A Biomimetic Compliance Control of Robot Hand by Considering Structures of Human Finger

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Abstract

In this paper, for an object grasped by a robot hand to work in compliance control domain, we first analyze necessary condition for successful stiffness modulation in the operational space. Next, we propose a new compliance control method for robot hands which consist of two steps. RIFDS (Resolved Inter-Finger Decoupling Solver) is to decompose the desired compliance characteristic specified in the operational space into the compliance characteristic in the fingertip space without inter-finger coupling, and RIJDS (Resolved Inter-Joint Decoupling Solver) is to decompose the compliance characteristic in the fingertip space into the compliance characteristic in the joint space without inter-joint coupling. According to the analysis results, the finger structure should be biomimetic in the sense that either kinematic redundancy or force redundancy are required to implement the proposed compliance control scheme. Five-bar fingered robot hands are treated as illustrative examples to implement the proposed compliance control method. To show the effectiveness of the proposed compliance control method, simulations are performed for two-fingered and three-fingered robot hands.

1 Introduction

Related to grasping and manipulation of robot hands, a coordinated dynamic hybrid position/force control method considering the dynamics of manipulator and object has been proposed for a set of robot arms or a multi-fingered robot hand handling one constrained object [1]. Hasegawa et al. [2] presented a multi-sensors-based manipulation of an object by multi-fingered hand equipped with three-axis force sensor in the fingertip. But, their explicit force-based fine motion control may practically not be easy because the real force signal is very noisy. It therefore has been pointed out that instead of explicit force signal the stiffness or compliance is an important quantity for characterizing the grasping and manipulation of robot hands in the case that it is specially dominated in approximated linear analysis where low velocities and small relative motions lead to small inertial forces.

Many approaches have been reported in the field of grasp stiffness or compliance. The stiffness of objects grasped by "virtual springs" was analyzed in cases of planar and three-dimensional space in [3]. In [4], the effective grasp stiffness was analyzed by considering the structural compliances in the fingers and fingertips, servo gains at the joints of finger, and small changes in the grasp geometry that affect the way in which the grasp forces act upon the object. However, the geometric condition for successful implementation of compliance control scheme has not been addressed so far. Reflecting geometric conditions for successful compliance control, Yi, et al. [5] proposed that an independent joint-based compliance control via redundant actuation is much adequate approach to modulate the operational stiffness comparing to the case of the kinematically redundant structured fingers or manipulators.

In this paper, we first analyze the geometric conditions for successful implementation of the stiffness control in the object operational space. Our proposed compliance control method consists of two steps: RIFDS and RIJDS. To show the effectiveness of the proposed compliance control method, simulations are performed for two-fingered and three-fingered robot hands.
2 Stiffness Analysis in Robot Hand

Consider a rigid object being manipulated by \( n_f \)-fingered robot hand as shown in Figure 1, where each finger has \( n_j \)-joints, the relation between the dynamic force vector in the operational space and the fingertip force vector is given by

\[
T_o = \{G' \}_{o}^T T_f, \tag{1}
\]

where \( T_o \in \mathbb{R}^{n \times 1} \) denotes the dynamic forces and moments in the operational space including the inertial load and external load, \( T_f \in \mathbb{R}^{m \times 1} \) fingertip force vector in the fingertip space, and \( \{G' \}_{o} \in \mathbb{R}^{m \times n} \) Jacobian matrix. Here, \( m \) denotes the dimension of wrench transmitted through each contact point.

![Figure 1: A multi-fingered robot hand.](image)

When the object motion trajectory is pre-specified, the task of load distribution can be classified to determine the fingertip forces and moments in order to achieve a desired motion of object and to maintain the grasp. The general solution of (1) is given by

\[
T_o = \{G' \}_{o}^T T_f + \{I_{m \times n} \} - \{G' \}_{o}^T \{G' \}_{o} T_f \xi_f, \tag{2}
\]

where \( \{G' \}_{o}^T \) is a pseudo-inverse of \( \{G' \}_{o}^T \) and \( \xi_f \) is an arbitrary \( m \times 1 \) vector.

Using (2), we can perform explicit force control of robot hand by using force sensor signal, but the fine finger motion control is practically hard because force measurement at the fingertip is not easy and the real force signal is very noisy. The explicit force control method applied to robot hand may not be biomimetic. It is therefore worth studying the compliance control method of robot hand which is believed to be more human-like.

2.1 Stiffness Relations

By taking the partial derivative of (1) with respect to \( u_q \), the \( n \times n \) stiffness matrix in the operational space including the effect of the change of contact configuration can be expressed as follows:

\[
[K_o] = \{G' \}_{o}^T [K_f][G' \}_{o} - ([T_f]_o \circ [H_{o}]_{ff}), \tag{3}
\]

and we define

\[
[K_o] = [K'_o] + ([T_q]_o \circ \{H' \}_{f}), \tag{4}
\]

where \( [K_f] \) represents the \( n_f \times n_f \) stiffness matrix in the fingertip space, and \( \{o\} \) and \( \{H_{o}]_{ff} \) represent the Generalized Scalar Dot Product [6] and the second-order kinematic influence coefficient matrix which is induced by the change of contact configuration[7], respectively.

To find the stiffness relation between the joint space and the fingertip space, it is first needed to consider the structure of finger in the hand. Note that the forward or backward Jacobian mapping between the joint space and the fingertip space may not be unique due to the structure of finger used in robot hands. For example, the forward mapping is unique in the case of the serial structured finger, but in the case of the closed-loop structured finger such as the finger with five-bar mechanism described in [8], the backward mapping is unique.

In this paper, we treat a robot hand with five-bar finger mechanism and then the stiffness relation between the fingertip space and joint space is described by using the backward Jacobian mapping. The \( m \times m \) stiffness matrix in the \( i \)th fingertip space including the effect of the change of joint configuration can be represented as follows [5]:

\[
[K'_f] = ([G' \}_{i}^T [K_q][G'_f] - ([T_q]_i \circ \{H' \}_{f}), \tag{5}
\]

and we define

\[
[K_f] = [K'_f] + ([T_q]_i \circ \{H' \}_{f}) = \{G' \}_{i}^T [K_q][G' \}_{i}, \tag{6}
\]

where \( [K_q] \) represents the \( n_j \times n_j \) stiffness matrix in the joint space of each finger, \( \{G' \}_{i} \) denotes a Jacobian relating the joint space \( q \) to the fingertip space \( f \), and \( \{H' \}_{f} \) represents the second-order kinematic influence coefficient matrix induced by the change of joint configuration described in [6], and more detail for the second-order kinematic influence coefficient matrix is described in [7].

2.2 Necessary Condition for Compliance Control

In a robot hand system, the components transmitted through the contact between the fingertip and the
contact point of object are limited by the contact constraint defined according to the contact types. It is therefore very important to investigate how many fingers and how many joints for each finger are required to modulate the desired compliance characteristic in the operational space.

In this section, we first investigate a necessary condition for an independent finger based compliance control of robot hand in terms of the number of fingers and contact types. Particularly, we treat the case of point contact with friction in the two- and three-dimensional space. In general robot hand system, there exist inter-finger coupling. If the inter-finger coupling can be eliminated, each finger can be independently controlled which makes the hand control relatively easy. The equation (4) can be rearranged as

\[
\begin{bmatrix}
K_{oo} \\
0
\end{bmatrix} =
\begin{bmatrix}
M_1 \\
M_2
\end{bmatrix}
K_{ff},
\]  

(7)

where \( K_{oo} \) and \( K_{ff} \) denote the vector which consists of independent elements of \( K_o \) and \( K_f \), respectively. The zero vector \( \overrightarrow{0} \) means that the corresponding parameters in \( K_f \) are equal to 0. \( M_1 \) denotes the relation between the independent elements of fingertip stiffness matrix and those of object stiffness matrix, and \( M_2 \) denotes a selection matrix, where each row corresponds to one of the coupling elements of \( K_f \). From (7), the necessary condition for successful implementation of the stiffness control in the operational space can be analyzed.

Table 1. Necessary condition for stiffness control (2D)

<table>
<thead>
<tr>
<th>Finger</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6</td>
<td>6</td>
<td>10</td>
<td>-2</td>
<td>( K_o(3 \times 3) )</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>6</td>
<td>10</td>
<td>1</td>
<td>( K_o(2 \times 2) )</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>15</td>
<td>21</td>
<td>0</td>
<td>( K_o(3 \times 3) )</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>28</td>
<td>36</td>
<td>2</td>
<td>( K_o(3 \times 3) )</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>45</td>
<td>55</td>
<td>4</td>
<td>( K_o(3 \times 3) )</td>
</tr>
</tbody>
</table>

In Table 1, (a), (b), and (c) denote the number of independent elements of \( K_o \), the number of finger coupling elements of \( K_f \), and the number of independent elements of \( K_f \), respectively; (d) = (c) - (a) - (b) means the remaining degree of freedom for \( K_f \). For example, in the case of two-fingered hand, the specified \( 3 \times 3 \) object stiffness matrix consists of six independent elements, and the fingertip stiffness matrix has ten independent elements in which six coupling elements exist. Thus we can acknowledge that the dimensions of \( M_1 \) and \( M_2 \) are 6 x 10 and 6 x 10, respectively, and then the input parameters are not enough to obtain the solution of (7). It is therefore confirmed that two-fingered hand cannot implement \( 3 \times 3 \) object stiffness characteristic in the two-dimensional space. Hence a robot hand should have at least three fingers to modulate \( 3 \times 3 \) object stiffness characteristic in two-dimensional space as shown in Table 1. Full stiffness modulation is possible when (d) is equal or greater than 0.

A similar analysis can be performed for three-dimensional operational space, but it is omitted here due to limit of space.

2.3 New Stiffness Decomposition Algorithm

2.3.1 RIFDS Algorithm

In this section, we describe the procedure for computing the fingertip stiffness without inter-finger coupling so as to achieve the desired stiffness in the operational space.

![Figure 2: Compliance control by two-fingers.](image)

We particularly treat the case of two-fingered robot hand, where its contact type is assumed a point contact with friction and its workspace is confined in two-dimensional space. When a two-fingered robot hand manipulates an object in the two-dimensional space as shown in Figure 2, the necessary condition for stiffness control is satisfied. Let the desired \( 2 \times 2 \) object stiffness matrix in the operational space be given as

\[
K_o = \begin{bmatrix}
K_{oo} & K_{oy} \\
K_{yo} & K_{yy}
\end{bmatrix},
\]  

(8)

and the \( 4 \times 4 \) stiffness matrix in the fingertip space is generally represented by

\[
K_f = \begin{bmatrix}
K_{fxx} & K_{fyy} & K_{fzx} & K_{fzy} \\
K_{fxy} & K_{fyy} & K_{fzy} & K_{fyy}
\end{bmatrix}.
\]  

(9)
There exist a grip Jacobian matrix as the following form

$$[G_g^f] = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \\ b_{41} & b_{42} \end{bmatrix}$$ \hspace{1cm} (10)

and in order to eliminate the inter-finger coupling, we design the stiffness matrix in the fingertip space as follows;

$$[K_f^f] = \begin{bmatrix} K_{f_{xx}} & 0 & 0 & 0 \\ 0 & K_{f_{yy}} & 0 & 0 \\ 0 & 0 & 2K_{f_{xx}} & 0 \\ 0 & 0 & 0 & 2K_{f_{yy}} \end{bmatrix}$$ \hspace{1cm} (11)

Then, the equation (4) can be rearranged as a vector form

$$K_{oo} = [B_f^f] K_{ff},$$ \hspace{1cm} (12)

where

$$K_{oo} = [K_{oxx} \ K_{oyy} \ K_{oxy}]^T,$$ \hspace{1cm} (13)

$$K_{ff} = [K_{f_{xx}} \ K_{f_{yy}} \ 2K_{f_{xx}} \ 2K_{f_{yy}}]^T,$$ \hspace{1cm} (14)

and

$$[B_f^f] = \begin{bmatrix} (b_{11})^2 & (b_{21})^2 & (b_{31})^2 & (b_{41})^2 \\ b_{11}b_{12} & b_{21}b_{22} & b_{31}b_{32} & b_{41}b_{42} \\ (b_{12})^2 & (b_{22})^2 & (b_{32})^2 & (b_{42})^2 \end{bmatrix}.$$ \hspace{1cm} (15)

Then, the problem of computing the fingertip stiffness for the given object stiffness can be transferred as that of solving a linear programming problem of the form

$$K_{oo} = [B_f^f] K_{ff},$$ \hspace{1cm} (16)

subject to

$$K_{ff} \geq 0.$$

The procedure of computing the fingertip stiffness for the given object stiffness can be consequently summarized as below.

**RIFDS algorithm:**
1. Specify $[K_o]$ and construct $K_{oo}$.
2. Check the necessary conditions from Table 1.
3. Determine the grip Jacobian $[G_g^f]$.
4. Rearrange (4) as (12).
5. Solve the linear programming problem given in (16) to determine $K_{ff}$.
6. Find $[K_f]$ by reconstructing $K_{ff}$.

### 2.3.2 RIFDS Algorithm

Next step aims at determination of joint stiffness for the fingertip stiffness $K_f$ calculated in RIFDS. To find the stiffness in the joint space of each finger, first we have to compute the unique Jacobian relation; particularly, the forward mapping is unique in the case of the serial structured finger, but in the case of the closed-loop structured finger, the backward mapping is unique [5].

A recent work has shown that a finger or manipulator should have at least the same number of active joints as the number of independent elements of the desired operational compliance matrix to modulate the desired compliance characteristic in the operational space [5]. To satisfy this condition, the finger structure should be biomimetic. That is, it should have either redundant joints or redundant actuators. However, in serial-chain structures the number of required joints increases exponentially according to the degree of freedom in the operational space and thus, a finger employing redundant actuation mechanism such as five-bar finger used in [8] is more appropriate than a serially structured finger in the design viewpoint.

In this paper, we treat a closed-loop structured finger with redundant actuators as shown in Figure 3, where the inverse relation between the fingertip motion and the joint motion is uniquely given by

$$\delta^i q = [G_f^f]^T \delta^i u_f,$$ \hspace{1cm} (17)

or using duality property, the force relation between the joint space and the fingertip space is obtained by

$$^iT_f = [G_f^f]^T T_q,$$ \hspace{1cm} (18)

where $^iT_q$ and $\delta^i q$ denote $^n_j \times 1$ vectors of the joint torques and infinitesimal joint motions, $^iT_f$ and $\delta^i u_f$ also denote $m \times 1$ generalized force and infinitesimal motion vectors in the ith fingertip, respectively; $[G_f^f]$ is the $^n_j \times m$ Jacobian matrix relating the fingertip motion to the joint motion of the ith finger.

We assume that a $2 \times 2$ stiffness matrix at the ith fingertip space in two-dimensional space is given by

$$[K_f] = \begin{bmatrix} K_{f_{xx}} & K_{f_{xy}} \\ K_{f_{yx}} & K_{f_{yy}} \end{bmatrix}.$$ \hspace{1cm} (19)

The backward Jacobian relationship for the ith finger with 4 active joints located in joint 1, 3, 4, and 5 can be uniquely defined as [5]

$$[G_f^f] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{bmatrix}.$$ \hspace{1cm} (20)
And we design the joint stiffness matrix as

\[
[K_q] = \begin{bmatrix}
    iK_{q1} & 0 & 0 & 0 \\
    0 & iK_{q2} & 0 & 0 \\
    0 & 0 & iK_{q4} & 0 \\
    0 & 0 & 0 & iK_{q5}
\end{bmatrix}
\]

so that each joint can be independently controlled.

By rearranging (6) as a vector form, we have

\[
iK_{ff} = [iA_q]^+ iK_{qq},
\]

where

\[
iK_{ff} = \left[ iK_{fxx} \ iK_{fzy} \ iK_{fyy} \right]^T,
\]

\[
iK_{qq} = \left[ iK_{q1} \ iK_{q3} \ iK_{q4} \ iK_{q5} \right]^T
\]

and

\[
[iA_q]^+ = \begin{bmatrix}
    (a_{11})^2 & (a_{21})^2 & (a_{31})^2 & (a_{41})^2 \\
    (a_{12})^2 & (a_{22})^2 & (a_{32})^2 & (a_{42})^2
\end{bmatrix}
\]

From (22), the general solution is given by

\[
iK_{qq} = ([iA_q]^+ iK_{ff} + ([iA_q]^+ [iA_q])^{-1} iK_{ff} \xi_q
\]

where \([iA_q]^+\) denotes the pseudo-inverse of \([iA_q]\) and \(iK_{ff}\) is an arbitrary \(n_j \times 1\) vector.

In robot hands with closed-loop structured fingers, the procedure of computing the joint stiffness for the given fingertip stiffness can be consequently summarized as below.

**RIJDS algorithm**:

1. Determine the structure of finger to implement the \([K_f]\) determined by RIFDS.
2. Determine the finger Jacobian \([G_f]\) for each finger.
3. Rearrange (6) as (22).
4. Solve (22) to determine \(iK_{qq}\).
5. Find \([iK_q]\) by reconstructing \(iK_{qq}\).

### 3 Compliance Control of Grasped Object by Multi-fingered Robot Hand

#### 3.1 Case I: Two-Fingered Hand

Consider a dual finger shown in Figure 2. The physical parameters and the kinematics of the five-bar finger are described in [8]. In Figure 2, the given task of the object is to control the position along the x-direction and to control the force to the y-direction. The block diagram for the given task is shown in Figure 4.

![Figure 4: Block diagram for compliance control.](image)

Initially, the task planner in Figure 4 defines the desired x-directional trajectory, y-directional pushing force, stiffness matrix in the operational space, and the internal force in the fingertip space for stable grasp. Specifically, the virtual trajectory in the y-direction is determined from the relationship of the desired force and the planned stiffness of the y-direction. The equivalent stiffness matrix \([iK_q]\) in the joint space is derived from RIFDS followed by RIJDS described in the section 2. The trajectory in the ith fingertip space is determined as follows. The generalized force in the operational space is defined as

\[
T_o(t) = [M_o] \ddot{u}_o(t) + [B_o] \dot{u}_o(t) + [K_o] \delta u_o(t),
\]

where

\[
\delta u_o(t) = u_{od}(t) - u_{oa}(t)
\]

and \(u_{od}(t), u_{oa}(t)\) denote the desired and actual positions of the manipulated object, respectively. \([M_o]\) and \([B_o]\) denote the inertial and damping matrices of the object, respectively, and \(\ddot{u}_o\) denotes the acceleration vector of the object.

The actual force exerted by the object on the environment is indirectly obtained, in an online fashion, by

\[
K_{ovy} u_{sv}(t) : u_{sv}(t) > 0 \\
0 : u_{sv}(t) \leq 0
\]

\[
f_{ov}(t) = \begin{cases}
    K_{ovy} u_{sv}(t) : u_{sv}(t) > 0 \\
    0 : u_{sv}(t) \leq 0
\end{cases}
\]
\[ u_{oc}(t) = u_{oa}(t) - u_{env}(t). \]  

In (30), \( u_{env}(t) \) denotes the position of environment. The fingertip force is determined by (2) with consideration of the desired internal force.

For the change of \( T_o(t) \), the change of \( T_f(t) \) is obtained according to (1). Then, the induced trajectory in the fingertip space is calculated by

\[ \delta u_f = [K_f]^{-1} \delta T_f, \]  

where

\[ \delta u_f = \begin{bmatrix} \delta^1 u_f \\ \delta^2 u_f \end{bmatrix}. \]  

The small change of the joint angles for \( i \)th finger is given by

\[ \delta^i q = [G_f^i] \delta^i u_f. \]  

Thus we can update the next joint positions as follows

\[ \dot{q}(t + 1) = \dot{q}(t) + \delta^i q. \]  

Finally, the joint torque necessary to make the fingertip exhibit the desired stiffness characteristic in the fingertip space is given by

\[ T_q(t) = K_q \dot{q}(t) + K_q D \dot{q}(t), \]

where

\[ \dot{q}(t) = \dot{q}(t) - \dot{q}_a(t), \]

and the small damping gain \( K_q D \) is included for stable trajectory following.

In the simulation, a dynamic model of robot hand given by

\[ T_q(t) = [I_{qq}] \ddot{q}(t) + [P_{qqq}] \dot{q}(t), \]

is employed as a plant model, where \([I_{qq}]\) and \([P_{qqq}]\) are the inertia matrix and inertia power array of the \( i \)th finger, respectively [6].

In simulation, the desired stiffness in the operational space is specified as

\[ K_o = \begin{bmatrix} 50 & 0 \\ 0 & 10 \end{bmatrix} \text{[N/m]}. \]

We plan a desired force to the \( y \)-direction as 0.05[N], which is controlled by inducing a virtual displacement of 5 [mm] inside the environment; the \( x \)-directional velocity of the grasped object is set as 0.01[m/sec].

The geometrical structure of grasp is symmetric as shown in Figure 2. The \( x \)-directional trajectory and \( y \)-directional force are shown in Figures 5 and 6, respectively. It is shown that the motion and the force defined in the operational space can be controlled properly by eliminating the finger and joint couplings of the hand system.

![Figure 5: Trajectory to the x-direction in case I.](image1)

![Figure 6: Force trajectory to the y-direction in case I.](image2)

### 3.2 Case II: Three-Fingered Hand

A three-fingered robot hand with five-bar mechanism is shown in Figure 7, where the physical parameters for each finger are the same as those of the two-fingered hand. However, this finger system deals with 3 motion degrees (\( x, y, \) and \( \phi \)). The block diagram for the contact task is shown in Figure 8.

In this simulation, we give the desired force to the \( y \)-direction as 0.05[N] and the \( x \)-directional velocity of the grasped object as 0.01[m/sec], while controlling the orientation constant. The desired stiffness in the operational space is initially specified as

\[ K_o = \begin{bmatrix} K_{oxx} & K_{oxy} & K_{oxy} \\ K_{oyx} & K_{oyy} & K_{oyy} \\ K_{ozx} & K_{ozy} & K_{ozy} \end{bmatrix} = \begin{bmatrix} 50 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 3 \end{bmatrix}. \]  

From Table 1, geometric condition recommends
Figure 7: Compliance control by three-fingers.

Figure 8: Block diagram for compliance control.

three fingers for this task. In this work, the geometry of grasp is symmetric for finger 1 and 2, and the contact point of the third finger lies on the center of the bottom side of the object. The equation (16) can be symbolically expressed as

$$K_{oo} = \begin{bmatrix} 1.0 & 0.0 & 1.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ y_1 & 0.0 & y_2 & 0.0 & y_3 & 0.0 \\ 0.0 & 1.0 & 0.0 & 1.0 & 0.0 & 1.0 \\ 0.0 & -x_1 & 0.0 & x_2 & 0.0 & x_3 \\ y_1^2 & x_1^2 & y_2^2 & x_2^2 & y_3^2 & x_3^2 \end{bmatrix} K_{ff}, \quad (39)$$

where

$$K_{oo} = [K_{oxx} \ K_{oxy} \ K_{oxo} \ K_{oyy} \ K_{oyx} \ K_{oyo}]^T,$$

$$K_{ff} = [1K_{fxx} \ 1K_{fyy} \ 2K_{fzx} \ 2K_{fzy} \ 3K_{fzx} \ 3K_{fzy}]^T,$$

and $y_i$, $x_i$ denote the elements of position vectors directing from the $i$th finger contact position to the task position, and they are given all positive.

Note that the elements of the second row of the mapping matrix are calculated as zero. This is because we excluded the coupling terms $^iK_{fxy}(i=1,2,3)$ in the fingertip space for independent finger control. Thus, we have zero $K_{oxy}$, which, in fact, is a linear combination of $^iK_{fxy}(i=1,2,3)$. Also, note that the third row corresponds to modulation of $K_{oxx}$. However, we can easily notice that zero $K_{oxx}$ cannot be achieved by all positive stiffness components $K_{ff}$ defined in the fingertip space since the three influence coefficients (i.e., $y_1$, $y_2$, and $y_3$) are always positive in this grasped configuration. Therefore, we first solve for the diagonal fingertip stiffness elements $K_{ff}$ by linear programming technique without considering the third row of (39). Then, we calculate $K_{oxx}$ term by resubstituting the solution of $K_{ff}$ into (39). Finally, the stiffness matrix given in the operational space is changed as

$$K_o = \begin{bmatrix} 50 & 0 & 3.69 \\ 0 & 10 & 0 \\ 3.69 & 0 & 0.3 \end{bmatrix} [N/m] \quad (40)$$

for $y_1=0.06[m]$, $y_2=0.06[m]$, $y_3=0.1[m]$, $x_1=0.03[m]$, $x_2=0.03[m]$, and $x_3=0[m]$. In such posture of grasp, the coupling term between $x$-directional stiffness and $\phi$-directional stiffness always exist, while the other off-diagonal terms can be made zero.

Figure 9 shows a modified posture of grasp, in which the contact position of the third finger lies above the task position $O$. In this case, the $(3, 5)$ element of

Figure 9: Compliance control by three-fingers.
object are shown in Figures 10, 11, and 12, respectively. Note that a small perturbation of the orientation angle exist in its initial stage. The grasp configuration of Figure 7 is similar to that of hand writing by three fingers. In such operation, we can experience that the x-directional trajectory yields a small angle perturbation of the pencil grasped by the three-fingers. Therefore, existence of $K_{o\theta}$ seems to be natural. In this sense, the result of (40) is biomimetic. On the other hand, the grasp configuration given in Figure 9 represents a force closure type in which angle perturbation is not allowed (i.e., zero $K_{o\theta}$).

4 Concluding Remarks

A new compliance control method for robot hands was proposed by employing two step algorithms named RIFDS and RIJDS. Also, a necessary condition for stiffness control was obtained with which a desired compliance in the operational space could be examined to be achieved by a given hand mechanism. Through the analysis, it is concluded that both the number of fingers and the structure of each finger are important for fulfillment of compliance control scheme and that the geometric configuration of the given grasp should be carefully considered to satisfy the stiffness characteristic specified in the operational space. The effectiveness of the proposed algorithm was shown through hybrid control simulations.

References