Proceedings of Advanced
Intelligent Mechatronics
AIM'97, Tokyo,Japan,1997

A Geometric Dexterous Motion Control of Redundant Robot Manipulators by using Nonlinear Optimization Method

W.K. Hyun[1], I.H.Suh[2], K.T.Park[3] and B.J. Lee[4]

1. Dept. of Electronics Eng., Honam Univ., KwangJu, KOREA
TEL: +82-62-940-5482, FAX: +82-62-940-5482
E-mail: wkhyun@honam.honam.ac.kr
2. Dept. of Electronics Eng., Hanyang Univ., KOREA
TEL: +82-345-400-5172, FAX: +82-345-408-5803
E-mail: ihsuh@shira.hanyang.ac.kr
3. Dept. of Automatic Control Lab., KIMM, TaeJeon, KOREA
TEL: +82-42-868-7131, FAX: +82-42-868-7149
4. Dept. of Control and Instrumentation Eng., Hanyang Univ., KOREA
TEL: +82-345-400-5218

ABSTRACT

A dexterous motion control method of redundant robot manipulators based on nonlinear optimization method is proposed to satisfy multi-criteria such as singularity avoidance, minimizing energy consumption ,and avoiding physical limits of actuator, while performing a given task. The method employs a neural optimization network with parallel processing capability as a nonlineae optimization method, where only a simple geometric analysis for resolved motion of each joint is required instead of computing of the Jacobian and its pseudo inverse matrix. For dexterous motion, a joint geometric manipulability measure(JGMM) is proposed. JGMM evaluates a contribution of each joint differential motion in enlarging the length of shortest axis among principal axes of the manipulability ellipsoid volume approximately obtained by a geometric analysis. Redundant robot manipulators is then controlled by neural optimization networks in such a way that (1) linear combination of the resolved motion by each joint differential motion should be equal to the desired velocity, (2) physical limits of joints are not violated, and (3) weighted sum of the square of each differential joint motion is minimized where weightings are adjusted by JGMM.

This paper was supported by Korea Research Fundation

To show the validity of the proposed method, several numerical examples are illustrated.

1. Introduction

Redundant robot arms, designed to have more joints than necessary for given tasks, have received much attention due to their flexibility in task execution, provided by the availability of infinite number of joint motions that lead to the same end-effector trajectory. Typically, the kinematic component of a redundant manipulator control scheme must generate a set of joint angle trajectories, from the infinite set of possible trajectories, while optimizing performance functions, such as singularity avoidance, collision avoidance, or joint limit avoidance. For this, redundant robot control method should satisfy multi-criteria while taking given task. General approaches are nested projection methods which project a gradient of performance function to null space of Jacobian and project the other performance to the nested null space of the null space of the Jacobian[3]. However, this method has a difficulty in numerically and algebraically finding the nested null space of the Jacobian especially when the robot has high redundant D.O.F.

In this paper, optimal solution approach with equality and inequality constraints is employed in dexterous motion control the redundant robot manipulators, where physical limits of actuator and dexterous performance are represented as inequality conditions and weightings of the performance functions, respectively. Optimal solutions are achieved by neural optimization network which has a capability of parallel processing. For this, kinematic approach should be represented as a form of parallel processing, which is obtained by proposed simple geometric analysis of the resolved motion for each joint differential motion. An index for the motion can also be obtained by the geometric analysis on the following idea; For a dexterous or a nonsingular motion of a redundant robot manipulator, it is necessary for a robot manipulator to be in a configuration whose manipulability index is maximum, where manipulability index is mathematically defined as the volume of ellipsoid in the m-dimensional task space when the Euclidean norm of joint velocity is less than or equal to unity[1]. The length of each principal axis of the ellipsoid volume implies how well the robot end effect can move toward the axis. Therefore, if the shape of the ellipsoid volume is similar to a sphere, the end-effector of manipulator is able to moving well toward any direction. The principal axes of the manipulability ellipsoid can be obtained by the singular value decomposition of the manipulator Jacobian matrix. And since the product of all singular values is equal to $\sqrt{det(JJ^T)}$ [1], it is necessary to maximize the minimum singular value[2,3], or to minimize condition number[4] for the enhancement of the volume of the ellipsoid. These optimization processes require partial differentiation of the square root of $det(JJ^T)$ or singular values of Jacobian matrix with respect to joint variables, which seems to be so complicated, especially in the case of highly redundant manipulators.

To cope this problem, a joint geometric manipulability measure(JGMM) is proposed for nonsingular motions of a redundant robot. Specifically, each length of principal axis of the manipulability ellipsoid volume is approximately obtained by a geometric analysis at time $t-\delta t$. And then joint motion is determined in such a way that the shortest principal axis at time $t-\delta t$ is enlarged by a neural optimization network, where no explicit partial differentiation is required.

2. Resolved Motion Control of Redundant Robot Manipulators by using optimization solution approach based on neural networks.

In controlling redundant robot manipulators, neural networks have been effectively utilized as in [5,6,7]. Among them, Hyun et al[3] proposed a neural optimization network to resolve the motion of redundant manipulators. And also they proposed a model for description of differential motion not to explicitly use the pseudo inverse of the Jacobian. Now, such a modelling technique in [5] is briefly reviewed as follows; First of all, a task coordinate is assigned in such a way that the direction of desired velocity vector of the end-effector is to be the z-axis, and then contributions of end-effector motion due to each differential joint motion are mapped onto the task coordinate. This process is sketched in Fig.1.

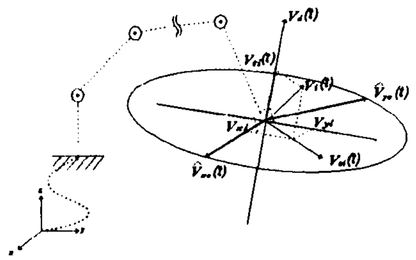

Fig.1. The decomposed components of $V_i(t)$ with respect to $V_d(t)$.

To be more specific, suppose that the i-th joint rotates as much as δq and the other joints locked up at time interval δt, where δq is constant value which is small enough to guarantee the movement of the end-effector to be linealized. Then the end-effector moves with a velocity. Let the velocity of end-effector resulting from the i-th joint differential motion be denoted by 6×1 vector $Z_i(t)=[\ V_i^T(t)\ \mid\ W_i^T(t)]^T$, where $V_i(t)$ and $W_i(t)$ represent the translational and rotational velocities, respectively. Let 6×1 vector $Z_d(t)=[\ V_d^T(t)\ \mid\ W_d^T(t)]^T$ be the desired velocity of end-effector. And let $\hat{v}_{nd}(t)$ and $\hat{w}_{nd}(t)$ be the unit vectors given as $V_d(t)/\| V_d(t)\|$ and $W_d(t)/\| W_d(t)\|$, respectively. Also let $V_{ti}(t)$ and $W_{ti}(t)$ be the vectors which are obtained by the projection $V_i(t)$ onto $V_d(t)$ and by the projection $W_i(t)$ onto $W_d(t)$, respectively, and let $V_{oi}(t)$ and $W_{oi}(t)$ be the vectors which are obtained by the projection of $V_i(t)$ onto the surface orthogonal to $V_d(t)$ and by the projection of $W_i(t)$ onto the surface orthogonal to $W_d(t)$, respectively. Further let vJf) = v,(f) . $\hat{v}_{nd}(t)$, where · implies the inner product of two vectors. Then $V_{ti}(t)$ can be written as

$$V_{ti}(t) = v_{zi}(t)\ \hat{v}_{nd}(t). \quad (1)$$

And let $\hat{v}_{ox}(t)$ be the unit vector on the surface orthogonal to $V_d(t)$ given as $\hat{v}_{ox}(t) \equiv V_{o1}(t)/\| V_{o1}(t)\|$, and let $\hat{v}_{oy}(t)$ be the unit vector orthogonal to both $\hat{v}_{ox}(t)$ and $\hat{v}_{nd}(t)$. Then if we let $v_{x_i}(t)$ and $v_{y_i}(t)$ be the projections of $V_{oi}(t)$ onto $\hat{v}_{ox}(t)$ and $\hat{v}_{oy}(t)$, respectively, $V_{oi}(t)$ can be expressed as

$$V_{oi}(t) = v_{x_i}(f)\ \hat{v}_{ox}(t) + v_{y_i}(t)\ \hat{v}_{oy}(t). \quad (2)$$

In a similar way, let $w_{zi}(t) \equiv W_i(t) \cdot$ &f(f), $\hat{w}_{ox}(t) \equiv W_{o1}(t)/\| W_{o1}(t)\|$ and $\hat{w}_{oy}(t)$ be the unit vector orthogonal to both $\hat{w}_{nd}(t)$ and $\hat{w}_{ox}(t)$. Then $W_{ti}(t)$ and $W_{oi}(t)$ can be written as

$$W_{ti}(t) = w_{zi}(t)\ \hat{w}_{nd}(t), \quad (3)$$

and

$$W_{oi}(t) = w_{x_i}(t)\ \hat{w}_{ox}(t) + w_{y_i}(t)\ \hat{w}_{oy}(t), \quad (4)$$

where $w_{x_i}(t)$ and $w_{y_i}(t)$ are the projections of $W_{oi}(t)$ onto $\hat{w}_{ox}(t)$ and $\hat{w}_{oy}(t)$, rcsgectively, and $\hat{w}_{oz}(t)$ is that of $W_i(t)$ onto &(t>.

It is noted that if linear combinations of u.,(f) and $w_{iz}(t)$ are equal toll $V_d(t)\|$ and II $W_d(t)\|$, respectively, and linear combinations of $v_{x_i}(t)$, $v_{y_i}(t)$, $w_{x_i}(t)$ and $w_{y_i}(t)$ are equal to zero, respectively, for $i = 1, 2, \ldots, n$, then the robot can follow the desired trajectory. This can be mathematically summarized as

$$g_1(u(t)) = \sum_{i=1}^{n} u_i(t) v_{zi}(t) - \| V_d(t) \| = 0, \quad (5)$$

$$g_2(u(t)) = \sum_{i=1}^{n} u_i(t) v_{x_i}(t) = 0, \quad (6)$$

$$g_3(u(t)) = \sum_{i=1}^{n} u_i(t) v_{yi}(t) = 0, \quad (7)$$

$$g_4(u(t)) = \sum_{i=1}^{n} u_i(t) w_{zi}(t) - \| W_d(t) \| = 0, \quad (8)$$

$$g_5(u(t)) = \sum_{i=1}^{n} u_i(t) w_{x_i}(t) = 0, \quad (9)$$

and

$$g_6(u(t)) = \sum_{i=1}^{n} u_i(t) w_{yi}(t) = 0, \quad (10)$$

where $u(t) = [u_1(t)\ u_2(t)\ \ldots\ u_n(t)]^T$ and $u_i(t)$ is a scalar variable to be multiplied to a prespecified small joint angle δq during a fixed time interval δt so that the velocity of the I-th joint is obtained by $\dot{q}_i(t) = u_i(t)\delta q/\delta t$.

The resolved motion of the redundant robot manipulators is here shown to be obtained by solving an optimization problem with equality and inequality constraints. For this, let $h_j(u(t))$ for inequality constraint of joint velocity be given by

$$h_j(u(t)) = \begin{cases} u_i(t) - U_{\max i}, & j = 2i, \\ U_{\min i} - u_i(t), & j = 2i - 1, \end{cases} \quad (11)$$

where $U_{\max i}$ and $U_{\min i}$ are maximum and minimum limit values for the i-th joint velocity, respectively. Then, the optimization problem can bc formulated to resolve the motion of redundant robot manipulators as Problem 1(P_1).

P_1:

$$\min_{u(t)} f(u(t)) \quad (12)$$

subject to the constraints

$$g_k(u(t)) = 0,\ k = 1, 2, \ldots, 6, \quad (13)$$

$$h_j(u(t)) \le 0, \quad j = 1, 2, \ldots, 2n, \quad (14)$$

where

$$f(u(t)) = \sum_{i=1}^{n} \frac{1}{2} \omega_i(t) u_i^2(t). \quad (15)$$

In Eq.(15), $w_i(t)$ is the weighting factor for the i-th joint motion. In problem P_1, the inequality constraints can be converted into equality constraints using slack variables, Namely, the constraints of the form $h_j(u(t)) \le 0$ can be expressed as

$$C_j(u(t), z(t)) = h_m(u(t)) + z_m^2(t), \quad (16)$$
$$j = 1, 2, \ldots 2n,$$

where $z(t) = [z_1(t) z_2(t) \ldots z_n(t)]^T$. Since $z_j^2(t)$'s are positive, the constraints C_j(u(t), $z(t)$) = o make $h_j(u(t))$ be negative. Therefore, a neural optimization network with anti-symmetry connections among the neurons can be easily constructed to solve Problem 1 by applying the Platt and Barr's approach[19] as follows;

For i-1.2, $\ldots, n$, j=1,2, ..., 2 n and k=1,2,. . . . 6.

$$\frac{\partial u_i}{dt} = -\frac{\partial f(u)}{\partial u_i} - \sum_{k=1}^{6} \beta_k \frac{\partial g_k(u)}{\partial u_i} - \sum_{j=1}^{2n} \gamma_j \frac{\partial C_j(u, z)}{\partial u_i}, \quad (17)$$

$$\dot{z}_j = -\sum_{j=1}^{2n} \gamma_j \frac{\partial C_j(u, z)}{\partial z_j}, \quad (18)$$

$$\dot{\beta}_k = g_k(u), \quad (19)$$

$$\dot{\gamma}_i = C_j(u, z), \quad (20)$$

and

$$\gamma_j \ge 0, \quad (21)$$

where β_k's and γ_j's imply the neurons corresponding to Lagrange multipliers for the equality constraints, and the z_i's imply the neurons for slack variables to convert the inequality constraints to the equality constraints.

3. Joint Geometric Manipulability Measure(JGMM).

In this section, a joint geometric manipulability measure is proposed to adjust $u_i(t), i=1,2,\ldots,n,$ in Eq.(15) for nonsingular motions of the redundant robot. Specifically, each length of principal axis of the manipulability ellipsoid volume is approximately obtained by a geometric analysis at time $t-\delta t$. And then joint motion is determined in such a way that the shot-test principal axis at time $t-\delta t$ is enlarged by a neural optimization network proposed in[5], where no explicit partial differentiation is required.

For this, let the translational velocity of the end-effector resulting from i-th joint differential motion at time $t-\delta t$ be denoted as $V_i(t-\delta t)$. Note that $u_i(t-\delta t)V_i(t-\delta t)$ is considered as an approximated translational velocity of end-effector due to $\dot{q}_i(t)$. And then an ellipsoid volume can be approximately generated in three dimensional space by a vector sum of $u_i(t-\delta t)V_i(t-\delta t)$ for $i=1.2, \ldots n$ under the constraint of $\|u(t-\delta)\|\le 1$. Let such an ellipsoid volume be expressed as $\zeta_v(t-\delta t)$ implying manipulability of the translational movement of end-effector at time $t-\delta t$. *Now,* the the longest principal axis of $\zeta_v(t-\delta t), V_{\max}(t-\delta t)$, can be obtained as

$$V_{\max}(t-\delta t)=Arg\{\max\| V_i(t-\delta t)\|,\ i-1,2,\ \ldots,n\} \quad (22)$$

Let second longest principal axis be denoted as $V_{omax}(t-\delta t)$. Since the shape of surface generated by the vector sum of two orthogonal vectors with respect to $V_{\max}(t-\delta t)$ becomes the 2-dimensional ellipsoid $\Gamma(t)$ as shown in Fig. 2. $V_{omax}(t-\delta t)$ can be obtained as

$$V_{Omax}(t-\delta t)=Arg\{\max\|S(V_{\max}(t-\delta t))\ V_i(t-\delta t)\|,\ i=1,2,\ \ldots,n\}$$
, (23)

where $S(V_i(t))$ is skew symmetric matrix of $V_i(t)$ and can be expressed as

$$S(V_i(t))=\begin{bmatrix} 0 & -v_{zi}(t) & v_{yi}(t) \\ v_{zi}(t) & 0 & -v_{xi}(t) \\ -v_{yi}(t) & v_{xi}(t) & 0 \end{bmatrix}. \quad (24)$$

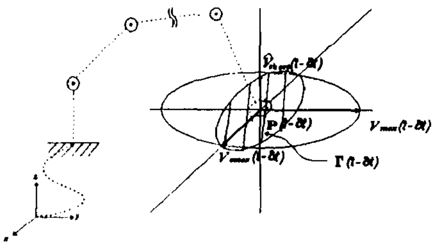

Fig. 2 An approximated manipulability ellipsoid volume represented by $V_{\max}(t-\delta t)$, $V_{Omax}(t-\delta t)$ and $\hat{V}_{short}(t-\delta t)$.

Let the unit vector of 3rd principal axis, the shortest principal axis of the ellipsoid volume, be denoted as $\hat{V}_{short}(t-\delta t)$. *Note* that the 3rd principal axis is orthogonal to both $V_{\max}(t-\delta t)$ and $V_{Omax}(t-\delta t)$. Thus $\hat{V}_{short}(t-\delta t)$ can be obtained by

$$\hat{V}_{short}(t-\delta t)=\frac{S(V_{\max}(t-\delta t))\ V_{Omax}(t-\delta t)}{\|S(V_{\max}(t-\delta t))\ V_{Omax}(t-\delta t)\|}. \quad (25)$$

It is remarked that the translational movement toward the direction of $\hat{V}_{short}(t-\delta t)$ is harder than that toward the direction of other principal axes in the sense of manipulability. To make the translational movement toward $\hat{V}_{short}(t-\delta t)$ be easy, the length of axis of $\hat{V}_{short}(t-\delta t)$ should be enlarged. For this, a joint geometric manipulability measure(JGMM) for translational velocity, $Q_{iv}(t)$, is given as

$$Q_{iv}(t)=\frac{Q_{ivM}(t)}{\sum_{k=1}^{n} Q_{kvM}(t)}, \quad (26)$$

where $Q_{ivM}(t)$ is defined as $Q_{ivM}(t)=\eta(i,t)/\mu(i,t)$, and where $\eta(i,t)$ and $\mu(i,t)$ *are* given by

$$\eta(i,t)=\frac{|\hat{V}_{short}(t-\delta t)\cdot V_i(t)|}{\|S(\hat{V}_{short}(t-\delta t))\ V_i(t)\|}, \quad (27)$$

and

$$\mu(i,t)=\frac{|\hat{V}_{short}(t-\delta t)\cdot V_i(t-\delta t)|}{\|S(\hat{V}_{short}(t-\delta t))\ V_i(t-\delta t)\|}. \quad (28)$$

In Eq.(27), $\eta(i,t)$ denotes the ratio of tangential component of *Vi(t)* with respect to $\hat{V}_{short}(t-\delta t)$ to orthogonal components of $V_i(t)$ with respect to $\hat{V}_{short}(t-\delta t)$. And thus, if $\eta(i,t)$ is large, a robot manipulator is in a configuration of which $V_i(t)$ is similar to $\hat{V}_{short}(t-\delta t)$. And a large value of

$\mu(t)$ implies that a robot manipulator is in a configuration of which $V_i(t-\delta t)$ is similar to $\hat{V}_{short}(t-\delta t)$. Thus, a large value of $Q_{ivM}(t)$ implies that i-th joint differential motion can change the configuration of the robot in such a way that the length of axis of $\hat{V}_{short}(t-\delta t)$ becomes longer. Since $Q_{ivM}(t)$ can be extremely large or small, $Q_{iv}(t)$ in Eq.(26) is employed instead as a normalized form of $Q_{kvM}(t)$ for $k=1,2,\ldots,n$.

It is remarked that if $Q_{jv}(t)$ is larger than $Q_{iv}(t)$ as shown in Fig.3, the j-th joint differential motion helps a redundant robot manipulator to be in a nonsingular configuration more than the i-th joint differential motion does.

In a similar way, JGMM for rotational velocity of end-effector can be given as follows;

$$Q_{iw}(t)=\frac{Q_{iwM}(t)}{\sum_{k=1}^{n} Q_{kwM}(t)}, \tag{29}$$

where $Q_{iwM}(t)$ is defined as

$$Q_{iwM}(t)=\frac{|\hat{W}_{short}(t-\delta t)\cdot W_i(t)|}{\|S(\hat{W}_{short}(t-\delta t))\,W_i(t)\|}\Big/\frac{|\hat{W}_{short}(t-\delta t)\cdot W_i(t-\delta t)|}{\|S(\hat{W}_{short}(t-\delta t))\,W_i(t)\|} \tag{30}$$

In Eq.(30), $\hat{W}_{short}(t-\delta t)$ is defined as

$$\hat{W}_{short}(t-\delta t)=\frac{S(W_{\max}(t-\delta t))\,W_{Omax}(t-\delta t)}{\|S(W_{\max}(t-\delta t))\,W_{Omax}(t-\delta t)\|}, \tag{31}$$

where $\hat{W}_{short}(t-\delta t)$ and $W_{\max}(t-\delta t)$ are given as

$$W_{\max}(t-\delta t)=Arg\{\max\|W_i(t-\delta t)\|,\ i=1,2,\cdots,n\}, \tag{32}$$

and

$$W_{Omax}(t-\delta t)=Arg\{\max\|S(W_{\max}(t-\delta t))\,W_i(t-\delta t)\|,\ i=1,2,\cdots,n\} \tag{33}$$

If all joint velocities can be continuously adjusted according to JGMM, $Q_{iv}(t)$ and $Q_{iw}(t)$, redundant robot manipulators are expected to avoid singular postures. For this, motions of redundant robot manipulators are controlled by adjusting weighting of the neural optimization network. Specifically, weighting factor $w_i(t), i=1,2,\ldots,n$, is adjusted by $Q_{iv}(t)$ and $Q_{iw}(t)$ as follows;

$$w_i(t)=\frac{2}{\pi}\tan^{-1}\left(\frac{1}{Q_{vi}(t)}+\frac{1}{Q_{wi}(t)}\right)+x_i, \tag{34}$$

where the weighting factor $w_i(t)$ is selected so that it is inversely proportional to the measures $Q_{iv}(t)$ and $Q_{iw}(t)$. In this way, the joint which contributes largely to the desired motion may move with large velocity. In Eq.(34), x_i is the off-set weighting factor.

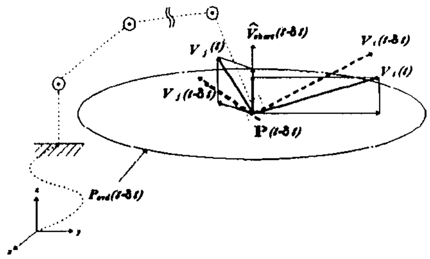

Fig.3 Decomposed orthogonal and tangential components of $V_i(t)$ and $V_j(t)$ respect $\hat{V}_{short}(t-\delta t)$.

4. Simulation Results

A planar redundant robot manipulator with three degrees of freedom is employed, where the lengths of the links l_1, l_2, and l_3 are chosen to be the same as 400(mm).

The purpose of our simulation is to show the validities of the proposed measure JGMM. For this, consider the case when a redundant robot manipulator becomes a near-singular as shown in Fig.6. For this case, several measures such as manipulability[2], condition number[6], singular value decomposition[1] and JGMM are applied to avoid the singularity. The task is to follow the linear path from start point to the goal point with the speed of 44.5 (mm/sec). The initial configuration is $q(0)=[90°\ -135\ 135°]^T$ and goal position is $x=[-300,00.0]^T$. In this example, a configulation of the robot manipulators is to be near singular posture. Fig.4 shows resultant motions for the approaches using (a) our proposed JGMM, (b) Yoshikawa's Manipulability(YM), (c) condition number(CN), (d) singular value decomposition(SVD) measures and (e) pseudo inverse(PI). It can be observed from Fig.4 that our proposed JGMM and YM generate similar smooth motion trajectories in the sense that maximum accerations for two motion trajectories are measured to be almost same, but the approaches using CN, SVD and PI show a rather big accelerations when compared with JGMM and YM. Especially, joint velocities for pseudo inverse method shown in Fig.4.(e) changed abruptly at about 6 sec, since the robot manipulator was in to be a near singular configuration at that time. Thus, our proposed method can be a good alternative to the conventional methods in [1,2,3], where complex computations including partial derivatives of subgoals such as manipulability are required.

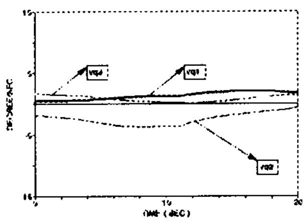

Fig.4(a)

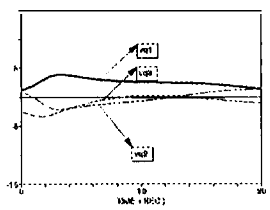

Fig.4(b)

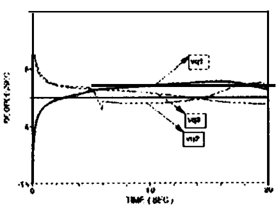

Fig.4(c)

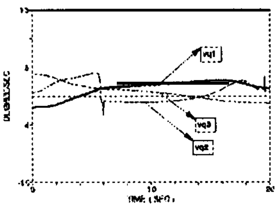

Fig.4(d)

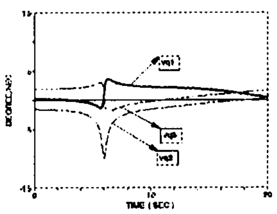

Fig.4(e)

Fig.4 Trajectories of joint velocities.
(a) JGMM (b) YM (c) CN (d) SVD (e) PI

5. Conculsion

An effective dexterous motion control method of redundant robot manipulators based on neural optimization network was proposed to satisfy multi-criteria. Optimal solution approach with equality and inequality constraints is employed in dexterous motion control of the redundant robot manipulators, where physical limits of actuator and dexterous performance are represented as inequality conditions and weightings of the performance function of optimal problem, respectively. Optimal solutions are achieved by neural optimization network which has a capability of parallel processing. For this, kinematic approach was represented as a form of parallel processing by proposed simple geometric analysis for the resolved motion of each joint differential motion. For dexterous motion, a joint geometric manipulability measure(JGMM) was proposed. JGMM evaluates a contribution of each joint differential motion for enlarging the shortest length of principal axis of the manipulability ellipsoid volume approximately obtained by a geometric analysis. The proposed method was shown to be valid from several numerical examples.

Reference

[1] T. Yoshikawa, "Manipulability of robotic Mechanisms", *Intl.J. of Robotics Research,* Vol. *4, No. 2,* pp. 3-9, summer 1985.

[2] C.A. Klein and B.E. Blaho, "Dexterity Measure for the Design and Control of Kinematically Redundant Manipulators," *Intl. J. of Robotics Research,* Vol. 6, No. 2, pp.72-83, 1987.

[3] Y. Nakamura, Advanced Robotics, Redundancy and Optimization, addison- Wesley Publishing Company, 1991.

[4] J.K Salisbury and J.J Craig, "Articulated hand: force control and kinematics," Intl. J. Robotics Res. Vol.1, No.1, pp14-17, 9182

[5] W.K. Hyun, and I.H. Suh, and J.H. Lim, "Resolved Motion Control of Redundant Robot Manipulators by Neural Optimization Network," *IEEE Intl. Conf. of IROS '90*, pp. 627-634, Japan, 1990.

[6] R.K. Elsley, "A learning Architecture for Control based on back-propagation Neural Network," *Proc. of IEEE IJCNN-88,* San Diego, Cal, Vol. 2, pp. 587-594, July, 1988.

[7] K. Tsutsumi and H. Matsumoto, "Neural Computation and Learning Strategy for Manipulator Position Control", *Proc. of IEEE ICCN,* Vol. *4,* pp. 525-534, 1987.

[8] J. Luh, M. Walker and R. Paul, "Resolved-Acceleration Control of Mechanical Manipulator," *IEEE Trans. on Automatic Control,* Vol. AC-25, No. 3, pp. 468-474, 1980.

[9] J.C. Platt and A.H. Rarr, "Constrained differential Optimization", *Neuml &formation processing system,* American institute of physics, pp. 612-621, NEW YORK 1988.