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Synthesis and Analysis of Digital Multiple Repetitive Control Systems

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Abstract

This paper newly proposes the so-called digital 'multiple' repetitive control systems which can regulate efficiently 'multiple repetitive errors' which contain multiple dominant fundamental frequencies and their harmonics. Our multiple repetitive controller has not only a very simple structure but also such advantages as smaller memory requirement and faster error convergence speed than the conventional 'single' repetitive controller. This paper also presents a robust stability condition and an error convergence condition for our multiple repetitive control systems in a mathematically rigorous way. The performance of our multiple repetitive control system is demonstrated through simulations.

1 Introduction

A lot of effort has been recently devoted to the development of the digital repetitive control scheme which is regarded as one of the most powerful methods to regulate the repetitive errors whose fundamental frequencies are priory known [2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 15, 16]. In these prior researches, the repetitive errors with just only one fundamental frequency and its harmonics were considered.

However, unfortunately, repetitive errors might have multiple fundamental frequencies and their harmonics in practice. Hereinafter, we will call this kind of errors as multiple repetitive errors. One of such examples can be found in head positioning servo control systems for hard disk drives [1, 8, 12, 14, 15]. Another typical example can be found in joint gear systems which consist of gears more than one whose angular velocities are different from each others [9]. In order to regulate such multiple repetitive errors by using the repetitive control schemes developed so far, the dead-time length of the controller should be equal to the least common multiple (L.C.M.) of all the periods corresponding to fundamental frequencies in the error. Thus, the size of memory for implementation of the dead-time element should be very large. Besides, it takes a lot of time to regulate the error.

A simple technique to provisionally regulate multiple repetitive errors reduces only dominant single repetitive error. The other repetitive errors are called non-repetitive errors regardless of whether or not they are periodic. In this case, non-repetitive errors might be amplified [2, 12, 15] while the dominant single repetitive error is reduced. To alleviate this drawback, an approach based on gain adjusting algorithm and optimized repetitive function was suggested in [2]. However, this approach was not developed to actively reduce non-repetitive errors. On the other hand, the use of notch filter corresponding to the frequencies of non-repetitive errors was proposed in [12]. But incorporating notch filter is deviated from the original purpose of repetitive controller and the complexity of the controller can be excessively increased when the considered non-repetitive errors contain many non-negligible fundamental frequencies.

In this paper, we present a novel approach to regulate effectively the multiple repetitive errors. The proposed controller, the so-called 'multiple repetitive controller' has a simple structure in which the prior repetitive controllers corresponding to the fundamental frequencies in the error are plugged in parallel. As the result, the dead-time length of our controller is dramatically reduced to the summation of all the periods corresponding to fundamental frequencies in the error. Therefore, the multiple repetitive controller can not only be implemented with memory much less than those required in the prior systems but also provide the error regulation speed much faster than those provided in the prior systems. In this paper, we show the robust stability of the multiple repetitive controller in a mathematically rigorous way and the error convergence condition for multiple repetitive errors is also found. The advantage of the multiple repetitive controller is demonstrated through simulation results on a typical hard disk head positioning servo control system.

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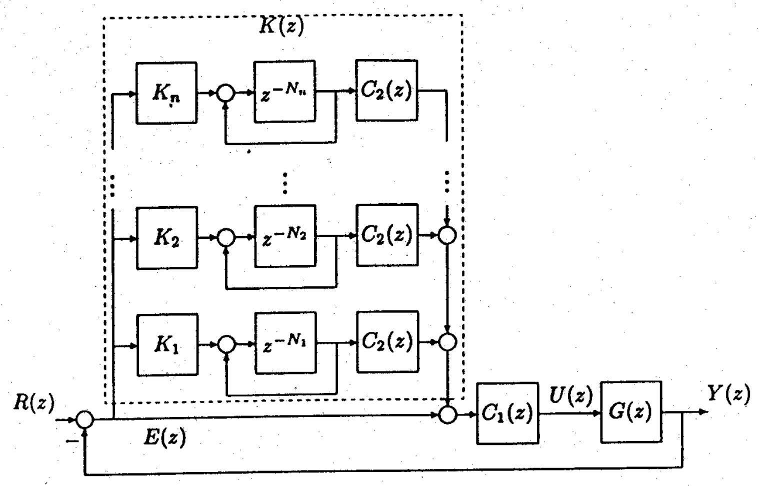

Figure 1: Digital Multiple Repetitive Control System

2 Main Results

Let's consider the following SISO linear discrete system.

$$Y(z) = G(z)U(z), \tag{1}$$

where $G(z)$, $Y(z)$, and $U(z)$ are the system transfer function, output variable, and input variable, respectively. We assume here that $G(z)$ is minimum phase. Now, suppose that, in this system, we are to regulate a periodic input and/or periodic disturbances whose fundamental frequencies are given by $\omega_1, \cdots \omega_n$. Here, with respect to the sampling period, there exists a sequence $\{N_{i,\ i=1,\cdots,n}\}$ such that

$$\omega_i = \frac{2\pi}{N_i T}, \quad i = 1, \cdots, n. \tag{2}$$

Prior repetitive controllers which can regulate this entire signal should contain a dead-time element z^{-N} where N is given by

$$N = LCM\{N_1, \cdots, N_n\}. \tag{3}$$

This implies that not only do prior repetitive controllers require much more memory but also it will take longer time for prior repetitive controllers to regulate the error as n is getting larger. To overcome these problems, we suggest here the following multiple repetitive controller $K(z)$, which is also depicted in Fig.1.

$$K(z) = C_2(z)\left(\sum_{i=1}^{n} K_i \frac{z^{-N_i}}{1 - z^{-N_i}}\right), \tag{4}$$

The controller $C_1(z)$ in Fig.1 is chosen so that

$$H(z) \triangleq \frac{C_1(z)G(z)}{1 + C_1(z)G(z)} \tag{5}$$

is asymptotically stable and minimum phase. The existence of such a $C_1(z)$ is obvious. Then, it is immediate that there exists an inverse function C_2 of H such that

$$C_2(z)H(z) \ = \ 1. \tag{6}$$

In practice, however, it is impossible to exactly achieve the above property because of model uncertainty and so on. That is, there exists a cancellation error $\Delta(z)$ such that

$$C_2(z)H(z) \ = \ 1 \ + \ \Delta(z). \tag{7}$$

We assume that there exists a constant ϵ such that

$$|\Delta(z)| \le \epsilon, \quad {}^{\forall}\, z \text{ such that } |z| \ge 1. \tag{8}$$

Now, we are ready to state the following theorem.

Theorem 1 *The overall closed loop system shown in Fig.1 is asymptotically stable if the control gains $K_{i,\ i=1,\cdots,n}$ satisfy the following inequalities.*

$$K_i > 0, \quad i = 1, \cdots, n, \tag{9}$$

$$\textstyle\sum_{i=1}^{n} K_i < \frac{2}{1+\epsilon}. \tag{10}$$

Proof : *It is immediate from (4), (5), (7), and Fig.1 that the overall closed loop transfer function G_c is given by*

$$G_c(z) = \frac{H(z)\left(1 + C_2(z)\sum_{i=1}^{n} K_i \frac{z^{-N_i}}{1 - z^{-N_i}}\right)}{1 + (1 + \Delta(z))\sum_{i=1}^{n} K_i \frac{z^{-N_i}}{1 - z^{-N_i}}}. \tag{11}$$

Now, we show that all zeros of the denominator of (11) are inside the unit circle $|z| = 1$. First, note from (9), *(10), and (8) that*

$$\begin{aligned}
&\min_{|z|\geq 1} Re[\sum_{i=1}^{n} K_i z^{-N_i}/1 - z^{-N_i}] \\
= &\min_{|z|\geq 1} \sum_{i=1}^{n} K_i Re[z^{-N_i}/1 - z^{-N_i}] \\
\geq & -\frac{1}{2}\min_{|z|\geq 1} \sum_{i=1}^{n} K_i \\
> & -\frac{1}{2}\frac{2}{1+\epsilon} \\
\geq & -\left|\frac{1}{1+\Delta(z)}\right|, \quad {}^{\forall}|z| \geq 1. \qquad (12)
\end{aligned}$$

This implies that

$$\frac{1}{1+\Delta(z)} + \sum_{i=1}^{n} K_i z^{-N_i}/1 - z^{-N_i} \neq 0, \quad {}^{\forall}|z| \geq 1. \quad (13)$$

Thus, all zeros of the denominator of (11) are inside the unit circle $|z| = 1$. Finally, since H is asymptotically stable, it is immediate that the overall closed loop system shown in Fig.1 is asymptotically stable. □

It is easy to see that Theorem 3.2 in [10] is a special case of our theorem. Indeed, they considered the case of $n = 1$, $\epsilon = 0$.

Now, we are in the order to show the convergence of the error.

Theorem 2 *If the overall closed loop system shown in Fig.1 is asymptotically stable, then the error $e(k)$ in Fig.1 converges asymptotically to zero.*

Proof : *The error transformation $T(z)$ is given by*

$$T(z) = \frac{1}{1 + C_1(z)G(z)} \times \frac{1}{1 + (1+\Delta(z))\sum_{i=1}^{n} K_i \frac{z^{-N_i}}{1 - z^{-N_i}}}. \quad (14)$$

Since H and G_c are asymptotically stable, it is immediate that

$$|T(z)| = 0, \quad {}^{\forall}z \text{ such that } z^{N_i} = 1. \quad (15)$$

Therefore the error $e(k)$ in Fig.1 converges asymptotically to zero. □

Note here that multiple repetitive errors are completely rejected by our multiple repetitive controller even under modeling uncertainty.

3 Numerical Examples

Consider a typical disk drive head positioning servo control system dealt with in [2] where $H(z)$ defined in (5) is given by

$$H(z) = \frac{5.01 - 14.0z^{-1} + 14.2z^{-2} - 6.0z^{-3} + 0.88z^{-4}}{1 - 0.874z^{-1} - 0.992z^{-2} + 0.882z^{-3}} \quad (16)$$

And the sampling time T is given by

$$T = 416.66\mu s. \quad (17)$$

To show the validity of the proposed multiple repetitive control system, it is assumed that errors consist of multiple dominant fundamental frequency components. Note that these multiple repetitive errors are usually observed in many practical control systems, for an instance, hard disk drive control systems where the multiple repetitive errors are primarily caused by mechanical characteristics of ball bearings in the spindle motor [1, 14]. Now, we assume that the fundamental frequencies of the considered multiple repetitive error are given as follows.

$$60Hz, \quad 96Hz, \quad 160Hz. \quad (18)$$

By this along with (2), (3), and (17), we have

$$N_1 = 40, \quad N_2 = 25, \quad N_3 = 15, \quad N = 600. \quad (19)$$

Note here that $N \gg N_1 + N_2 + N_3$. This implies that the memory size of the conventional single repetitive controller should be almost 8 times larger than that of the multiple repetitive controller.

On the other hand, we also assume that the unknown modeling uncertainty is given by

$$\Delta(s) = \frac{1 + s/2293}{1 + s/2751} - 1. \quad (20)$$

This uncertainty can cause at most 20% magnitude variation and 5.2° phase variation. The corresponding z-transformation of this transfer function is given by

$$\Delta(z) = \frac{0.902 - 0.287z^{-1}}{1 - 0.385z^{-1}} - 1. \quad (21)$$

According to Theorem 1 along with (21), K_i, $i = 1, 2, 3$ are determined as follows.

$$K_1 = K_2 = K_3 = 0.3. \quad (22)$$

Fig.2 shows the closed loop error profile obtained when we 'turn off' all the gains, that is, let all the gains be zeros. We can see the explicit effect of the considered multiple repetitive error. Fig.3 and Fig.4 show the error profiles when we 'turn on' K_1 only and then both

K_1 and K_2 only, respectively. Here, these gains are chosen to produce the best performance in each simulation. From these figures, it can be seen that each component of our multiple repetitive controller reduces respectively its corresponding error frequency as expected. Now, Fig.5 shows the error profile when all the gains are 'turned on'. It can be seen from Fig.5 that multiple repetitive errors can be effectively reduced by employing our multiple repetitive controller. On the other hand, Fig.6 shows the best error profile that can be achieved by using the conventional single repetitive controller. It is obvious from Fig.5 and 6 that the error convergence speed obtained by the multiple repetitive controller is much faster than that of the single repetitive controller case.

4 Concluding Remarks

Multiple repetitive errors were shown to be effectively reduced by employing our proposed multiple repetitive controllers. Multiple repetitive controllers were simply plugged in parallel in a feedback control system and each repetitive controller could reduce each fundamental frequency error component and its harmonics. Our proposed multiple repetitive control system can save dramatically the required memory size and reduce errors much faster than the conventional single repetitive control system. It is believed that the multiple repetitive control system can be practically and usefully applied to any linear SISO digital control system with multiple repetitive errors and/or disturbances in industry.

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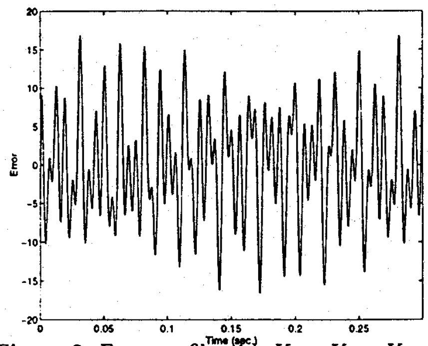

Figure 2: Error profile with $K_1 = K_2 = K_3 = 0$

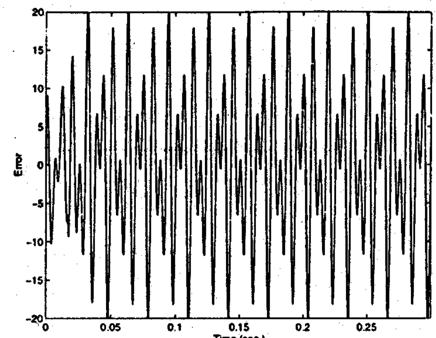

Figure 3: Error profile with $K_1 = 1,\ K_2 = K_3 = 0$

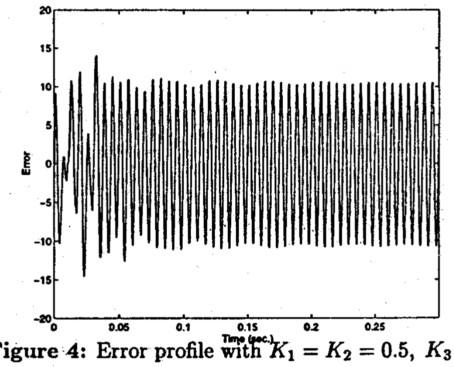

Figure 4: Error profile with $K_1 = K_2 = 0.5,\ K_3 = 0$

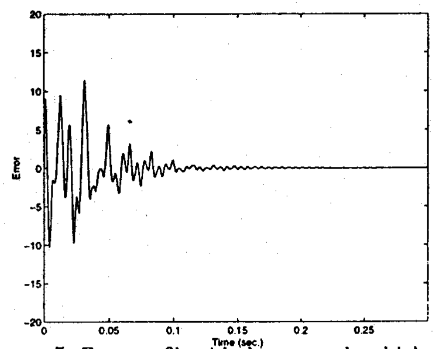

Figure 5: Error profile with the proposed multiple repetitive controller, that is, with $K_1 = K_2 = K_3 = 0.3$

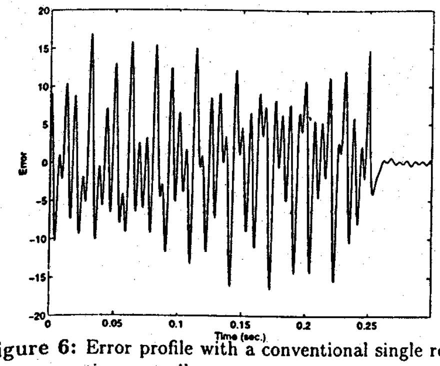

Figure 6: Error profile with a conventional single repetitive controller

References

[1] G. Bouchard, L. Lau and F.E. Talke, "An investigation of non-repeatable spindle runout," IEEE Trans. Magnetics, Mag-23, pp. 3687-3689, 1987.

[2] W. S. Chang, I. H. Suh and T. W. Kim, "Analysis and design of two types of digital repetitive control systems," Automatica, vol. 31, no. 5, pp. 741-746, 1995.

[3] K. K. Chew and M. Tomizuka, "Digital control of repetitive errors in disk drive systems," Proc. American Control Conf., pp. 540-548, 1989.

[4] K. K. Chew and M. Tomizuka, "Digital control of repetitive errors in disk drive systems," IEEE control Syst. Mag., Jan. pp. 16-20, 1989.

[5] S. Hara, Y. Yamamoto, T. Omata and M. Nakano, "Repetitive control system: a new type servo system for periodic exogenous signals," IEEE Trans. Autom. Control, AC-33, pp. 659-667, 1988.

[6] J. Hu and M. Tomizuka, "A digital segmented repetitive control algorithm," ASME Journal of Dynamic Systems, Measurement and Control, vol. 116, pp. 577-582, 1994

[7] T. Inoue, M. Nakano, T. Kubo, S. Matsumoto and H. Baba, "High accuracy control of a proton synchrotron magnet power supply," Proc. 8th IFAC World Congress, vol. 6, pp. 3137-3142, 1981.

[8] D. C. Mee and E. D. Daniel, Magnetic Recording vol. 2 Computer Data Storage, McGraw-Hiil, New York, USA, 1988.

[9] R.B. Randall, "A new method of modeling gear faults," ASME Journal of Mechanical Design, vol. 104, pp. 259-265, 1982.

[10] M. Tomizuka, T. Tsao and K. K. Chew, "Discrete time domain analysis and synthesis of repetitive controller," Proc. American Control Conf., pp. 860-866, 1988.

[11] T. Tsai, G. Anwar and M. Tomizuka, "Discrete time repetitive control for robot manipulator," Proc. IEEE International Conf. on Robotics and Automation, pp. 1341-1346, 1988.

[12] L. Guo, "Reducing Manufacturing Cost Associated with Hard Disk Drives-A New Disturbance Rejection Control Scheme," IEEE/ASME Trans. on Mechatronics, Vol. 2, No. 2, pp. 77-85, 1997.

[13] H. G. M. Dotsch and H. T. Smakman, "Adaptive Repetitive Control of a Compact Disc Mechanism," Proceedings of the 34th Conference on Decision and Control, pp. 1720-1725, 1995.

[14] L. Nagarathnam, H. Leuthold and D. Jennings, "Mechanical Vibration Caused by Ball Bearings in Spindle Motors," Proceedings of the 24th Annual Symp. Incremental Motion Control & Devices, pp. 251-255, 1995.

[15] J. A. Tenney and M. Tomizuka, "Effects of Non-Periodic Disturbances on Repetitive Control Systems," Proceedings of the 13th IFAC Triennial World Congress, pp. 7-12, 1996.

[16] G. Hillerstrom and K. Walgama, "Repetitive Control Theory and Applications-A Survey," Proceedings of the 13th IFAC Triennial World Congress, pp. 1-6, 1996.