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Optimal Design of a Five-bar Finger with Redundant Actuation

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Abstract

In order to develop a human hand mechanism, a 5-bar finger with redundant actuation is suggested. Optimal sets of actuator locations and link lengths for the cases of minimum actuator, one, two, and three redundant actuators are obtained by employing a composite design index which simultaneously consider several performance indices such as workspace, isotropic index, and force transmission ratio. Eventually, several finger-configurations optimized for special performance indices are illustrated.

I. Introduction

Robot hands have been employed for fine motion control and assembling parts. Most of existing robot hands employ tendon-driven power transmission. However, frictions existing in the transmission line require more effort on control. In light of this fact, we propose a five-bar finger mechanism which is directly driven by ultra-sonic motor at joints of the mechanism. Since the five-bar finger mechanism has many potential joint locations for attaching actuators, redundant actuation mode can be achieved[1-2]. Redundant actuation prevails in general biomechanical systems, such as the human body, the bodies of mammals and insects. Redundant actuation can be also found in many robotic applications. They includes multiple arms, dual arms, multi-fingered hands, walking machines, and so on[4-6].

Redundant actuation can be easily explained in terms of mobility. When mobility of a system is greater than the degree-of-freedom, the system is called "a kinematically redundant system". On the other hand, when the number of actuators is greater than the mobility (this situation usually happens in a closed-chain system), the system is called "*redundantly actuated system*". For example, the mobility of the human upper-extremity (arm) can be considered as 7, while it has 29 human actuators (i.e, muscles)[9]. Accordingly, it has 22 redundant actuator.

The purpose of this paper is the optimum design and development of a five-bar finger employing redundant motors. Section 2 introduces the kinematic modeling for a five-bar finger. Optimal design for the five-bar finger is treated in section 3. According to the optimization result, a five-bar finger with two redundant actuators has been developed and explained in section 4. Finally, we draw conclusion.

2. Kinematic Modeling

2.1 Open-chain kinematics

Consider a 5-bar finger mechanism shown in Fig. 1. This system has one closed-kinematic chain. The closed-kinematic chain is formed by connecting the two open-chains at the given location of the second link of the left open-chain, as shown in Fig. 1. In order to enlarge the area encompassed by the finger, the folded-in configuration of the right open-chain is chosen. Since two chains of the 5-bar mechanism have a common kinematic relation at the end-point of the system, the components of the end-point vector $\boldsymbol{u}$ are described by

$$x = l_1 c_1 + l_2 c_{12} = l_3 c_3 + l_4 c_{34} + l_5 c_{345}, \qquad (1)$$

$$y = l_1 s_1 + l_2 s_{12} = l_3 s_3 + l_4 s_{34} + l_5 s_{345}, \qquad (2)$$

and

$$\Phi = \theta_1 + \theta_2 = \theta_3 + \theta_4 + \theta_5. \qquad (3)$$

Adopting the standard Jacobian representation for the velocity of a vector of N dependent (output) parameters $\boldsymbol{u}$ in terms of a set of P independent input coordinates ${}_r\dot{\boldsymbol{\phi}}$ of *rth* open-chain, one has

$$\dot{\boldsymbol{u}} = [\,{}_rG^u_\phi\,]\ {}_r\dot{\boldsymbol{\phi}}_a. \qquad (4)$$

Here,

$$[\,{}_rG^u_\phi\,] = [\,\frac{\partial u}{\partial\,{}_r\phi_1}, \frac{\partial u}{\partial\,{}_r\phi_2}, \cdots, \frac{\partial u}{\partial\,{}_r\phi_P}\,] \qquad (5)$$

is the Jacobian relating the coordinates $\boldsymbol{u}$ to ${}_r\dot{\boldsymbol{\phi}}$, and is of dimension of $N \times P$, with the *mth* column being of dimension of $N \times 1$. Jacobians of the first and second open-chain, respectively, are given by

$$[{}_1G_\theta^u] = \begin{bmatrix} -(l_1s_1+l_2s_{12}) & -(l_2s_{12}) \\ (l_1c_1+l_2c_{12}) & (l_2c_{12}) \\ 1 & 1 \end{bmatrix}, \quad (6)$$

and

$$[{}_2G_\theta^u] = \begin{bmatrix} -l_3s_3-l_4s_{34}-l_5s_{345} & -l_4s_{34}-l_5s_{345} & -l_5s_{345} \\ l_3c_3+l_4c_{34}+l_5c_{345} & l_4c_{34}+l_5c_{345} & l_5c_{345} \\ 1 & 1 & 1 \end{bmatrix} \quad (7)$$

2.2 Internal kinematics for 5-bar Finger mechanism

Since the mobility of this mechanism is two, at least two actuators are required to control the mechanism. There exist several choices in the selection of independent joints (i.e., actuator locations). In general, the base joints have been chosen as the actuator locations in previously developed 5-bar systems, primarily to minimize the dynamic effect due to floating actuators. However, from a kinematic point of view, inclusion of one or two floating actuators may be promising. For example, a better manipulability, isotropy, or load handling capacity can be achieved by using a certain floating actuator[1]. An internal kinematic relationship between dependent joints and independent joints is required to deal with the problem addressed in the above.

The equivalent velocity relation is given by

$$\dot{\boldsymbol{u}} = [{}_1G_\theta^u]\,{}_1\dot{\theta} = [{}_2G_\theta^u]\,{}_2\dot{\theta}. \quad (8)$$

Choosing the joints θ_1 and θ_3 as the independent joints($\boldsymbol{\theta}_a$) and the joints θ_2, θ_4, and θ_5 as the dependent joints ($\boldsymbol{\theta}_p$), Eq. (8) can be rearranged according to the following form

$$[A]\,\dot{\boldsymbol{\theta}}_p = [B]\,\dot{\boldsymbol{\theta}}_a \quad (9)$$

where

$$[A] = [-[{}_1G_\theta^u]_{;2}\ [{}_2G_\theta^u]_{;2,3}], \quad (10)$$

$$[B] = [[{}_1G_\theta^u]_{;1}\ -[{}_2G_\theta^u]_{;1}], \quad (11)$$

$$\dot{\boldsymbol{\theta}}_p = (\dot{\theta}_3\ \dot{\theta}_4\ \dot{\theta}_5)^T, \quad (12)$$

and

$$\dot{\boldsymbol{\theta}}_a = (\dot{\theta}_1\ \dot{\theta}_2)^T. \quad (13)$$

Now, premultiplying the inverse of the matrix [A] to both sides of Eq. (8) yields

$$\dot{\boldsymbol{\theta}}_p = [G_a^p]\,\dot{\boldsymbol{\theta}}_a, \quad (14)$$

where $[G_a^p]$ denotes the first-order *KIC* matrix relating $\boldsymbol{\theta}_p$ to $\boldsymbol{\theta}_a$.

According to the duality existing between the velocity vector and force vector, the force relation between the independent joints and the dependent joints is described by

$$\boldsymbol{T}_a = [G_a^p]^T\boldsymbol{T}_p. \quad (15)$$

Then, the effective load referenced to the independent joints is given by

$$\boldsymbol{T}_a^* = \boldsymbol{T}_a + [G_a^p]^T\boldsymbol{T}_p = [G_a^\phi]^T\boldsymbol{T}_\phi \quad (16)$$

where

$$[G_a^\phi] = \begin{bmatrix} I \\ [G_a^p] \end{bmatrix}, \quad (17)$$

$$\boldsymbol{T}_a = (T_1\ T_2)^T. \quad (18)$$

In Eq. (15), $\boldsymbol{T}_\phi$ denotes a force vector consisting of $\boldsymbol{T}_a$ and the whole set or subset of the joint torque at the dependent joints.

2.3 Forward Kinematics for 5-bar mechanism

Since the joints(${}_r\boldsymbol{\phi}$) of the *rth* chain is composed of some of the independent and dependent joints, ${}_r\dot{\boldsymbol{\phi}}$ can be expressed in terms of the independent joints by

$${}_r\dot{\boldsymbol{\phi}} = [{}^rG_a^\phi]\,\dot{\boldsymbol{\phi}}_a \quad (19)$$

where the matrix $[{}^rG_a^\phi]$ is formed using elements of $[G_a^p]$ augmented with a unity in the *ith* row and *jth* column and with zeros in all other elements of the *ith* row if ${}_r\phi_i = \phi_{a_j}$. Thus, the forward kinematics for the common object is obtained by embedding the first-order internal KIC into one of the *rth* pseudo open-chain kinematic expressions as follows :

$$\dot{\boldsymbol{u}} = [{}_rG_\phi^u]\,{}_r\dot{\boldsymbol{\phi}} = [G_a^u]\,\dot{\boldsymbol{\phi}}_a, \quad (20)$$

where the forward Jacobian is determined by

$$[G_a^u] = [{}_rG_\phi^u][{}^rG_a^\phi]. \quad (21)$$

3. Kinematic Optimal Design for Five-Bar Finger with Redundant Actuator

3.1 Optimization Methodology

To deal with a nonlinear optimization with constrains, three numerical methods are used. The exterior penalty function method is employed to transform the constrained optimization problem into an unconstrained optimal problem. Powell's method is applied to obtain an optimal solution for the unconstrained problem, and quadratic interpolation method is utilized for uni-directional minimization[3].

3.2 Kinematic Design Indices

Based on the effective force relationship between the operational force vector and the input force vector, the ratio of the 2-norm of the output load to that of the input load can be expressed as

$$\frac{\| T_u \|}{\| T_\phi \|} = \left\{ \frac{T_\phi^{\ T}[G_u^\phi][G_u^\phi]^T T_\phi}{T_\phi^{\ T} T_\phi} \right\}^{\frac{1}{2}}, \tag{22}$$

where $\|T_\phi\|$ and $\|T_u\|$ are defined as

$$\|T_\phi\|^2 = T_\phi^{\ T} T_\phi, \tag{23}$$

$$\|T_u\|^2 = T_u^{\ T} T_u. \tag{24}$$

Based on the Rayleigh quotient, the output bounds with respect to the input loads are given as

$$\sigma_{\min} \| T_\phi \| \le \| T_u \| \le \sigma_{\max} \| T_\phi \|, \tag{25}$$

where $\sigma_{\min}$ and $\sigma_{\max}$ are the square root of minimum and maximum singular values of $[G_u^\phi][G_u^\phi]^T$, respectively. Since the nonzero eigenvalues of $[G_u^\phi]^T[G_u^\phi]$ is the same as those of $[G_u^\phi][G_u^\phi]^T$, the nonzero eigenvalues are obtained in terms of $[G_u^\phi]^T[G_u^\phi]$, and these singular values are used in determining the bounds of the force transmission ratio. An alternative expression of Eq. (25) is

$$\frac{1}{\sigma_{\max}} \le \frac{\| T_\phi \|}{\| T_u \|} \le \frac{1}{\sigma_{\min}}, \tag{26}$$

where $\sigma_F(=\frac{1}{\sigma_{\min}})$ is defined as the maximum force transmission ratios (actuator capacities for an unit operational load of $\|T_u\|$.

3.2.1 Single design index

The operating region or workspace of the five-bar finger will be characterized by a reachable workspace. Also, a manipulator should be designed so that it has well-conditioned workspace which allows its end-effector to move from one regular value to another without passing through a critical value (singularity). An isotropic index is a criterion to measure such phenomenon. The isotropic index, σ_I, is defined as

$$\sigma_I = \frac{\sigma_{\min}}{\sigma_{\max}}, \tag{27}$$

The global isotropic index is defined with respect to the entire workspace of the manipulator as

$$\Sigma_I = \frac{\int_W \sigma_I \, dW}{W}, \tag{28}$$

where the workspace of manipulators is denoted as

$$W = \int_W dW. \tag{29}$$

Maximum force transmission ratio is defined as the required actuator capacity for an unit operational load of $\|T_u\|$. The global maximum force transmission ratio is defined with respect to the entire workspace of the manipulator as

$$\Sigma_F = \frac{\int_W \sigma_F \, dW}{\int_W dW}. \tag{30}$$

The design of a manipulator system can be based on any particular criterion. However, the single criterion-based design does not provide sufficient control on the range of the design parameters involved. Therefore, multi-criteria based design has been proposed[8]. However, the previous multi-criteria methods such as weighed sum did not provide any systematic design procedure and flexibility in design. To consider these facts, we employ a composite design index[8].

3.2.2 Composite design index

As an initial step to this process, preferential information should be given to each design parameter and each design index. Then, each design index is transferred to common preference design domain which ranges from zero to one. Here, the preference given to each design criterion is very subjective to the designer. Preference can be given to each criterion by weighting. This provides flexibility in design. For σ_I, the best preference is given the minimum value, and the least preference is given the maximum value of the criterion. Then, the design index is transferred into common preference design domain as below

$$\tilde{\Sigma}_I = \frac{\Sigma_I - \Sigma_{I\min}}{\Sigma_{I\max} - \Sigma_{I\min}}, \tag{31}$$

where '$\sim$' implies that the index is transferred into the common preference design domain. Since workspace is also in favor of maximum value, the design index transferred into common preference design domain is given as

$$\tilde{W} = \frac{W - W_{\min}}{W_{\max} - W_{\min}}. \tag{32}$$

On the other hand, force transmission ratio is in favor of minimum value, the design index transferred into common preference design domain is given as

$$\tilde{\Sigma}_F = \frac{\Sigma_{F\max} - \Sigma_F}{\Sigma_{F\max} - \Sigma_{F\min}}. \tag{33}$$

Note that each composite design index is constructed such that a large value represents a better design. A set of optimal design parameters is obtained based on max-min principle[7]. Initially the minimum values among the design indices for all set of design parameters are obtained, and then a set of design parameters, which has the maximum of the minimum values, is chosen as the optimal set of design parameters. Based on this principle, the composite global design index(*CGDI*) is defined as the minimum value of the above mentioned design indices at a set of design parameters, and given as

$$CGDI = \min\{ \tilde{W}^{\alpha},\ \tilde{\Sigma}_I{}^{\beta},\ \tilde{\Sigma}_F{}^{\gamma} \}. \quad (34)$$

The upper Greek letters (α, β, etc) represent the degree of weighting, and usually large value implies large weighting, and usually large value implies large weighting. Now, a set of optimal design parameters is chosen as the set that has the maximum *CGDI* among all *CGDI*'s calculated for all set of design parameters.

3.3 Kinematic Optimization

The link lengths and the base width of the five-bar mechanism can be cited as kinematic design parameters. Initially, we assume that the workspace of the five-bar mechanism is the first quadrant of the x-y plane. That is,

$$0.01m \le x,\ y \le 0.3m. \quad (35)$$

Now, kinematic constraints associated with these parameters are given as

$$l_1 + l_2 = 0.3m, \quad (36)$$

$$l_3,\ l_4 \ge 0.07m\ ,\ l_5 \ge 0.02m, \quad (37)$$

where the sum of l_1 and l_2 are decided based on the range of the workspace. Also, l_3 and l_4 should be greater than the minimum link length which is decided based on the size of the transmission system embedded inside the link. l_5 requires a minimum length to attach a finger-tip at the end of the link.

Kinematic optimization for the five-bar mechanism has been performed for the case of $\alpha = 1$, $\beta = 4$, $\gamma = 1$ in which a large weighting is given the isotropic index, and for the case of $\alpha = 1$, $\beta = 1$, $\gamma = 4$ in which a large weighting is given the maximum force transmission ratio. In Table 1, simulation result for the case of minimum actuation is shown. Characteristics of the kinematic design indices resulting from the optimization procedure have been improved in comparison to those of non-optimized case in which all link lengths are chosen as unit length. Similar to Table 1, Table 2, Table 3, and Table 4 illustrate the simulation results for the case of one, two, and three redundant actuation, respectively. We can conclude that for minimum actuation case, actuation of the first and fourth joints(here, we denote it as 14) has best performance in both isotropic and maximum force transmission characteristics, and that for one redundant actuation case (i.e., three actuators), actuation of the first, fourth, and fifth joints(here, we denote it as 145) has the best performance in both characteristics, and that for two redundant actuation case (i.e., four actuators), actuation of the first, third, fourth, and fifth joints(here, we denote it as 1345) has the best performance in both characteristics. As the number of actuators increase, characteristics of kinematic isotropy and maximum force transmission ratio are enhanced except the case of full actuation(i.e., three redundant actuation) in which only the force transmission ratio is improved a little bit, while the isotropic characteristic deteriorates. Figure 2 illustrates optimal five-bar configurations for 14, 145, and 1345. The black dots denote the positions of actuators. Figure 3 and 4 represent the kinematic isotropic index for optimized and nonoptimized cases, respectively, and then Figure 5 and 6 represent the maximum force transmission ratio for optimized and nonoptimized cases, respectively. As expected, optimization results in reduction of maximum force transmission ratio and improvement of the kinematic isotropy throughout the workspace.

Though both kinematic isotropy and maximum force transmission ratio are considered in the above, maximum force transmission ratio is believed to be much important factor than kinematic isotropy and workspace because fingers in multi-fingerd hands usually require large payload, and is operated in a small workspace. Specifically, the value of force transmission ratio for 145 joints has been reduced as much as 15.3 percents of that of 14 joints, and the value of force transmission ratio for 1345 joints has been reduced as much as 37.3 percents of that of 134 joints, and the value of force transmission ratio for full actuation(i.e., 12345 joints) has been reduced much smaller that the two previous cases. Conclusively, two redundant actuation is suggested to enhance the force transmission ratio of the five-bar mechanism.

4. Development of Five-Bar Finger with Redundant Actuator

4.1 Structure of Five-Bar Finger

Figure 7 shows the prototype of the five-bar mechanism. According to the optimization result, four actuators are placed to 1345 joints. Each joint of the finger is driven by a compact actuator mechanism having ultrasonic motor and a gear set with potentiometer, and the system is controlled by VME Bus-based Control system. The ultra-sonic motors have high torque/size ratio as compared to DC motor with a similar size. The quantitative specifications of the ultrasonic motor are shown in Table 5. A gear transmission having about 15:1 speed reduction ratio is employed. Particularly, the gear transmission consisting of series of spur gears and the potentiometer, as shown in Fig. 8, are embedded inside the link, which yields compact and modular design of the finger mechanism.

5. Conclusions

In this paper, we proposed employment of redundant actuation in finger design on the purpose of enhancing the kinematic isotropic characteristic and maximum force transmission ratio of the finger mechanism. Using the concept of composite design index which allows multi-purpose and multi-variable optimization, optimal sets of actuator locations and link lengths for the cases of using minimum number of actuators, one-, two-, and three-redundant actuators are obtained. Three design indices such as workspace, isotropic index, and force transmission ratio were simultaneously optimized with consideration of their relative weighting factors. Eventually, several finger-configurations optimized for special performance index are suggested. Future work involves experimental work associated with internal force control[1,2] and development of a three-fingered hand made of five-bar finger mechanism.

Acknowledgment

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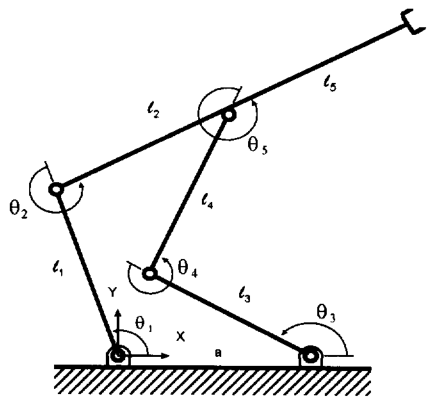

Figure 1. Five-bar Finger Mechanism

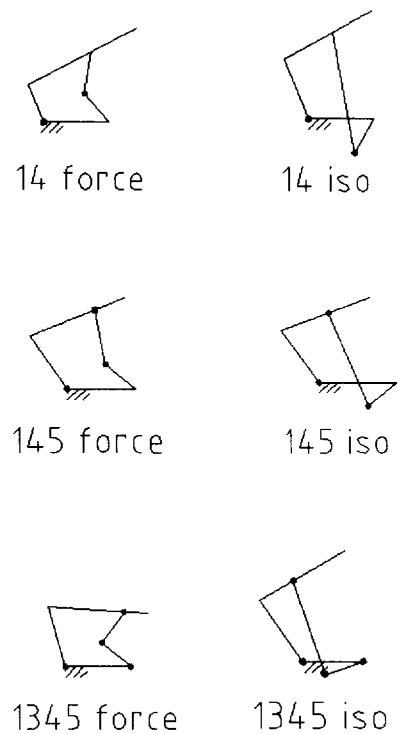

Figure 2. Optimal Finger Configurations

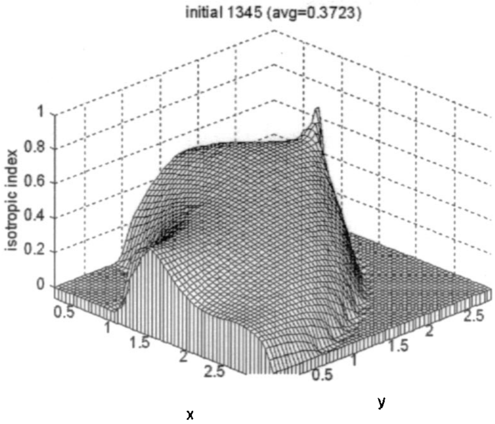

Figure 3. Kinematic Isotropic Index
(All links lengths are given unity)

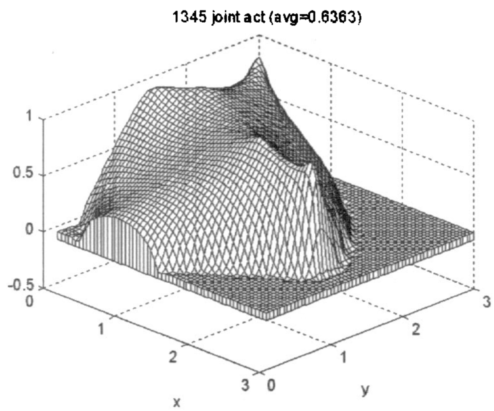

Figure 4. Kinematic Isotropic Index After Optimization

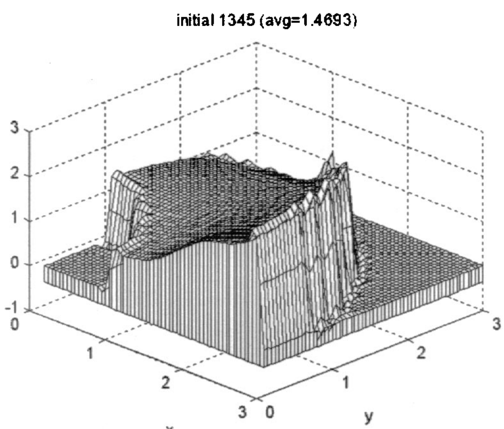

Figure 5. Max. Force Transmission Ratio
(All links lengths are given unity)

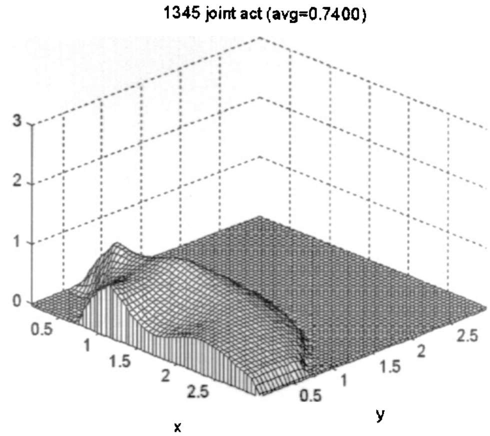

Figure 6. Max. Force Transmission Ratio After Optimization

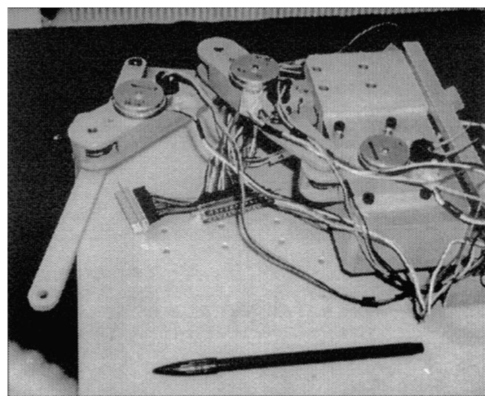

Figure 7. Proto-type of Five-Bar Finger Mechanism

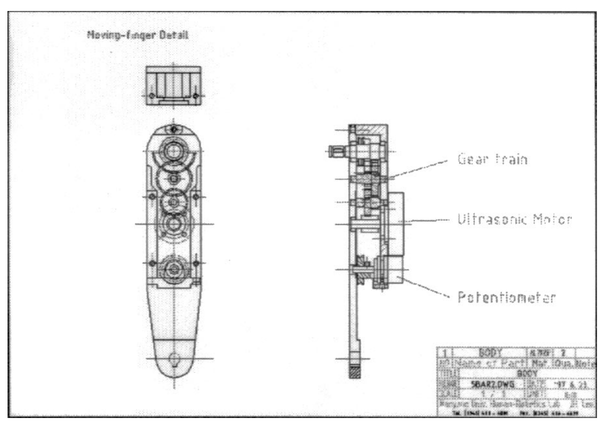

Figure 8. Gear Transmission System

Table 1. Optimization Result for Minimum Actuation

actuation	case	W_{area}	Σ_I	Σ_F
12	initial case	5.76	0.1970	2.8904
	isotropic opt.	4.39	0.5454	1.9072
	force trans. opt.	3.18	0.5054	1.8774
13	initial case	5.76	0.1757	4.5294
	isotropic opt.	5.19	0.4990	2.2001
	force trans. opt.	3.10	0.4328	1.9885
14	initial case	5.76	0.4468	1.9720
	isotropic opt.	2.76	0.6676	1.2828
	force trans. opt.	3.17	0.4759	1.1810
15	initial case	5.76	0.1325	6.5652
	isotropic opt.	4.53	0.6258	2.5587
	force trans. opt.	4.67	0.4989	1.9974
23	initial case	5.76	0.0620	7.9667
	isotropic opt.	2.59	0.5179	1.8201
	force trans. opt.	3.53	0.4959	1.7580
24	initial case	5.76	0.3508	2.3534
	isotropic opt.	6.09	0.5368	1.8513
	force trans. opt.	5.19	0.5230	1.6325
25	initial case	5.76	0.1424	5.2939
	isotropic opt.	3.27	0.5544	1.5538
	force trans. opt.	3.57	0.5029	1.5507
34	initial case	5.76	0.4129	2.2236
	isotropic opt.	5.12	0.6967	1.9118
	force trans. opt.	5.50	0.5583	1.5601
35	initial case	5.76	0.1109	7.5883
	isotropic opt.	5.98	0.6155	2.1639
	force trans. opt.	5.58	0.4624	2.0856
45	initial case	5.76	0.4373	2.5583
	isotropic opt.	6.12	0.6058	1.5071
	force trans. opt.	5.33	0.5869	1.3651

Table 2. Optimization Result for One-Redundant Actuation

actuation	case	W_{area}	Σ_I	Σ_F
123	initial case	5.76	0.2002	2.4400
	isotropic opt.	4.92	0.5775	1.5406
	force trans. opt.	2.49	0.5014	1.4832
124	initial case	5.76	0.3666	1.5429
	isotropic opt.	6.43	0.5752	1.3838
	force trans. opt.	3.70	0.5432	1.2076
125	initial case	5.76	0.2171	2.3956
	isotropic opt.	3.24	0.5762	1.5985
	force trans. opt.	4.14	0.4961	1.5166
134	initial case	5.76	0.3920	1.5632
	isotropic opt.	2.78	0.5713	1.2562
	force trans. opt.	4.30	0.4598	1.1634
135	initial case	5.76	0.1874	4.2490
	isotropic opt.	1.99	0.5838	1.4320
	force trans. opt.	5.42	0.5794	1.2906
145	initial case	5.76	0.4243	1.5554
	isotropic opt.	4.75	0.6471	1.0702
	force trans. opt.	4.40	0.5812	1.0008
234	initial case	5.76	0.3052	1.9618
	isotropic opt.	6.05	0.5172	1.5416
	force trans. opt.	4.07	0.4888	1.2885
235	initial case	5.76	0.1243	5.0690
	isotropic opt.	1.79	0.4403	1.4665
	force trans. opt.	3.50	0.4011	1.2577
245	initial case	5.76	0.3475	1.7242
	isotropic opt.	6.11	0.5682	1.1147
	force trans. opt.	3.55	0.5415	1.0291
345	initial case	5.76	0.3740	1.8138
	isotropic opt.	6.17	0.6560	1.3368
	force trans. opt.	4.37	0.4975	1.1462

Table3. Optimization Result for Two-Redundant Actuation

actuation	case	W_{area}	Σ_I	Σ_F
2345	initial case	5.76	0.3034	1.6341
	isotropic opt.	4.03	0.6199	0.9745
	force trans. opt.	3.09	0.5088	0.7488
1345	initial case	5.76	0.3723	1.4693
	isotropic opt.	5.66	0.6363	1.0538
	force trans. opt.	3.13	0.5318	0.7400
1245	initial case	5.76	0.3597	1.3759
	isotropic opt.	4.57	0.5715	0.8527
	force trans. opt.	3.07	0.5416	0.7562
1235	initial case	5.76	0.2091	2.1979
	isotropic opt.	1.78	0.4913	1.2516
	force trans. opt.	4.56	0.4495	1.2347
1234	initial case	5.76	0.3336	1.4096
	isotropic opt.	4.44	0.6219	1.0100
	force trans. opt.	2.74	0.4566	0.8467

Table 4. Optimization Result for Three-Redundant Actuation

actuation	case	W_{area}	Σ_I	Σ_F
12345	initial case	5.76	0.3277	1.3235
	isotropic opt.	5.26	0.6397	0.8204
	force trans. opt.	3.29	0.5289	0.6996

Table 5. Specifications of Ultra-sonic Motor

Driving Frequency	50KHz
Drive Voltage	110Vrms
Maximum Torque	0.1Nm(1Kgf · cm)
Rated Speed	250rpm
Weight	20g