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Analysis and Design of Digital Dual-Repetitive Controllers

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Abstract-A design method of dual-repetitive controllers is newly proposed to reduce dual-repetitive errors which consist of dual dominant fundamental frequency components and their harmonics in the frequency domain. The proposed control system can save memory size and reduce errors faster than the conventional single repetitive control system with dual-repetitive errors. Sufficient conditions for the stability and error convergence of the proposed control systems are given. To show the validity of the proposed systems, computer simulation results are illustrated for a typical disk drive head positioning control system.

I. INTRODUCTION

In order to effectively reduce repetitive and/or harmonic errors with a known period in various control systems performing repetitive tasks, many repetitive control systems, based on the internal model principle, have been developed [3, 4, 11]. Repetitive control systems have been originally studied in the continuous domain by Inoue *et al.* [7] and Hara *et al.* [5]. Digital repetitive controllers were also proposed [3, 4, 10]. Several modified repetitive controllers were proposed to obtain robustness under modeling uncertainties [3-5]. However, when such controllers are employed to reduce harmonic errors in real systems, non-harmonic error components are often amplified. Gain adjusting algorithm or optimized repetitive function suggested by Chang *et al.* [2] could reduce the amplification of non-harmonic errors, but could not actively reduce non-harmonic errors. In all such cases, repetitive error components consisting of only a single fundamental frequency and its harmonics were considered to be reduced by employing repetitive controllers. Such type of errors will be hereinafter called "*single repetitive errors* (*SRE*)". However, in many practical situations, repetitive errors to be reduced actively may consist of multiple dominant frequency components and their harmonics in the frequency domain. This type of errors will be hereinafter called "*multi-repetitive errors* (*MRE*)". As an example of the control system with *MRE*, head positioning control system can be considered [1, 3, 4]. It is well known that error components in the system are primarily due to not only repeatable run-out which corresponds to one dominant fundamental frequency and its harmonics, but also non-repeatable run-out which has also periodic components whose fundamental frequencies are different from that of the dominant frequency of the repeatable run-out [8]. In the case of head positioning servo control for hard disk drives, a conventional repetitive controller has been usually employed to reduce only the *SRE* due to repeatable run-out. We will call this type of repetitive controller "*single repetitive controller* (*SRC*)". Unfortunately, non-repeatable errors cannot be actively reduced by using the *SRC* only for the rejection of repeatable run-out, though the amplification of non-repeatable errors can be reduced by gain adjusting algorithm or optimized repetitive function [2]. The only way to reduce the *MRE* by employing the *SRC* is to choose the dead-time element in such a way that the dead time length should be equal to the least common multiplier of all dead-time lengths of fundamental components of the *MRE* in discrete time case. Therefore, it requires relatively large memory size. Also, fast error convergence speed cannot be obtained since the least common multiplier of several dead-time lengths are much longer than dead-time lengths of any fundamental components of the *MRE*. As another example of the physical system with multi-repetitive mechanical disturbances, a gear system with a driving gear and a joined gear can be considered, where multi-repetitive mechanical disturbances appear whenever angular velocity of a driving gear, ω_1, is different from that of the joined gear, ω_2 [9].

In this paper, multi-repetitive controllers (*MRC*) are newly proposed to effectively and simply reduce such *MRE*. In the proposed system, *MRC* are simply plugged in parallel in a feedback control system, which is similar to the conventional repetitive control system where a repetitive controller is plugged in a feedback control system [3, 4]. Each repetitive controller has its own dead-time length which is equal to the period of each fundamental component of *MRE*. And the number of

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repetitive controllers to be employed is equal to the number of dominant fundamental frequencies of *MRE* to be reduced in the frequency domain. It is remarked that our *MRC* does not require relatively large memory size, and the speed of error convergence can be made to be fast even for the reduction of *MRE*.

In the following section, for easy understanding, dual-repetitive controllers (*DRC*) are proposed for the system whose errors consist of dual repetitive error (*DRE*) components. And sufficient conditions for the stability of proposed dual-repetitive control systems (*DRCS*) and the error convergence of proposed systems are given. In Section 3, computer simulation results are illustrated for a typical disk drive head positioning control system to verify the validity of the proposed systems.

II. DESIGN OF DUAL-REPETITIVE CONTROLLERS

Consider a conventional linear SISO digital repetitive control system shown in Fig. 1(a), where $R(z)$ is the z-transform of the reference input, $C(z)$ the z-transform of the controlled output, $E(z)$ the z-transform of the error, $G_r(z)$ the transfer function of the repetitive controller, $G_o(z)$ the transfer function of the controlled system, and $G_i(z)$ a stable rational function of z. $G_i(z)$ is often chosen as the inverse transfer function of $G_{so}(z) \equiv G_o(z)/[1+G_o(z)]$, if perfect identification of the controlled system is available. K_r, z^{-N}, T and NT, are the gain of the repetitive controller, the dead-time element, the sampling time, and the dead-time length, respectively. NT is chosen to be equal to the period of the fundamental component of the repetitive errors. In order to obtain robustness of the repetitive control system under modeling uncertainty, a modified repetitive control system has been proposed in which FIR low pass filter, $q(z)$, is usually employed as shown in Fig. 1(b) [3, 4]. A signal will be called harmonic if it is periodic and has no frequency component other than a fundamental frequency $\omega_0=2\pi/NT$ (in rads^{-1}) and its harmonics, $k\omega_0=2\pi k/NT$, for $k\in I$, where I denotes the set of integers.

In general, since a repetitive controller is employed for reducing only the *SRE*, active reduction of the *MRE* is not guaranteed unless the dead-time element of the *SRC* is chosen in such a way that the dead time length should be equal to the least common multiplier of all dead-time lengths of fundamental components of the *MRE* in discrete time case. However, in case of the reduction of *MRE* by employing *SRC*, a relatively large memory size should be required since the memory size is directly proportional to the dead-time length [6]. And also, fast error convergence speed cannot be obtained.

To effectively and simply reduce such *MRE*, we newly propose multi-repetitive control systems (*MRCS*), where *MRC* are simply plugged in parallel in a feedback control system and each repetitive controller can reduce each fundamental frequency error component and its harmonics in *MRE*. And thus, the number of repetitive controllers to be employed should be equal to the number of dominant fundamental frequencies of *MRE* for a designer to reduce all *MRE*. For the sake of simplicity, a prototype dual repetitive control system (*DRCS*) is considered without loss of generality to reduce *DRE* components as shown in Fig. 2(a). Let the components of *DRE* have two fundamental frequencies, ω_1 and ω_2, defined as $\omega_1=2\pi/(N_1T)$ and $\omega_2=2\pi/(N_2T)$ and their harmonics. In the *DRCS*, two repetitive controllers are simply plugged in parallel in a feedback control system. (K_{r1}, z^{-N_1}) and (K_{r2}, z^{-N_2}) are gains and dead-time elements of repetitive controllers $G_{r1}(z)$ and $G_{r2}(z)$, respectively. First of all, to simply show that *DRC* can reduce *DRE*, assume that perfect modeling of $G_{so}(z)$, which is defined as $G_{so}(z) \equiv G_o(z)/[1+G_o(z)]$ be available. Then the closed loop transfer function of the prototype *DRCS*, $G_{clp}(z)$, can be obtained as

$$G_{clp}(z)=\frac{[1+G_{r1}(z)+G_{r2}(z)]G_o(z)}{1+[1+G_{r1}(z)+G_{r2}(z)]G_o(z)} = \frac{[1+\left\{\dfrac{K_{r1}z^{-N_1}}{1-z^{-N_1}}+\dfrac{K_{r2}z^{-N_2}}{1-z^{-N_2}}\right\}G_i(z)]G_{so}(z)}{1+\left\{\dfrac{K_{r1}z^{-N_1}}{1-z^{-N_1}}+\dfrac{K_{r2}z^{-N_2}}{1-z^{-N_2}}\right\}G_i(z)G_{so}(z)} \quad (1)$$

Theorem 2.1 (Stability of prototype *DRCS*): The prototype *DRCS*, $G_{clp}(z)$ in (1), is stable if $G_{so}(z)$ is asymptotically stable and if K_{r1} and K_{r2} satisfy the inequalities given by

$$K_{r1}>0, \quad \text{(2-a)}$$

$$K_{r2}>0, \quad \text{(2-b)}$$

and

$$K_{r1}+K_{r2}<2. \quad \text{(2-c)}$$

Proof. Since $G_i(z)G_{so}(z)=1$ under the assumption that the perfect modeling of $G_{so}(z)$ be available, the characteristic equation in (1) can be written as

$$\alpha(z)\{1-\beta(z)\}=0, \quad (3)$$

where

$$\alpha(z)=\{1-(1-K_{r1})z^{-N_1}\}\{1-(1-K_{r2})z^{-N_2}\}, \quad (4)$$

$$\beta(z)=\left\{\frac{K_{r1}K_{r2}z^{-N_1}z^{-N_2}}{\alpha(z)}\right\}. \quad (5)$$

From inequalities (2-a), (2-b) and (2-c), we can know that K_{r1} and K_{r2} satisfy $0<K_{r1}<2$ and $0<K_{r2}<2$, and thus, all roots of $\alpha(z)$ lie within unit circle. Now, to show that all roots of $(1-\beta(z))$ lie within unit circle, the principle of the argument is employed as follows;

First of all, note that all poles and zeros of $\beta(z)$ lie within the unit circle. Thus, if the contour of $(1-\beta(z))$ does not encircle the origin, and all zeros (roots) of (1-$\beta(z)$) lie within the unit circle. To show this, let $z=\exp(j\omega T)$ then $\beta(z)|_{z=\exp(j\omega T)}$ in (5) can be rewritten as

$$\beta(z)|_{z=\exp(j\omega T)} = \frac{K_{r1}K_{r2}}{a(\omega)+jb(\omega)}, \quad (6)$$

where $a(\omega)$ and $b(\omega)$ are given as

$$a(\omega) = (1-K_{r1})(1-K_{r2})-(1-K_{r1})\cos\theta_2-(1-K_{r2})\cos\theta_1+\cos(\theta_1+\theta_2), \quad (7)$$

and

$$b(\omega) = -(1-K_{r1})\sin\theta_2-(1-K_{r2})\sin\theta_1+\sin(\theta_1+\theta_2). \quad (8)$$

In (7) and (8), $\theta_1=j\omega TN_1$ and $\theta_2=j\omega TN_2$. Let ω_c be a frequency at which the imaginary part of $\beta(j\omega)$, $b(\omega)$, becomes zero. Then we can obtain two equations given as

$$1-K_{r1}=\cos(j\omega_c TN_1), \quad (9)$$

and

$$1-K_{r2}=\cos(j\omega_c TN_2). \quad (10)$$

And from (6), (7), (9) and (10), $|\mathrm{Re}\{\beta(j\omega)\}|^2$ at $\omega=\omega_c$ can be obtained as

$$|\mathrm{Re}\{\beta(j\omega)\}|_{\omega=\omega_c}|^2 = \left|\frac{K_{r1}K_{r2}}{a(\omega)}\right|^2_{\omega=\omega_c} \quad (11)$$
$$= \left|\frac{K_{r1}K_{r2}}{\sin(j\omega_c TN_1)\sin(j\omega_c TN_2)}\right|^2$$
$$= \left|\frac{K_{r1}K_{r2}}{(2-K_{r1})(2-K_{r2})}\right|.$$

From (2-a), (2-b) and (2-c), we can know that $|\mathrm{Re}\{\beta(j\omega_c)\}|<1$. This implies that $\{1-\beta(z)\}$ cannot encircle the origin. Thus, all roots of $\alpha(z)\{1-\beta(z)\}$ lie within unit circle. This completes the proof.

It is remarked from Theorem 2.1 that if $K_{r2}=0$, in other words, if $G_{r2}(z)$ is inactive, then (2) becomes $0<K_{r1}<2$ which is the same condition given in a conventional linear SISO digital single repetitive control system (*SRCS*) [10]. In this sense, our proposed *DRCS* can be considered as a generalized and extended version of a *SRCS*. In general, for a *MRCS* where the number of repetitive controllers is M, it is observed from extensive computer simulation results that (2) becomes

$$K_{ri}>0 \text{ and } \sum_{i=1}^{M} K_{ri} < 2 \text{ for } i\in[1, 2,.., M]. \quad (12)$$

The proof is under investigation.

In order to obtain robustness of the repetitive control systems under modeling uncertainty, a modified dual repetitive control system (*MDRCS*) can be also proposed in which a low pass filter is usually employed as shown in Fig. 2(b). To be specific, let a low pass filter, $q(z)$, be an FIR filter which satisfies the conditions given by $0<q(z)\leq 1$, and $\angle(q(z))=0$. Then the transfer function of the *MDRCS* denoted by $G_{clpm}(z)$ can be obtained by replacing z^{-N_1} with $q(z)z^{-N_1}$ and z^{-N_2} with $q(z)z^{-N_2}$ in (1).

Theorem 2.2 (Stability of *MDRCS*): The *MDRCS*, $G_{clpm}(z)$, is stable if $G_{so}(z)$ is asymptotically stable, and if K_{r1} and K_{r2} satisfy the inequalities given by

$$|K_{r1}-1|<\frac{1}{|q(z)|}, \qquad |K_{r2}-1|<\frac{1}{|q(z)|},$$

and

$$2K_{r1}K_{r2}\left\{(K_{r1}+K_{r2}-1)-\frac{1}{q(z)^2}\right\}<\left\{1-\frac{1}{q(z)^2}\right\}\left\{(K_{r1}+K_{r2}+1)^2-\frac{1}{q(z)^2}\right\}. \quad (13)$$

The proof can be easily done by a similar way to the case of the prototype *DRCS*, and thus is omitted here.

It is remarked from Theorem 2.2 that if $q(z)=1$, then (13) becomes (2). And it is also worthwhile to note that the stable region of K_{r1} and K_{r2} in (2) for a prototype *DRCS* can be extended by employing FIR filter. This implies that the reduced stable region of K_{r1} and K_{r2} due to modeling uncertainty can be compensated by the use of FIR filter as in the case of a conventional *SRCS* [10].

Now to consider the modeling uncertainties, let the unmodeled dynamics $\Delta(z)$ be represented as a multiplicative modeling error. Then the relation between the true system transfer function $G_s(z)$, and the nominal system transfer function $G_{so}(z)$, which is defined as $G_{so}(z) \equiv G_o(z)/[1+G_o(z)]$ can be written as

$$G_s(z) = G_{so}(z)[1+\Delta(z)]. \quad (14)$$

Assume that $G_{so}(z)$ has no uncancelable zeros. Then the relation between $G_i(z)$ and $G_s(z)$ is given by

$$G_i(z)G_s(z) = 1 + \Delta(z). \quad (15)$$

Theorem 2.3 (Error convergence of *MDRCS*): *DRE* are asymptotically decreased if the *MDRCS*, $G_{clpm}(z)$ with modeling uncertainty, is asymptotically stable, and if $\mathrm{Re}[1+\Delta(z)]>0$ at dual harmonic frequencies, and if $\mathrm{Re}\left\{\frac{K_{r2}z^{-N_2}}{1-z^{-N_2}}[1+\Delta(z)]\right\}>0$ at the first harmonic frequencies whose fundamental frequency $\omega_1=2\pi/(N_1T)$, and if

$\text{Re}\left\{\frac{K_{r1}z^{-N_1}}{1-z^{-N_1}}[1+\Delta(z)]\right\}>0$ at the second harmonic frequencies whose fundamental frequency $\omega_2=2\pi/(N_2T)$.

Proof. Let $G_{repm}(z)$ be the relative error transfer function of *MDRCS* defined as the ratio of the error transfer function with modified dual repetitive controller (*MDRC*) to the error transfer function without *MDRC*. Then $|G_{repm}(z)|$ at the first harmonic frequencies can be obtained as

$$\left|G_{repm}(z)\right|_{z=\exp(j\omega_1 kT)}=\left|\frac{1}{1+\{G_{r1}(z)+G_{r2}(z)\}G_s(z)}\right|_{z=\exp(j\omega_1 kT)}$$

$$=\left|\frac{1-|q(z)|}{1-|q(z)|+K_{r1}|q(z)|(1+\Delta(z))+\left\{\frac{K_{r2}z^{-N_2}|q(z)|}{1-z^{-N_2}|q(z)|}\right\}(1-|q(z)|)(1+\Delta(z))}\right|_{z=\exp(j\omega_1 kT)} \quad (16)$$

Since $0<|q(z)|\leq 1$, $\text{Re}[1+\Delta(z)]>0$, and $\text{Re}\left\{\frac{K_{r2}z^{-N_2}}{1-z^{-N_2}}[1+\Delta(z)]\right\}>0$ at the first harmonic frequencies, $|G_{repm}(z)|<1$ at the first harmonic frequencies. In a similar way, it can be easily shown that $|G_{repm}(z)|<1$ at the second harmonic frequencies.

It is remarked from Theorem 2.3 that if $|q(z)| = 1$, then $|G_{repm}(z)|$ at dual harmonic frequencies in (16) is zero, that is, *DRE* are completely rejected. However, $|G_{repm}(z)|$ at dual harmonic frequencies is no longer zero if $|q(z)| \neq 1$, which implies that for the *MRCS*, low pass filtering degrades the harmonic error rejection performance of repetitive control action. In this sense, gains K_{ri} of *MRC* may be required to be adjusted in order to reduce the infinite norm of errors in the frequency domain since the repetitive error rejection performance of repetitive control action depends on the magnitude of the gain of a repetitive controller [2].

III. NUMERICAL EXAMPLES

Consider a typical disk drive head positioning servo control system, where input-output transfer function $G_{so}(z)$ in (1) given by Chang *et al.* [2].

$$G_{so}(z)=\frac{5.01-14.0z^{-1}+14.2z^{-2}-6.0z^{-3}+0.88z^{-4}}{1-0.874z^{-1}-0.992z^{-2}+0.882z^{-3}}. \quad (17)$$

The sampling time, T, and the number of sectors of the hard disk drive system, N, giving the dead-time length are chosen as 416.66 [μs] and 40, respectively. To show the validity of the proposed prototype of the *MRCS*, it is assumed that errors consist of multiple dominant fundamental frequency components. Note that the multiple frequency components are sufficient to set up the practical situation of controlling a hard disk drive since dominant non-repeatable errors often occur at a few non-repeatable run-out frequencies, primarily due to the mechanical characteristics of ball bearings in the spindle motor [1].

To simply investigate the validity of the proposed *MRCS*, let us consider *DRE* which consist of two fundamental frequency components locating at 60 and 96 Hz. And let $G_{r1}(z)$ and $G_{r2}(z)$ be *DRC* to reduce error components locating at 60 Hz and 96 Hz, respectively. Then the dead time length of $G_{r1}(z)$, N_1T, is given as 40T, and the dead time length of $G_{r2}(z)$, N_2T, is given as 25T. Fig. 3 shows time response of errors for the control system (a) without repetitive controller, (b) with *SRC* for reducing 60 Hz component error only (K_{r2}=0 case), (c) with *SRC* for reducing 96 Hz component error only (K_{r1}=0 case) and (d) with proposed *DRC* for reducing both 60 and 96 Hz component errors. It can be seen from Fig. 3 that *DRE* can be effectively reduced by employing *DRC* while *SRC* can reduce only *SRE*. The only way to reduce *DRE* by the use of a conventional *SRC* is to choose the dead-time element in such a way to that the dead-time length should be equal to the least common multiplier of the two dead-time lengths. The dead-time length for such a *SRC* is then chosen as 200T (12 Hz) which is the least common multiplier of 40T (60 Hz) and 25T (96 Hz). Fig. 4 shows how errors converge for cases of (a) the proposed *DRCS* and (b) a conventional *SRCS* with dead-time length 200T for reducing both 60 and 96 Hz component errors. It can be observed that the speed of error convergence in our proposed *DRCS* can be made to be fast even for the reduction of *DRE* since the dead-time lengths of each repetitive controller in *DRCS* are much shorter than that of least common multiplier of the two dead-time lengths. In this sense, our proposed *MRCS* can play a more critical role in reducing *MRE* in case that the number of dominant fundamental frequency components is large and/or least common multiplier of dead-time lengths is longer than a conventional *SRCS*.

IV. CONCLUDING REMARKS

Dual-repetitive errors were shown to be effectively reduced by employing our proposed dual-repetitive controllers. Dual repetitive controllers were simply plugged in parallel in a feedback control system and each repetitive controller could reduce each fundamental frequency error component and its harmonics in dual-repetitive errors. Our proposed dual-repetitive control system can save memory size and reduce errors faster than the conventional single repetitive control system with dual-repetitive errors. It is believed that the dual-repetitive control system can be practically and usefully applied to any linear SISO digital control system with dual-repetitive errors and/or disturbances in industry.

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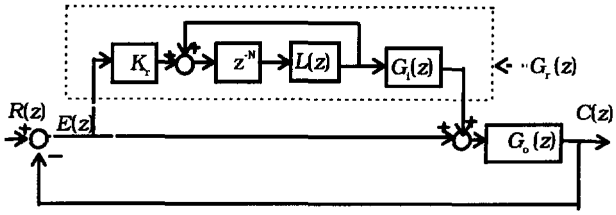


Fig. 1. Block diagram representing two conventional repetitive control systems: (a) a prototype single repetitive control system [$L(z)=1$]; (b) a modified single repetitive control system [$L(z)=q(z)$].

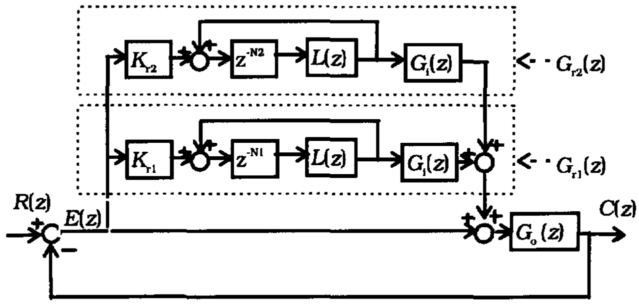
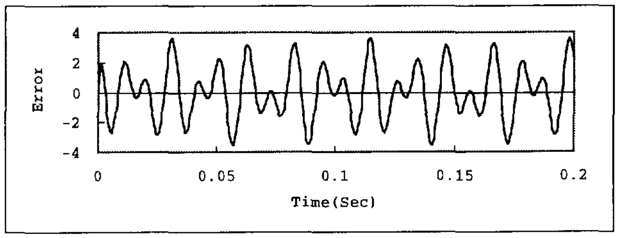
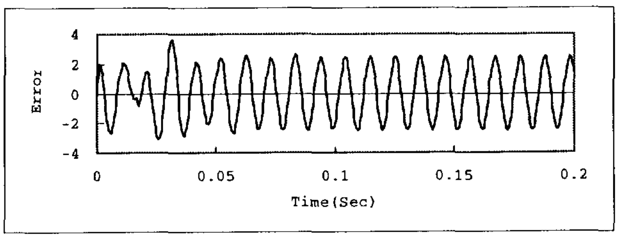


Fig. 2. Block diagram representing two proposed dual-repetitive control systems: (a) a prototype dual-repetitive control system [$L(z)=1$]; (b) a modified dual-repetitive control system [$L(z)=q(z)$].



(a)



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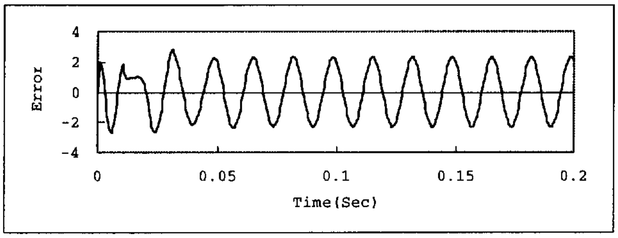

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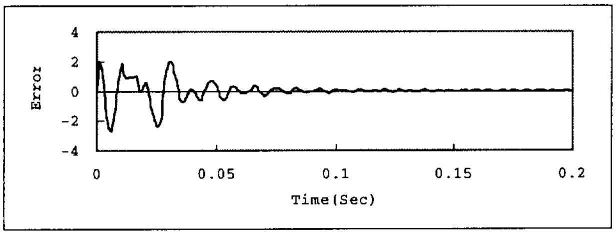
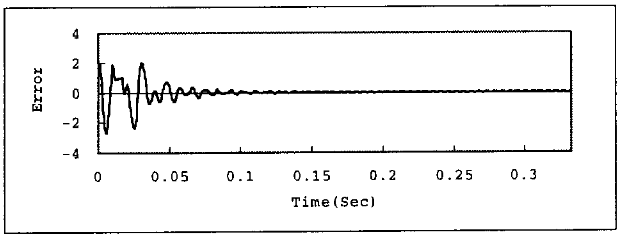
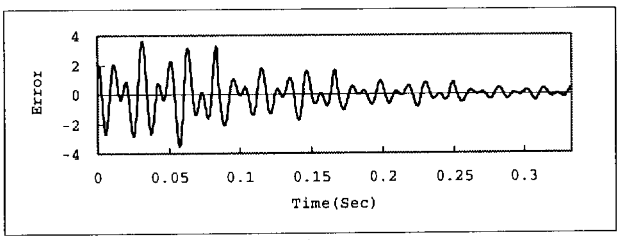


(d)

Fig. 3. Time response of errors in the control system (a) without repetitive controller, (b) with single repetitive controller for reducing 60 Hz component error only (K_{r2}=0 case), (c) with single repetitive controller for reducing 96 Hz component error only (K_{r1}=0 case) and (d) with proposed dual repetitive controller for reducing both 60 and 96 Hz component errors.



(a)



(b)

Fig. 4. Error convergence performance for cases of (a) the proposed dual repetitive control system and (b) a conventional single repetitive control system to reduce dual repetitive errors.