# Robust Internal Loop Compensator Design for Motion Control of Precision Linear Motor

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***Abstract***— **We propose a robust internal loop compensator and its optimal design method based on $\mathcal{H}_\infty$ control. The controller consists of two parts, internal and external loop. Internal loop is used as a compensator for canceling disturbances including difference between reference model and real system, and external loop is designed to meet the performance criterion. The property of the internal loop compensator is compared with that of disturbance observer. Different from the disturbance observer, controller gains and $Q$ filter can be systematically designed, instead of heuristically selected, in the proposed internal loop compensator structure. The proposed controller structure of internal and external loop is characterized by compensator design using reference model, insensitivity of the compensated system to modeling inaccuracies and unknown external disturbances, and satisfaction of desired performance specifications. The performance of proposed controller is demonstrated by experiments of a twin-servo mechanism using two brushless DC linear servo motors.**

## I. Introduction

Recently, high speed and high accuracy motion control is one of the most interesting research field as the products become smaller and faster such as data storage or semi-conducting devices. To meet these high performance specifications, conventional optimal control methods such as linear quadratic(LQ) control have been widely used. However, mathematical description of the system can not represent real plant exactly, and unpredictable disturbances affect the performance if they are not properly attenuated.

This paper focuses on disturbance estimation and attenuation scheme in order to guarantee desired performance specifications. Various methods for this purpose have been proposed. Time delay control[1, 2] based on direct estimation of disturbance using time delay, and disturbance observer[3, 4] which makes the behavior of real system as that of given nominal system in low frequency region using low pass filter are good examples. But these methods are not so generic and have not optimality. Adaptive robust control[5] was proposed to preserve the advantages of both adaptive control and robust control based on sliding mode concept.

In this paper, we propose a robust controller synthesis method. The objective is to design robust compensator for disturbance attenuation and to provide optimization concept in this compensator design procedure. Therefore the controller has two independent control loop, namely, internal and external loop. The internal loop is used as a compensator rather than a controller, so we call this *internal loop compensator* and external loop is used as a controller, we call this *external loop controller*. Internal loop compensator is for the rejection of uncertain disturbances, and then controllers of advanced motion planning can be integrated in the external loop after the internal loop design. So their characteristics can be designed independently to meet each objective.

In the next section, robust internal loop compensator and optimization method based on $\mathcal{H}_\infty$ control is proposed and compared with conventional disturbance observer. In section III, we deal with the design of external loop controller using twin-servo system with two brushless DC linear motors. Experimental results of the system are shown in section IV, and conclusion follows.

## II. Robust Internal Loop Compensator Design and Optimization

The most important objective of a control system is to achieve certain performance specifications as well as providing stability of the system. Generally, conventional controller is determined by only one controller, so it is difficult to meet these two design requirements at the same time. In comparison with this, the well known 2-degree-of-freedom(DOF) controller has the feature that one can design the command input response and the closed loop characteristics independently[6]. In this section, we propose a robust internal loop compensator based on model following scheme with 2-DOF control structure, and we focus on optimal design method of the compensator in $\mathcal{H}_\infty$ control framework.

Firstly, we consider multivariable system described by the following dynamic equation

$$\text{Plant } P: \quad \begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \\ \boldsymbol{y} &= \boldsymbol{C}\boldsymbol{x} \end{aligned} \tag{1}$$

$$\text{Model } P_m: \quad \begin{aligned} \dot{\boldsymbol{x}}_m &= \boldsymbol{A}_m\boldsymbol{x}_m + \boldsymbol{B}_m\boldsymbol{u}_m \\ \boldsymbol{y}_m &= \boldsymbol{C}_m\boldsymbol{x}_m \end{aligned} \tag{2}$$

where $\boldsymbol{x} \in \mathbb{R}^n$ is plant state, $\boldsymbol{x}_m \in \mathbb{R}^n$ is model state, $\boldsymbol{u} \in \mathbb{R}^r$ is plant input, $\boldsymbol{u}_m \in \mathbb{R}^r$ is model input, $\boldsymbol{y} \in \mathbb{R}^r$ is plant output, $\boldsymbol{y}_m \in \mathbb{R}^r$ is model output, and $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{A}_m$, $\boldsymbol{B}_m$, $\boldsymbol{C}_m$ are matrices of appropriate dimension. Without loss of generality, we assume that $(\boldsymbol{A}, \boldsymbol{B})$ is controllable and $(\boldsymbol{C}, \boldsymbol{A})$ is observable. The reference model has piecewise continuous and uniformly bounded input $\boldsymbol{u}_m$ and state $\boldsymbol{x}_m$.

To analyze stability and performance, we define the model error $\boldsymbol{e}_m$ as the difference between the reference model state and the plant state,

$$\boldsymbol{e}_m = \boldsymbol{x}_m - \boldsymbol{x}. \tag{3}$$

From (1), (2), we can get the following error dynamic equation

$$\dot{\boldsymbol{e}}_m = \boldsymbol{A}_m\boldsymbol{x}_m + \boldsymbol{B}_m\boldsymbol{u}_m - (\boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}). \tag{4}$$

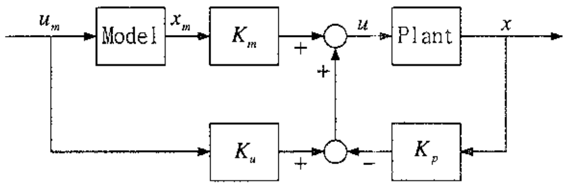



Fig. 1. Model Following Control System

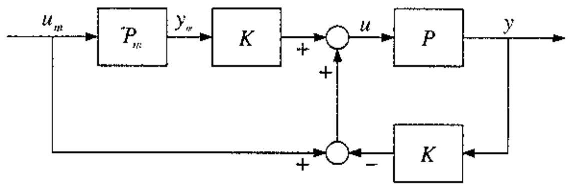



Fig. 2. RIC Structure Based on Model Following Controller

Now, we use model following controller[7] in Fig. 1, and define control action $\boldsymbol{u}$ that has the following form:

$$\boldsymbol{u} = \boldsymbol{K}_u\boldsymbol{u}_m + \boldsymbol{K}_m\boldsymbol{x}_m - \boldsymbol{K}_p\boldsymbol{x}. \tag{5}$$

Then we can get the equation of error $\boldsymbol{e}_m$ as follows

$$\begin{aligned}\dot{\boldsymbol{e}}_m &= (\boldsymbol{A}_m - \boldsymbol{B}\boldsymbol{K}_m)\boldsymbol{e}_m + (\boldsymbol{B}_m - \boldsymbol{B}\boldsymbol{K}_u)\boldsymbol{u}_m \\ &+ \{\boldsymbol{A}_m - \boldsymbol{A} + \boldsymbol{B}(\boldsymbol{K}_p - \boldsymbol{K}_m)\}\,\boldsymbol{x}.\end{aligned} \tag{6}$$

To achieve perfect model following, following equations have to be satisfied:

$$\begin{aligned}\boldsymbol{K}_m - \boldsymbol{K}_p &= \boldsymbol{B}^+(\boldsymbol{A}_m - \boldsymbol{A}), \\ \boldsymbol{K}_u &= \boldsymbol{B}^+\boldsymbol{B}_m.\end{aligned} \tag{7}$$

Therefore sufficient conditions for perfect model following are obtained as

$$\begin{aligned}(\boldsymbol{I} - \boldsymbol{B}\boldsymbol{B}^+)(\boldsymbol{A}_m - \boldsymbol{A}) &= 0, \\ (\boldsymbol{I} - \boldsymbol{B}\boldsymbol{B}^+)\boldsymbol{B}_m &= 0.\end{aligned} \tag{8}$$

(8) holds if the matrix $(\boldsymbol{I} - \boldsymbol{B}\boldsymbol{B}^+)$ is orthogonal to $(\boldsymbol{A}_m - \boldsymbol{A})$ and to $\boldsymbol{B}_m$. If reference model and the plant have a Luenberger controllable canonical form, then this property is always satisfied[7, 8] and perfect model following is achieved.

### *A. Robust Internal Loop Compensator Design*

In most control problem, however, (8) can not be satisfied because we cannot accurately describe uncertainties and parameter variations of plant in real environments. Consider the following multivariable dynamic system with parametric uncertainty

$$\begin{aligned}\dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} + \triangle\boldsymbol{A}(\boldsymbol{x}) + \triangle\boldsymbol{B}(\boldsymbol{u}), \\ \boldsymbol{y} &= \boldsymbol{C}\boldsymbol{x} + \triangle\boldsymbol{C}(\boldsymbol{x}),\end{aligned} \tag{9}$$

where $\triangle\boldsymbol{A}(\boldsymbol{x})$, $\triangle\boldsymbol{B}(\boldsymbol{u})$ and $\triangle\boldsymbol{C}(\boldsymbol{x})$ are nonlinear time-varying parametric uncertainties with the following known upper norm bounds.

$$\begin{aligned}||\triangle\boldsymbol{A}(\boldsymbol{x})|| &\le \beta_1\,||\boldsymbol{x}|| \\ ||\triangle\boldsymbol{B}(\boldsymbol{u})|| &\le \beta_2\,||\boldsymbol{u}|| \\ ||\triangle\boldsymbol{C}(\boldsymbol{x})|| &\le \beta_3\,||\boldsymbol{x}||\end{aligned} \tag{10}$$

In order to design robust compensator, we firstly select gains in (5) as follows

$$\boldsymbol{K}_m = \boldsymbol{K}_p = \boldsymbol{K}, \qquad \boldsymbol{K}_u = \boldsymbol{I}. \tag{11}$$

It is sometime impractical to use full state of plant in real environments, hence we need to derive compensator equation using only plant output signal. Therefore the dynamic controller is represented as follows

$$\begin{aligned}\boldsymbol{u} &= \boldsymbol{u}_m + \boldsymbol{K}(\boldsymbol{y}_m - \boldsymbol{y}) \\ &= \boldsymbol{u}_m + \boldsymbol{K}\boldsymbol{C}_m\boldsymbol{x}_m - \boldsymbol{K}\boldsymbol{C}\boldsymbol{x} - \boldsymbol{K}\triangle\boldsymbol{C}(\boldsymbol{x}).\end{aligned} \tag{12}$$

This structure can be described in Fig. 2, and we will derive various properties of this controller structure to get robustness. This structure is called as RIC(robust internal-loop compensator) based on model following controller.

Combining (2), (9), and (12), we get the differential equation of error $\boldsymbol{e}_m$, which can be written as

$$\dot{\boldsymbol{e}}_m \triangleq \boldsymbol{E}\boldsymbol{e}_m + \boldsymbol{w}, \qquad \boldsymbol{e}_m(0) = \boldsymbol{e}_{m0}, \tag{13}$$

where

$$\begin{aligned}\boldsymbol{E} &= \boldsymbol{A}_m - \boldsymbol{B}\boldsymbol{K}\boldsymbol{C}_m, \\ \boldsymbol{w} &= (\boldsymbol{A}_m - \boldsymbol{A} - \boldsymbol{B}\boldsymbol{K}(\boldsymbol{C}_m - \boldsymbol{C}))\boldsymbol{x} + (\boldsymbol{B}_m - \boldsymbol{B})\boldsymbol{u}_m \\ &\quad - (\triangle\boldsymbol{A}(\boldsymbol{x}) + \triangle\boldsymbol{B}(\boldsymbol{u}) - \boldsymbol{B}\boldsymbol{K}\triangle\boldsymbol{C}(\boldsymbol{x})).\end{aligned}$$

From the above equation, we can get the boundedness of $\boldsymbol{w}$ using the assumption of (10)

$$||\boldsymbol{w}|| \le \eta_1\,||\boldsymbol{e}_m|| + \eta_2, \tag{14}$$

where

$$\begin{aligned}\delta_1 &= ||\boldsymbol{A}_m - \boldsymbol{A} - \boldsymbol{B}\boldsymbol{K}\boldsymbol{C}_m + \boldsymbol{B}\boldsymbol{K}\boldsymbol{C}||, \\ \delta_2 &= ||\boldsymbol{B}_m - \boldsymbol{B}||, \\ \eta_1 &= \delta_1 + \beta_1 + \beta_2\,||\boldsymbol{K}\boldsymbol{C}|| + \beta_2\beta_3\,||\boldsymbol{K}|| + \beta_3\,||\boldsymbol{B}\boldsymbol{K}||, \\ \eta_2 &= (\delta_1 + \beta_1 + \beta_2\,||\boldsymbol{K}\boldsymbol{C}|| + \beta_2\,||\boldsymbol{K}\boldsymbol{C}_m|| + \beta_2\beta_3\,||\boldsymbol{K}|| \\ &\quad + \beta_3\,||\boldsymbol{B}\boldsymbol{K}||)\,||\boldsymbol{x}_m|| + (\delta_2 + \beta_2)\,||\boldsymbol{u}_m||.\end{aligned}$$

Thus, the compensator design problem is to choose the parameters $\boldsymbol{A}_m$, $\boldsymbol{B}_m$, $\boldsymbol{C}_m$, and $\boldsymbol{K}$ in (2) and (12) such that the model error system (13) is stable, i.e., the uncertainties can be tolerated in the design and the compensator has a robustness.

*Theorem 1:* Suppose the nonlinear parametric uncertainties are bounded by (10) and if we choose the control parameters of (12) such that the nominal model error system (15) is asymptotically stable and the inequality of (16) is satisfied

$$\dot{\boldsymbol{e}}_m = \boldsymbol{E}\boldsymbol{e}_m, \tag{15}$$

$$\alpha > \mathrm{m}\,\eta_1, \tag{16}$$

where $\alpha$ is defined in Appendix, then the nonlinear parametrically perturbed error system of (13) is stable, and there exist $\alpha$, $T < \infty$ satisfying $||\boldsymbol{e}_m(t)|| \le \epsilon, \forall t \ge T$ for given allowable state error $\epsilon > 0$, i.e., the nonlinear perturbations $\triangle\boldsymbol{A}(\boldsymbol{x})$, $\triangle\boldsymbol{B}(\boldsymbol{u})$, and $\triangle\boldsymbol{C}(\boldsymbol{x})$ can be tolerated in the sense of bounded error.

*Proof.* The proof is given in Appendix. ∎

*Remark 1:* Suppose the parametric uncertainties in (9) can include the difference between real system and reference model, then we can set $\boldsymbol{A} = \boldsymbol{A}_m$, $\boldsymbol{B} = \boldsymbol{B}_m$, $\boldsymbol{C} = \boldsymbol{C}_m$, and then, $\delta_1$ and $\delta_2$ become zero.

*Remark 2:* Assume that the system of (9) is bounded input and bounded output(BIBO) stable, then $||\boldsymbol{w}|| \le \delta_c$ where $\delta_c > 0$ is some constant and we can choose $\alpha > \frac{m\delta_c}{\epsilon}$ to make stable system.

### B. Optimal RIC Design in the $\mathcal{H}_\infty$ Framework

In the previous section, we can see that the performance of internal loop compensator is determined by the $\boldsymbol{K}$. That is, if $\boldsymbol{K}$ is selected to satisfy (16) by using Theorem 1, RIC makes the output of real system track the output of reference model with allowable error. Although the high gain property in the internal structure can extend the bandwidth of control loop, this high gain has a limit by considering performance and robustness and should be optimized.

For the robustness, many disturbance compensating methods have been proposed using various concepts. Among them disturbance observer has easy conceptual basis and simple structure to real implementation, so this is being used widely in high accuracy and high speed control applications. Its main idea is that the difference between real system and nominal model is regarded as disturbance and it can estimate the magnitude of disturbance by applying inverse nominal model at the output of system. Therefore low pass filter $Q$ is appended for real implementation, the cut-off frequency of $Q$ is assigned to reject the disturbances. However the disturbance observer is weak to nonlinear disturbance because it is designed by linear control theory. Moreover the performance is affected by the shape of $Q$ and $Q$ is embedded in explicit function, then it is difficult to optimize $Q$ considering the performance and robustness criterion of given task.

In comparison with disturbance observer, the RIC has more general mechanism for the rejection of disturbances. As the $Q$ is included in the implicit function form in our formulation, the proposed structure also has similar properties of disturbance observer. $Q$ is represented by RIC parameters and its shape can be optimized by considering the characteristics of disturbance and real system. In addition to these, the limitation of disturbance observer such as performance degradation on the nonlinear disturbance can be overcome using additional adaptive or robust algorithms because our structure is using the model error defined by the difference between real system state and reference model state. In this section, we will deal with the optimization schemes of RIC.

We begin by assuming that the system can be represented by Luenberger controllable canonical form and disturbance comes through input channel. Then (1) is divided into $r$th block terms with $\mu_i$th order(Kronecker invariants) differential equation with disturbance $d_i$ as follows

$$\dot{\boldsymbol{x}}_i = \boldsymbol{A}_i\boldsymbol{x}_i + \boldsymbol{b}_i u_i + \boldsymbol{b}_i d_i, \qquad i = 1, 2, \ldots, r. \qquad (17)$$

And we design the reference model having the following form;

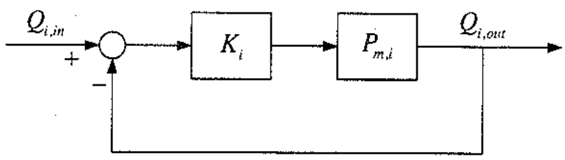
$$\begin{aligned} \boldsymbol{A}_m &= \mathrm{diag}\{\boldsymbol{A}_{m,1}, \boldsymbol{A}_{m,2}, \ldots, \boldsymbol{A}_{m,r}\}, \\ \boldsymbol{B}_m &= \mathrm{diag}\{\boldsymbol{b}_{m,1}, \boldsymbol{b}_{m,2}, \ldots, \boldsymbol{b}_{m,r}\}, \end{aligned} \qquad (18)$$

where $\boldsymbol{A}_{m,i} \in \mathbb{R}^{\mu_i \times \mu_i}(i = 1, \ldots, r)$ is in controllable canonical form and $\boldsymbol{b}_{m,i} \in \mathbb{R}^{\mu_i}$ is written as

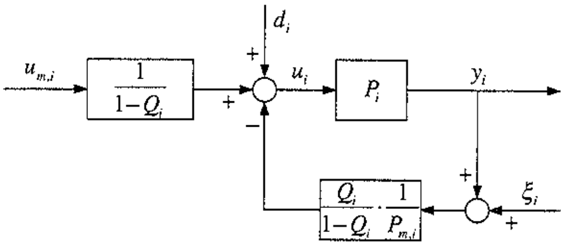
$$\boldsymbol{b}_{m,i} = [\, 0 \quad 0 \cdots \underset{\mu_i\text{th}}{1} \,]^T, \qquad (19)$$

Reference model is also divided into $rth$ SISO model

$$\dot{\boldsymbol{x}}_{m,i} = \boldsymbol{A}_{m,i}\boldsymbol{x}_{m,i} + \boldsymbol{b}_{m,i}u_{m,i}, \qquad i = 1, 2, \ldots, r, \quad (20)$$




(a) Transfer Function $Q$




(b) Equivalent Structure of RIC Using $Q$

Fig. 3. Function $Q$ and Equivalent Structure of RIC

and the controller can be designed for each system.

Using the formula (12), it is easy to show that the control input has the following representation with variable $s$ of Laplace transformation

$$\begin{aligned} u_i(s) = &\{1 + K_i(s)P_{m,i}(s)\}\, u_{m,i}(s) - K_i(s)y_i(s) \\ &+ d_i(s), \end{aligned} \qquad (21)$$

where $P_{m,i}$ is defined by

$$P_{m,i}(s) = \boldsymbol{C}_{m,i}(s\boldsymbol{I} - \boldsymbol{A}_{m,i})^{-1}\boldsymbol{b}_{m,i}.$$

Let's take a SISO example of Fig. 3(a) to help understanding of the property of this RIC compensator. We define the transfer function $Q_i$ which controls $P_{m,i}$ using feedback controller $K_i$ as shown in Fig. 3(a). $Q_i$ is described as

$$Q_i = \frac{P_{m,i}K_i}{1 + P_{m,i}K_i}. \qquad (22)$$

If we recalculate this equation to $K_i$, and plug this $K_i$ into Fig. 2, we obtain Fig. 3(b) with output sensor noise $\xi$, which is a well known structure of disturbance observer[4]. Therefore optimal RIC design problem becomes the optimization problem of $\boldsymbol{K}$ for reference model $\boldsymbol{P}_m$ because the performance is characterized by $\boldsymbol{K}$ and $\boldsymbol{P}_m$. Note also that the above structure of internal loop compensator can be expressed as a form of disturbance observer and our proposed internal compensator structure has very general characteristics although it was derived from model following concept. It is well known that the disturbance observer makes the system robust by using low pass filter $\boldsymbol{Q}$ which cuts off the external disturbances in low frequency region.

Needless to say, the optimal shape of $\boldsymbol{Q}$ is obtained based on how $\boldsymbol{P}_m$ and $\boldsymbol{K}$ are designed as shown above. Note also that this design of $\boldsymbol{Q}$ in conventional disturbance observer structure is very heuristic because in disturbance observer, it is very difficult to apply optimality concept in the design of $\boldsymbol{Q}$. However, in RIC structure, it is very simple. As mentioned previously, the RIC is determined by the reference model $\boldsymbol{P}_m$ and feedback controller $\boldsymbol{K}$ and also can be described by low-pass filter $\boldsymbol{Q}$ through (22) simultaneously. If we can

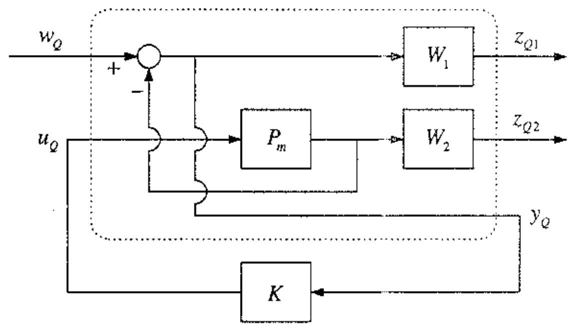



Fig. 4. Optimization of RIC by $H_\infty$ Mixed Sensitivity Method

design optimal $\boldsymbol{K}$ using the above properties of RIC, the performance can be enhanced.

From Fig. 3(a), the sensitivity $S_{Q_i}$ and complementary sensitivity $T_{Q_i}$ of $Q_i$ are obtained by

$$S_{Q_i} = \frac{1}{1 + P_{m,i}K_i}, \qquad T_{Q_i} = \frac{P_{m,i}K_i}{1 + P_{m,i}K_i}. \qquad (23)$$

We use $H_\infty$ mixed sensitivity method to determine optimal compensator gain $\boldsymbol{K}$ as shown in Fig. 4. This is another advantage of RIC structure other than robustness in that the structure accepts the optimality concept in designing $\boldsymbol{Q}$ or $\boldsymbol{K}$. In the mixed sensitivity problem formulation, nominal disturbance attenuation specifications and stability margin specification equations are combined into a single infinity norm specification. Now, the mixed $\mathcal{H}_\infty$ sensitivity problem is formulated as follows;

$$\min_{K_i} \left\| \begin{bmatrix} W_{1,i}(1 + P_{m,i}K_i)^{-1} \\ W_{2,i}P_{m,i}K_i(1 + P_{m,i}K_i)^{-1} \end{bmatrix} \right\|_\infty < 1. \qquad (24)$$

Since (24) uses reference model parameter, this equation can be solved easily and we can assign frequency characteristics of $Q_i$. The result of this design will be described in the experiment section. This concept for SISO example can be applied to the general system similarly.

*Remark 3:* To compensate nonlinear disturbances we can append additional control algorithm to (12) such that

$$\boldsymbol{u} = \boldsymbol{u}_m + \boldsymbol{K}(\boldsymbol{y}_m - \boldsymbol{y}) + \boldsymbol{F}(\boldsymbol{y}, \boldsymbol{y}_m) \qquad (25)$$

where $\boldsymbol{F}(\boldsymbol{y}, \boldsymbol{y}_m)$ is robust or adaptive algorithm to attenuate $\boldsymbol{e}_m$.

## III. Motion Controller Design for Twin-Servo Linear Motor System

After the robust internal loop compensator design, which is the main objective in this paper, we are to design external loop controller. As described in the previous section, the external controller can be easily designed using conventional advanced control schemes. This is possible due to the robust internal loop design.

The system we are dealing with in this paper is the twin-servo precision linear motor system to increase the payload capacity and speed as shown in Fig. 5.

Friction identification is the first important procedure of controller synthesis[9] to achieve precision positioning performance. However the structure itself has strong nonlinear friction characteristics, and irregular

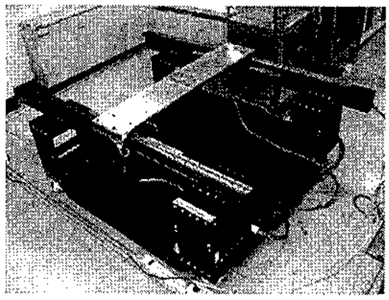


Fig. 5. Twin-Servo Precision Linear Motor System

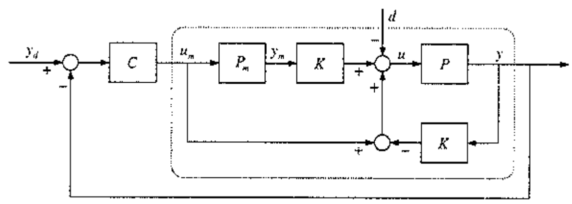



Fig. 6. Robust Motion Control Structure Based on RIC

gap between core and permanent magnet brings about parameter variation. Moreover force ripple caused by variation of magnetic force and inaccurate placement of the permanent magnet make control problem more difficult.

### A. Precision Motion Control

RIC is used to compensate different dynamic characteristics of two motors as shown in Fig. 6. In this figure the transfer function from reference input to position output is given by

$$G = \frac{C(1 + KP_m)P}{1 + PK + C(1 + KP_m)P}, \qquad (26)$$

where $C$ is an external loop feedback controller. When we express the plant having uncertainties as

$$P = P_m(1 + \triangle_P), \qquad (27)$$

then the robust stability of the above multiplicative uncertainties can be stated in the following theorem.

*Theorem 2:* If the plant is modeled with multiplicative uncertainties as shown in Eq.(27), the necessary and sufficient condition for stability of the closed loop system for the uncertainty $\triangle_P$ is

$$\|\triangle_P\| < \left\| \frac{1 + KP_m + C(1 + KP_m)P_m}{KP_m + C(1 + KP_m)P_m} \right\|. \qquad (28)$$

*Proof.* The proof can be easily derived, see Doyle *et al.*[11]. ∎

From Theorem 2, we can design external loop controller $C$ to stabilize two linear motor systems. To satisfy performance specifications, $C$ is designed based on reference model after the system is compensated by RIC.

## IV. Experimental Results

The proposed robust internal loop compensator algorithm is experimented with external control loop. Dynamic equation of the system is written as a second

TABLE I
PARAMETERS OF TWIN-SERVO SYSTEM

| Parameter | M | B |
|---|---|---|
| $X_1$ | 0.55 | 0.45 |
| $X_2$ | 0.4 | 0.3 |

order differential equation as

$$M\ddot{x} + u_{fric}(x, \dot{x}) + u_{ripple}(x) + d = u \qquad (29)$$

with inertia $M$, friction force $u_{fric}$, force ripple term $u_{ripple}$, position variable $x$, the control force $u$, and disturbance $d$. This equation includes motor actuator dynamics, and $u$ is an input voltage. Friction is described by linear viscous friction and nonlinear term as

$$u_{fric}(x, \dot{x}) = B\dot{x} + \bar{u}_{fric}(x, \dot{x}), \qquad (30)$$

where $B$ is viscous damping coefficient and $\bar{u}_{fric}$ is nonlinear friction force term as a function of position and velocity. Table I shows the estimated parameter value of inertia and damping coefficients of twin-servo motor system.

Firstly, we design optimal RIC in the $\mathcal{H}_\infty$ framework. Reference model is selected to minimize the difference between the dynamics of two liner motors as

$$P_m(s) = \frac{1}{0.5s^2}. \qquad (31)$$

The design goal is to find a feedback controller $K$ of internal loop compensator that has a control loop bandwidth 100 rad/s.

A mixed sensitivity problem is formulated as follows

$$\min_K \left\| \begin{bmatrix} W_1(1+P_mK)^{-1} \\ W_2P_mK(1+P_mK)^{-1} \end{bmatrix} \right\|_\infty < 1. \qquad (32)$$

and the problem of imaginary model poles can be solved via axis shifting technique. But there are still two model zeros at infinity, which are also on the imaginary axis. This can be treated via a $W_2$ weighting selection:

$$W_2 = \frac{s^2}{7 \times 10^4}. \qquad (33)$$

Inverse of $W_1$ is the desired shape of the sensitivity function. $W_1$ is selected as

$$W_1 = \frac{\beta(\alpha s^2 + 2\zeta_1\omega_c\sqrt{\alpha}s + \omega_c^2)}{(\beta s^2 + 2\zeta_2\omega_c\sqrt{\beta}s + \omega_c^2)} \qquad (34)$$

where $\beta = 250$ is DC gain which controls the disturbance rejection, $\alpha = 0.5$ is high frequency gain which controls the response peak overshoot, $\omega_c = 200$ is crossover frequency, and $\zeta_1 = \zeta_2 = 0.8$ is damping ratios of the corner frequencies. The structure has implicit $Q$ filter parameters of disturbance observer, so we can design the parameters using the desired criterion in the frequency domain.

From Eq.(31)-(34), we obtain optimal controller $K$ using MATLAB[13]:

$$K(z) = \frac{31088z^2 + 978.35z - 30109}{z^2 + 0.18390z - 0.79021}. \qquad (35)$$

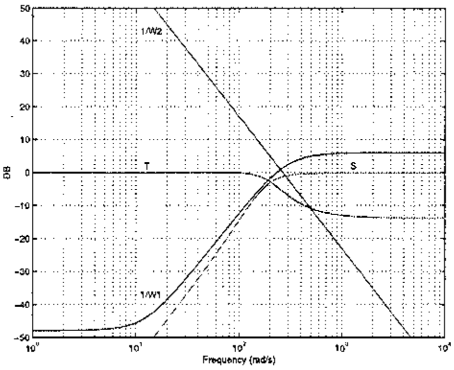



Fig. 7. Sensitivity Function, Complementary Sensitivity Function, and Weighting Functions

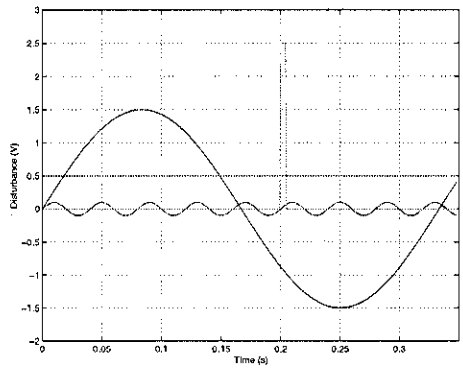



Fig. 8. External Disturbance applied to the Twin-Servo System

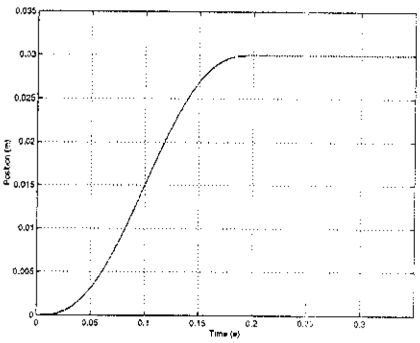
This controller is discretized by using the bilinear transformation and reduced to 2nd order through optimal Hankel minimum degree approximation. The resulting sensitivity function, complementary sensitivity function, and weighting functions are shown in Fig. 7.

For the trajectory tracking problems, conventional PID controller is used to stabilize whole system and track the desired position accurately. The fifth order polynomial function is used to specify the position, velocity, and acceleration at the beginning and end of path. The target position is 30mm. Control sampling frequency is 1000Hz and all controllers are discretized by using the bilinear transformation.

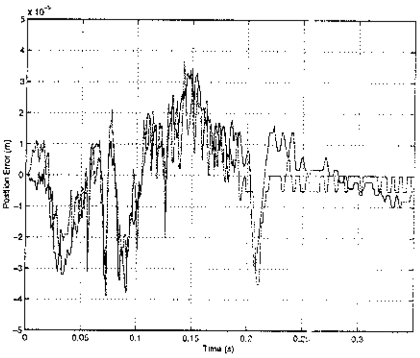
We intentionally apply external disturbance whose shape is shown in Fig. 8 with two sinusoidal plus constant input. This disturbance was applied to only one of two linear motors to examine robustness of the RIC. Fig. 9 shows the results with disturbance of Fig. 8. As can be seen here, the tracking error shows good performance within $\pm 40\mu$m and the trajectory shows the robustness of the proposed controller.

## V. CONCLUSIONS

We proposed a robust motion control scheme which consists of internal loop compensator and external loop controller. Internal loop compensator can be designed by many methods due to the generality of the controller structure and $H_\infty$ mixed sensitivity algorithm was used to optimize compensator gain in this paper. It makes the system stable under uncertainties and nonlinearities. External loop controller can also be arbitrarily designed to meet the specification of the system using the neat result of internal loop compensator. The effectiveness of the proposed algorithm is verified through tra-



(a) Position



(b) Position Error

Fig. 9. Experimental Results with Disturbance

jectory tracking control algorithms and the results show excellent performance under various nonlinear friction characteristics and disturbances for twin-servo brushless DC linear motor system.

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## Appendix

### I. Proof of Theorem 1

The transition matrix $\boldsymbol{\Phi}(t)$ of (15) is defined as

$$\boldsymbol{\Phi}(t) = \exp(\boldsymbol{E}t) \tag{36}$$

and suppose

$$\|\boldsymbol{\Phi}(t)\| = \mathrm{m}\exp(-\alpha t) \qquad t \geq 0 \tag{37}$$

for some constants $m > 0$, $\alpha > 0$.

From (13), we get

$$\boldsymbol{e}_m(t) = \boldsymbol{\Phi}(t)\boldsymbol{e}_{m0} + \int_0^t \boldsymbol{\Phi}(t-\tau)\boldsymbol{w}d\tau. \tag{38}$$

Performing the operator $\|\cdot\|$ to both sides, we get

$$\|\boldsymbol{e}_m(t)\| \leq \|\boldsymbol{\Phi}(t)\|\,\|\boldsymbol{e}_{m0}\| + \int_0^t \|\boldsymbol{\Phi}(t-\tau)\|\,\|\boldsymbol{w}(\tau))\|\,d\tau. \tag{39}$$

From (14), (37), and (39), we can get

$$\begin{aligned}\|\boldsymbol{e}_m(t)\|\exp(\alpha t) \leq \mathrm{m}\,\|\boldsymbol{e}_{m0}\| + \frac{\mathrm{m}\eta_2}{\alpha}\left(\exp(\alpha t) - 1\right)\\ + \int_0^t \mathrm{m}\eta_1 \exp(\alpha\tau)\,\|\boldsymbol{e}_m(\tau)\|\ d\tau.\end{aligned} \tag{40}$$

By the Gronwall-Bellman lemma, we get

$$\begin{aligned}\|\boldsymbol{e}_m(t)\|\exp(\alpha t) &\leq \mathrm{m}\,\|\boldsymbol{e}_{m0}\| + \frac{\mathrm{m}\eta_2}{\alpha}\left(\exp(\alpha t) - 1\right)\\ &+ \int_0^t \left[\mathrm{m}\,\|\boldsymbol{e}_{m0}\| + \frac{\mathrm{m}\eta_2}{\alpha}\left(\exp(\alpha\tau) - 1\right)\right]\cdot \mathrm{m}\eta_1 \exp\left(\int_\tau^t \mathrm{m}\eta_1\ ds\right) d\tau\end{aligned} \tag{41}$$

where

$$\begin{aligned}&\int_0^t \left[\mathrm{m}\,\|\boldsymbol{e}_{m0}\| + \frac{\mathrm{m}\eta_2}{\alpha}\left(\exp(\alpha\tau) - 1\right)\right]\cdot \mathrm{m}\eta_1 \exp\left(\int_\tau^t \mathrm{m}\eta_1\ ds\right) d\tau\\ &= \mathrm{m}\,\|\boldsymbol{e}_{m0}\|\left[-1 + \exp\left(\mathrm{m}\eta_1 t\right)\right] + \frac{\mathrm{m}^2\eta_1\eta_2}{\alpha}\left[\frac{1}{\alpha - \mathrm{m}\eta_1}\{\exp\left(\alpha t\right)\right.\\ &\left.\quad - \exp\left(\mathrm{m}\eta_1 t\right)\} + \frac{1}{\mathrm{m}\eta_1}\{1 - \exp\left(\mathrm{m}\eta_1 t\right)\}\right].\end{aligned}$$

From (41), we get

$$\begin{aligned}\|\boldsymbol{e}_m(t)\| &\leq \mathrm{m}\,\|\boldsymbol{e}_{m0}\|\exp\left((\mathrm{m}\eta_1 - \alpha)t\right) + \frac{\mathrm{m}\eta_2}{\alpha}\left[1 - \exp\left((\mathrm{m}\eta_1 - \alpha)t\right)\right]\\ &\quad + \frac{\mathrm{m}^2\eta_1\eta_2}{\alpha(\alpha - \mathrm{m}\eta_1)}\left[1 - \exp\left((\mathrm{m}\eta_1 - \alpha)t\right)\right]\\ &= \frac{\mathrm{m}\eta_2}{\alpha - \mathrm{m}\eta_1} + \left[\mathrm{m}\,\|\boldsymbol{e}_{m0}\| - \frac{\mathrm{m}\eta_2}{\alpha - \mathrm{m}\eta_1}\right]\exp\left((\mathrm{m}\eta_1 - \alpha)t\right)\\ &\leq \left\|\frac{\mathrm{m}\eta_2}{\alpha - \mathrm{m}\eta_1}\right\| + \left[\mathrm{m}\,\|\boldsymbol{e}_{m0}\| + \left\|\frac{\mathrm{m}\eta_2}{\alpha - \mathrm{m}\eta_1}\right\|\right]\exp\left((\mathrm{m}\eta_1 - \alpha)t\right).\end{aligned} \tag{42}$$

Therefore, for given $T$, (42) becomes

$$\left\|\frac{\mathrm{m}\eta_2}{\alpha - \mathrm{m}\eta_1}\right\| + \left[\mathrm{m}\,\|\boldsymbol{e}_{m0}\| + \left\|\frac{\mathrm{m}\eta_2}{\alpha - \mathrm{m}\eta_1}\right\|\right]\exp\left((\mathrm{m}\eta_1 - \alpha)T\right) \leq \epsilon. \tag{43}$$

Since $\mathrm{m}\,\|\boldsymbol{e}_{m0}\| + \left\|\frac{\mathrm{m}\eta_2}{\alpha - \mathrm{m}\eta_1}\right\|$ is bounded, and if we select

$$\alpha > \mathrm{m}\,\eta_1, \tag{44}$$

then the second term in (43) becomes zero as $T \to \infty$, and we can find $\alpha$ such that $\left\|\frac{\mathrm{m}\eta_2}{\alpha - \mathrm{m}\eta_1}\right\| \leq \epsilon$. If we choose $\alpha$ such that $\|\alpha\| \geq \frac{1}{\epsilon}\,\|\mathrm{m}\eta_2\| - \|\mathrm{m}\eta_1\|$, we can get (43) for given $T < \infty$. ■