \lambda_{2k}] \\ & \text{such that} & \varepsilon(\lambda_i) = \dfrac{1}{v} \quad (i = 1, 2, \cdots, 2k) \\ & \text{subject to} & \lambda_o < \lambda_i < \lambda_f \end{array} \tag{36}$$

The DTLP is essentially a root-solving problem for a single-variable equation. Many of numerical algorithms to find the roots of a equation can be applied to the DTLP. Also the algorithms are readily available from the commercial libraries. To find the values of λ from the equation (36), we utilize a Gauss-Newton routine [11] from the MATLAB library [10].

IV. Examples

The proposed method is applied to an example system as shown in Fig.1., which consists of a manipulator and a conveyor. The manipulator has two rotary joints and the axes of rotation are both directed along the z axis, thus the manipulator only generates movement on the $x - y$ plane. The masses are assumed to be concentrated at the middle of each link. However the method can be readily applied to non-redundant manipulators with more degrees of freedom. The link lengths denoted by l_1 and l_2, and the link masses denoted by m_1 and m_2 are given in Table I. The limit values of the joint velocities, accelerations and torques are also given in Table I. The robot workspace of the example system is determined from the joint angle ranges as $q_{1,min} \le q_1 \le q_{1,max}$ and $q_{2,min} \le q_2 \le q_{2,max}$, where the boundary values are given in Table I. The drawing of the robot workspace is shown in Fig. 1

The conveyor always moves in x direction, that is the directional vector $\boldsymbol{b}$ is set at $[1\ 0]^T$. The distance vector $\boldsymbol{d}$ is parallel to y axis, so it determines the distance of the straight-line path from the robot base. To investigate the characteristics of straight-line path over the robot workspace, we set the vector $\boldsymbol{d}$ as $[0\ d]^T$,

TABLE I. Parameters of the example system.

l_1 (m)	1	l_2 (m)	1
m_1 (Kg)	1	m_2 (Kg)	1
$q_{1,min}$ (rad)	$-\frac{\pi}{6}$	$q_{2,min}$ (rad)	0
$q_{1,max}$ (rad)	$\frac{7\pi}{6}$	$q_{2,max}$ (rad)	$\frac{4\pi}{5}$
$\dot{q}_{1,max}$ (rad/s)	0.8	$\dot{q}_{2,max}$ (rad/s)	1.0
$\ddot{q}_{1,max}$ (rad/s^2)	1.8	$\ddot{q}_{2,max}$ (rad/s^2)	1.6
$\tau_{1,max}$ (Nm)	0.5	$\tau_{2,max}$ (Nm)	0.2
$\boldsymbol{b}$	$[1\ 0]^T$	$\boldsymbol{d}$	$[0\ d]^T$

where d means the distance. The allowable values of d are then bounded by the robot workspace as

$$\sqrt{l_1^2 + l_2^2 - 2l_1l_2\cos(\pi - q_{2,max})} \le d \le l_1 + l_2 \tag{37}$$

For the straight-line path with d, we can obtain the boundary points of the geometric tracking line as

$$\lambda_o = -\sqrt{(l_1 + l_2)^2 - d^2}\ ,\ \ \lambda_f = \sqrt{(l_1 + l_2)^2 - d^2} \tag{38}$$

Then the geometric tracking line is represented by the set of λ between the two boundary points.

Fig.2 shows that the maximum permissible line-speed varies with the location along the straight-line path. In particular, it decreases suddenly and approaches to zero around both end-locations of the path. Fig.2 also illustrates the comparative concepts of the geometric tracking line and the dynamic tracking line. From (38), the geometric tracking line is obtained as $GTL = \{\lambda | -1.52 < \lambda < 1.52\}$. In the plot of $\overline{v}(\lambda)$, the range of λ between two cross points of λ axis specifies the geometric tracking line. The dynamic tracking line is the range of the straight-line path in which the maximum permissible line-speed is higher than the conveyor speed. So, in the plot of $\overline{v}(\lambda)$, $DTL(v)$ is specified as the range of λ in which $\overline{v}(\lambda) > v$. In case that $v = 0.4$ (m/s), the conveyor tracking can be achieved from λ=-1.35 to λ=1.33, that is $DTL(0.4) = \{\lambda | -1.35 \le \lambda \le 1.33\}$. However the dynamic tracking lines are variable according to the conveyor speed, for example, $DTL(0.7) = \{\lambda | -1.18 \le \lambda \le 1.02\}$.

For all values of d throughout the robot workspace, the dynamic tracking lines can be obtained. Extrapolating of their boundary points makes the feasible tracking region of the robot-conveyor system. Fig.3 shows the feasible tracking region for the cases of (a) $v = 0.5$ and (b) $v = 0.7$(m/s). The shadow regions in the robot workspace illustrate the feasible tracking regions against the conveyor speeds. Notify that the straight-line paths are always placed parallel to the x axis. As the conveyor speed increases, the area of the feasible tracking region decreases abruptly. It verifies the fact that the feasible trackinf region of the robot in the robot-conveyor system varies with the conveyor speed.

Finally, Fig.4 shows the contour map of maximum permissible line-speed for all locations of the robot workspace. The abscissa axis of the graph indicates the distance of the straight-line path from the robot base, and the ordinate axis indicates the locations of the end-effector along the same straight-line path. The labels on the contour lines indicate corresponding $\overline{v}$ in meters per second. The maximum value of $\overline{v}$ along a given path is obtained from the contour map, and the value can be regarded as the maximum permissible value of conveyor speed for the path of workpiece. For the case of d=1.3 (m), the maximum value of $\overline{v}$ is about 1 (m/s). That is, if the conveyor speed is higher than

1 (m/s), there is no feasible tracking region in the path. So, the contour map can be used as an aid to determining the conveyor speed, and it can help arrange the appropriate conveyor location with respect to the robot base.

V. Conclusion

In this paper, we have proposed a methodology for workspace specification of the conveyor tracking system. A new concept of dynamic tracking line has been presented as the feasible working region of the robot-conveyor system. Differently from the geometric tracking line, it has been determined from the dynamic charcteristics of the robot-conveyor system. The maximum permissible line-speed has been defined as a measure of robot capability for uniform straight-line motion, and has been derived in an analytic function of the end-effector location. By applying the analytic function to the workspace specification, the problem of finding the dynamic tracking line has been formulated as a simple root-solving problem. As illustrated by the examples in the paper, the method presented can be used as an aid to optimal operating and designing the robot workcell for conveyor tracking.

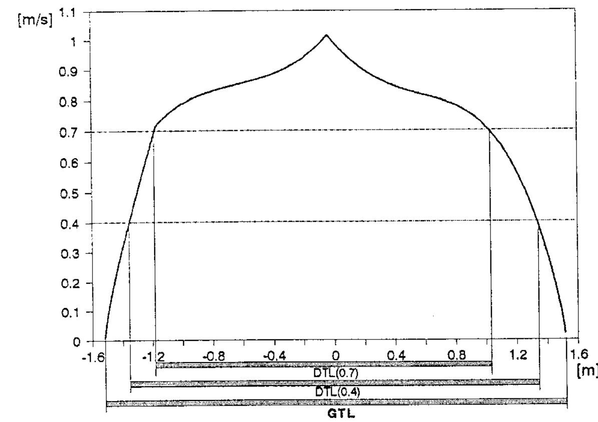
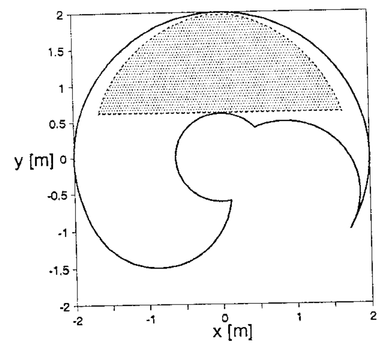
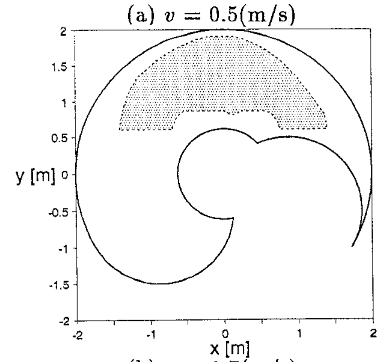


Fig.2. $\lambda - \overline{v}(\lambda)$ plot ($d = 1.3m$).



(a) $v = 0.5$(m/s)



(b) $v = 0.7$(m/s)

Fig.3. Feasible tracking region

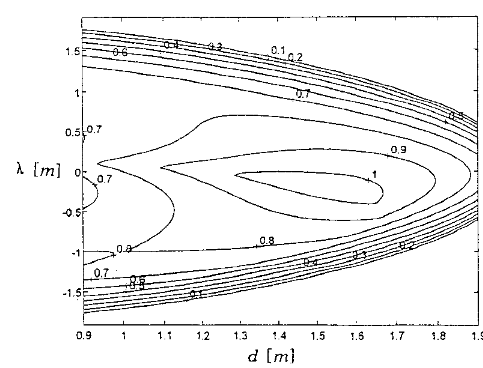


Fig.4. Contour map of $\overline{v}$.

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