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Fast Reinforcement Learning Using Stochastic Shortest Paths for a Mobile Robot

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Abstract— **Reinforcement learning (RL) has been used as a learning mechanism for a mobile robot to learn state-action relations without *a priori* knowledge of working environment. However, most RL methods usually suffer from slow convergence to learn optimum state-action sequence. In this paper, it is intended to improve a learning speed by compounding an existing Q-learning method with a shortest path finding algorithm. To integrate the shortest path algorithm with Q-learning method, a stochastic state-transition model is used to store a previous observed state, a previous action and a current state. Whenever a robot reaches a goal, a Stochastic Shortest Path(SSP) will be found from the stochastic state-transition model. State-action pairs on the SSP will be counted as more significant in the action selection. Using this learning method, the learning speed will be boosted when compared with classical RL methods. To show the validity of our proposed learning technology, several simulations and experimental results will be illustrated.**

I. Introduction

A mobile robot is often required to have an ability to learn what behaviors would be well suited for a given environmental situation for adaptation even with little or no *priori* task knowledge. To solve such an adaptation problem, many researchers have applied reinforcement learning to robot's action adaptation on little or no *priori* knowledge [1] [2] [3]. Reinforcement learning [4] refers to a class of problems in machine learning which postulates an agent perceiving its current state and taking actions in an environment. The environment, in return, provides a reward, which can be positive or negative. Reinforcement learning algorithms attempt to find a policy for maximizing cumulative reward for the agent over the course of the problem. The reinforcement learning algorithms selectively retain the outputs that maximize the received reward over time. In the general case of the reinforcement learning problem, the actions of an agent determine not only its immediate reward, but also the next state of the environment [5]. Q-learning is widely used as a learning method involving little or no *priori* knowledge [6]. Q-learning is able to compare the expected utility of the available actions without requiring a model of the environment. Although many of these methods are guaranteed to eventually find optimal policies and use very little computation time per experience, they make extremely inefficient use of the data they gather and often require a great deal of experiences to achieve good learning performance. Therefore, some model-based reinforcement learning methods, which would require a state-transition probability function, can be efficient for practical cases of a mobile robot. These algorithms are especially important for the applications in which computation is considered to be cheap and real-world experience costly [7] [8] [9] [10].

In our previous works [11] [12], we proposed a Monte-Carlo–based RL method involving a shortest path finding method to increase the probability of selecting the most appropriate behaviors as an effective learning technique. The main idea can be summarized as follows; Exploration for learning usually needs much cost, and may make the system be in danger; On the other hand, a shortest path finding method may usually cause a large number of computational or algorithmic complexity, but its cost is relatively low when compared with cost of exploration process in a real environment; By blending the shortest path finding method with an RL method, we can improve the learning performance in the sense of convergence speed.

However, in our previous works, some weakness has been observed for learning state-action pairs in uncertain environment. Firstly, shortest path may be incorrect because an observed state is often confused by noisy sensor data. Secondly, state-transitions are often non-deterministic because of inaccurate sensor, uneven floor and/or anti-symmetric actuator. In a real environment, such a non-deterministic state-transition can have critical effects on finding the optimal path.

To cope with the weakness of our previous approach, we reform our learning method by considering following perspectives; Firstly, the Long Term Memory(LTM) in our previous approach, which stores information on state-transitions composed of previous observed state, previous action and current state, is modified to a stochastic state-transition model. Secondly, a Stochastic Shortest Path(SSP) finding method is proposed to find the most relevant path in the stochastic state-transition model. To find an SSP, probabilistic cost function is defined by considering not only actual costs such as distances between two state or time during state-transition but also probabilities of state-transition. By applying probabilistic state-transition model and a stochastic shortest path-finding method, we will show implementation of both the learning speed and robustness of the proposing learning algorithm. The rest of this paper is organized as follows; A proposed learning algorithm combining Q-

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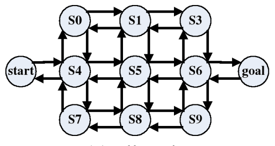
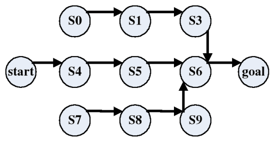
```
Initialize $Q(s,a)$ arbitrarily
Repeat (for each episode):
  Initialize previous observed state $s$
  Repeat (for each step of episode)
    Choose action $a$ from previous observed state $s$
        using policy derived from Q ($\varepsilon$-greedy)
    Take action $a$, observe current reward $r'$,current state $s'$
    $Q(s,a) \leftarrow Q(s,a) + \alpha[r' + \gamma max_{a'} Q(s',a') - Q(s,a)]$
    $s \leftarrow s'$ ;
  until s is terminal
```

Fig. 1. Q-learning algorithm



(a) all paths



(b) shortest paths

Fig. 2. An example of shortest path finding

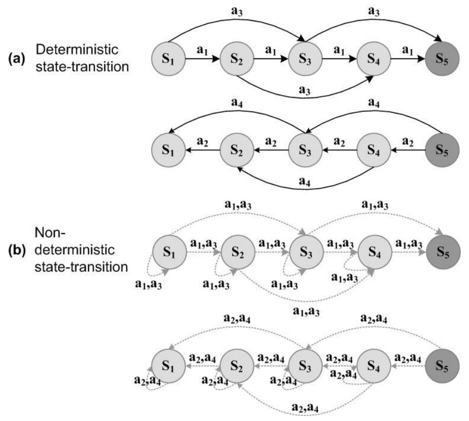

(a)
Deterministic state-transition
(b)
Non-deterministic state-transition
a1
a2
a3
a4
a1,a3
a2,a4
S1
S2
S3
S4
S5

Fig. 3. Examples of state-transitions. State-transition probabilities and cost function for the case (b) are given as follows: $P(s_{i+1}|s_i,a_1) = P(s_{i-1}|s_i,a_2) = 0.9$, $P(s_i|s_i,a_1) = 0.1$, $P(s_{i-1}|s_i,a_2) = 0.9$, $P(s_i|s_i,a_2) = 0.1$, $P(s_{i+2}|s_i,a_3) = 0.6$, $P(s_{i+1}|s_i,a_3) = 0.3$, $P(s_i|s_i,a_3) = 0.1$, $P(s_{i-2}|s_i,a_4) = 0.6$, $P(s_{i-1}|s_i,a_4) = 0.3$, $P(s_i|s_i,a_4) = 0.1$, $cost(s_i,a_1) = cost(s_i,a_2) = 1.0$, $cost(s_i,a_3) = cost(s_i,a_4) = 1.5$

learning with the stochastic shortest path finding method is explained in Sec. 2. Section 3 shows experimental setup for proposed learning algorithm. In Sec. 4, Experimental results of pushing a box into a goal position are presented. Finally, concluding remarks are given in Sec. 5.

II. SSPQL : Stochastic Shortest Paths based Q-Learning

A. Q-learning

Q-learning is a reinforcement learning technique to learn an action–value function that gives the expected utility of taking a given action in a given state [5]. Strength with Q-learning is that it is able to compare the expected utility of the available actions without requiring any model of environment. A summary of Q-learning algorithm is shown in Fig. 1. However, Q-learning may show some difficulties when applied to the reality: First, rewards may not be given immediately after an action is completed. Rewards are usually given after a goal is achieved by performing a sequence of actions. In general, this delayed reward causes the learning to be very time-consuming, or even to be not practical. To apply the Q-learning in real environment, it is necessary to reduce its long learning time mainly due to delayed rewards.

B. Stochastic Shortest Paths(SSP) in uncertain environment

By the state-transition model of environment, an agent can predict how environment will respond to its action. Given a goal and a state-transition model, an agent can infer paths from a current state to a goal by backtracking transitions of states from the goal. This is the basic idea of shortest path finding methods. In comparison to Q-learning, an agent can follow the optimal path in one trial because the shortest path finding method requires no exploration. Our objective is to find shortest paths from all states to a single goal in a directed graph. Fig. 2-(b) shows the shortest paths of Fig. 2-(a). Here, Dijkstra algorithm [13] is employed to find shortest paths. Dijkstra algorithm is a method to solve the single-source shortest path problem which is the problem of finding a path between two vertices such that the sum of the non-negative weights of its constituent edges is minimized. One of weaknesses of a shortest path finding method in a real world is to require a predefined state-transition model for working environment. Such a requirement can be mitigated by combining the shortest path finding method with Q-learning which does not require *a priori* knowledge. If exploration strategies use shortest paths, the frequency of unnecessary exploration can be decreased. As a result, the performance of learning is expected to be improved. As the shortest path finding methods such as Dijkstra method and A* are based on deterministic environment, which is a process in perfect observation and deterministic state-transitions, it is not assured that they can be successfully applied to robotic work in real environment. Here, a very low-cost path with thin probability may be obtained by noisy sensor data and/or non-deterministic state-transitions. Given a shortest path finding method in a deterministic state-transition model, the path may be misunderstood as an optimal path. Once such a low-cost path with thin probability is stored in the deterministic state-transition model, continuous adverse effect can be observed on exploration. To avoid this, it is necessary to use a SSP finding method.

Fig. 3 shows an example of state-transition under uncertain environment. As a result of action a_1, the probability of observing s_{i+1} in the situation of s_i is 0.9 and the probability of observing s_i is 0.1. Similarly, as a result of the action a_3, the probability of observing s_{i+2} is 0.6 and probability of observing other state is 0.4. The actual cost of an action can be regarded as the time to transit from a state to another. For example, let us find the shortest path from s_1 to s_5, which cost function includes a distance and a time in the deterministic environment as shown in Fig. 3-(a). In this case, shortest path is given as $(s_1, a_3) \Rightarrow (s_3, a_3) \Rightarrow s_5$. However, when considering stochastic state-transitions as in Fig. 3-(b), the action a_3 may not be the best choice because a_3 causes the next state to be more uncertain than a_1 does. To be more specific, firstly, by selecting a_3, the sequence may be $(s_1, a_3) \Rightarrow (s_2, a_3) \Rightarrow (s_3, a_3) \Rightarrow s_5$ and its cost is 4.5. Secondly, by selecting a_1, the sequence may be $(s_1, a_1) \Rightarrow (s_2, a_1) \Rightarrow (s_3, a_1) \Rightarrow (s_4, a_1) \Rightarrow s_5$ and its cost is 4.0. In this example, selecting a_3 requires more cost.

Unfortunately, the expected cost cannot be exactly computed because state-transitions are non-deterministic in real environment, and the computation of the expected cost requires a lot of recursive computations. Therefore, the expected cost here is approximately computed by both using distance between states and probability of state-transition. When uncertainty of a state-transition is low, the difference between the expected and actual cost will be small. On the contrary, the difference is large, when uncertainty is high. Thus, optimal paths in real environment should be varied by the probability of state-transitions. The expected cost of a state-transition from state s to s' is defined as

$$cost^*(s, s') = cost(s, s') \left(\sum_{i=0}^{\#\text{of action}} P(s'|s, a_i) \right)^{-\omega}, \qquad (1)$$

where ω is a weight to reflect uncertainty and $cost(s, s')$ is a pre-defined real cost function such as a minimum distance between two states. The equation of state-transition probability can be defined as

$$P(s_t|s_{t-1}, a_{t-1}) = \begin{cases} \frac{N(s_{t-1}, a_{t-1}, s_t)}{N(s_{t-1}, a_{t-1})} & \text{if } N(s_{t-1}, a_{t-1}) \neq 0 \\ \mu (\mu \approx 0) & \text{if } N(s_{t-1}, a_{t-1}) = 0, \end{cases} \qquad (2)$$

where $N(s_{t-1}, a_{t-1}, s_t)$ is the number of observations of s_t followed by action a_{t-1} in a state $s_{t-1}$and μ is a small positive value; $N_{cur}(s_{t-1}, a_{t-1})$ is the frequency of action a_t in a state s_t.

C. Stochastic Shortest Path(SSP)-based Q-Learning (SSPQL)

To integrate SSP with the Q-learning method, a previous observed state, a previous action and a current state will be stored in a probabilistic state–transition model at every step. After one episode ends, shortest paths reaching a goal state from every states can be found by using Dijkstra algorithm with a cost function of (1) in the stochastic state-transition model. By increasing probability of selecting the actions with maximum transition-probability between states in exploration strategy, the speed of convergence can be improved. The amount of probability of selecting actions can be computed by

```
Initialize $Q(s, a)$
Initialize a state-transition model $P(s'|s, a)$
Do until learning end
  Do until a robot reaches a goal
    Observe a current state s' and reward signal r'
    Choose action a s.t.
    $a = \text{argmax}_{a'}[Q(s', a') + w_{ssp}V_{ssp}(s', a')]$ togethter with $\varepsilon$-greedy policy
    Execute action a
    Update $Q$-Value with
    $Q(s, a) \leftarrow Q(s, a) + \alpha[r' + \gamma max_{a'} Q(s', a') - Q(s, a)]$
    Update state-transition model $P(s'|s, a)$
  S, A $\Longleftarrow$ stochastic shortest paths from all state to goal state
           using Dijkstra algorithm
  Update $V_{ssp}(s, a)$
S, A : states and actions consisting of stochastic shortest paths
```

Fig. 4. SSPQL algorithm

$$V_{ssp}(s_{i_1}, a_{j_1}) = \begin{cases} P(s_{i_2}|s_{i_1}, a_{j_1}) P(s_{i_3}|s_{i_2}, a_{j_2}) \ldots P(s_G|s_{i_N}, a_{j_N}) \\ \quad \text{if } s_{i_1}, a_{j_1} \text{ in stochastic shortest paths} \\ 0 \quad \text{else.} \end{cases} \qquad (3)$$

In (3), $s_{i_1}, s_{i_2}, \ldots, s_{i_N}$ are states on stochastic shortest path from s_{i_1} to a goal state s_G and $a_{j_1}, a_{j_2}, \ldots, a_{j_{N-1}}$ are actions with maximum transition probability between the states: For example, an action a_{j_1} is the action with maximum probability from s_{i_1} to s_{i_2}. If the uncertainty of state-transitions is small, V_{ssp} will be close to 1. Otherwise, it will be close to 0. In summary, an optimal policy function can be defined as

$$a = \text{argmax}_{a'}[Q(s', a') + w_{ssp_i}(s', a') V_{ssp}(s', a')]. \qquad (4)$$

Here, $w_{ssp_i}(s, a)$ is a weight value of of SSP. Algorithm of SSPQL is summarized in Fig. 4.

D. Convergence, Optimality, and Computational Complexity of the proposed algorithm

In classical Q-learning methods, convergence and optimality have been formally proved with appropriate parameters in the situation of Markov Decision Processs(MDPs) [14] under the following conditions; bounded rewards $|r| \leq const$, learning rates $0 \leq \alpha \leq 1$, and decreasing learning rate over time. As in Q-learning, our SSPQL can be shown to have convergence and optimality of following hold:

$$0 \leq w_{ssp_i}(s, a) \leq 1, \lim_{i \to \infty} w_{ssp_i}(s, a) = 0, \forall s, a. \qquad (5)$$

In (5), $w_{ssp_i}(s, a)$ is the weightings for the action a of the ith trials at state s. Under the conditions in (5), as i goes to infinity, 2nd and 3rd terms of (4) become null. Thus, (4) will eventually consist of Q-value and a random number. This is the same form of optimal action-value function for Q-learning. Thus, SSPQL can have optimal convergence property as Q-learning. It is remarked that $w_{ssp_i}(s, a) = N(s, a)^{-w}$ is a simple equation to satisfy (5). SSP-value is designed to put a relatively large effect on the action selection at early stages of learning. As the number of episodes are increased, effects of the term will be diminished. In this condition, SSP plays a role of assigning values believed to be close to an

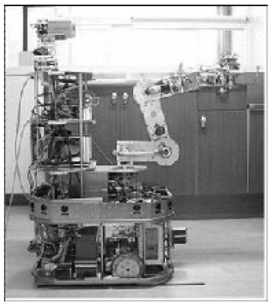


Fig. 5. A Photo of our experimental mobile robot

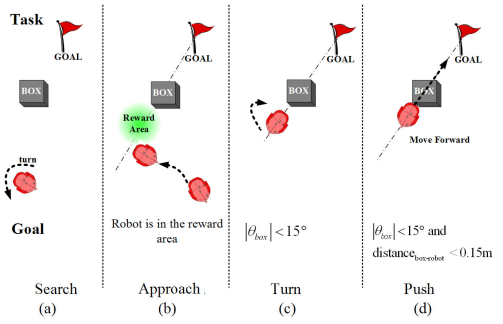


Fig. 6. A task description for box-pushing task

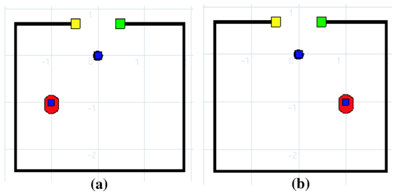


Fig. 7. Initial positions of robot with respect to the box and goal

unknown optimal Q-value at early stages of learning, which is expected to boost learning speed of SSPQL.

Next, computational complexity of SSPQL can be computed by considering complexity of finding a shortest path and computation of V_{ssp} to reach a goal state. First of all, running time of Dijkstra algorithm is $O(n^2)$ on a graph with n nodes and m edges. In our SSP, a node is mapped into a state, and an edge which is a transition between two nodes is mapped into a state-transition by an action. Thus, the number of nodes equals the number of state, and the number of edges equals the number of vector product of states and actions. For sparse graphs with edges much less than n^2, Dijkstra algorithm can be implemented much efficiently by storing the graph in the form of adjacency lists and using a binary or a Fibonacci heap as a priority queue to implement the Extract-Min function [13]. The algorithm requires $O((m+n)\log n)$ time in case of a binary heap, while it is $O(m+n\log n)$ in case of a Fibonacci heap. So, we can improve the computational complexity of SSP algorithm into $O(m+n\log n)$. Next, computational complexity of $V_{ssp}(s,a)$ as shown in (3) is $O(n^2)$. However, by ignoring state-action pairs which have low probability, the computational complexity can be decrease to $O(n\log n)$. Thus, total computational complexity of stochastic shortest path finding process is $O(m+2n\log n)$.

III. Experiment

A. Experimental Setup

To show the validity of our SSPQL algorithm, a task of pushing a box into a goal is considered for a mobile robot as shown in Fig. 5, where the robot has a differential-drive locomotion, a ring of 16 sonar sensors, a single CCD camera, and an Infra-Red(IR) range scanner to measure distances from objects and wall in the world.

Higher-level features such as relative positions of a box and a goal are computed from raw distance information and color segmentation of camera image. Due to the camera with restricted field of view, our robot is required to estimate locations for out-of-sight-objects. A Bayesian filter is used to estimate the posterior probability density locations of objects over the state–space on the data [15].

B. Task Description

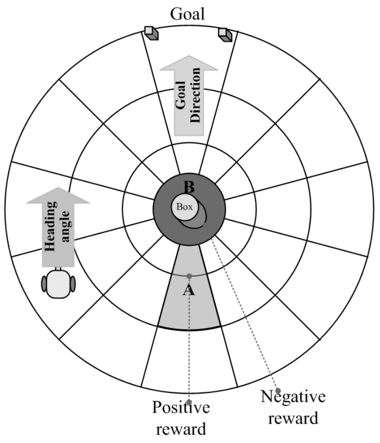
To show the validity of our proposed reinforcement learning method, there is given the task of pushing a box into a goal. To make our task learning more effective, the task is decomposed into more detailed four sub-tasks: search, approach, turn, and push sub-tasks. Firstly, the robot searches for positions of the box and the goal by using its camera and IR scanner. Secondly, the robot moves to the area in which the robot can get a reward as depicted in Fig. 6-(b). Thirdly, the robot aligns itself with the box and the goal so that they are in a line as shown Fig. 6-(c). Finally, the robot pushes the box into the goal. For subtasks of 'search' and 'push', we designed and used rules and thus, those two tasks need not to get any more learning. The sub-tasks of 'approach' and 'turn' are now being learned from the scratch.

C. States and primitive actions

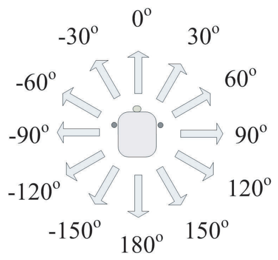
To apply reinforcement learning to the real robot task, it is necessary to design a discrete set of environmental states, primitive actions, and scalar reinforcement signals. If we define a state in such a way that the state implies object position with respect to the robot position in Cartesian coordinate, state-transitions would be too complex to manage. This causes learning process to be hard or to be a lot of time consuming process. Here, to reduce the number of states and to simplify state-transitions, we will set up a new coordinate system such that a box is located at the center of the coordinate, and the goal is faced to the 0° direction as depicted in Fig. 8. Transformed positions of the box, goal, and robot can be computed by

$$\begin{aligned}
x_{goal_{new}} &= (x_{goal}-\Delta x)\cos(\Delta\theta)-(y_{goal}-\Delta y)\sin(\Delta\theta),\\
y_{goal_{new}} &= (x_{goal}-\Delta x)\sin(\Delta\theta)+(y_{goal}-\Delta y)\cos(\Delta\theta),\\
x_{robot_{new}} &= (-\Delta x)\cos(\Delta\theta)-(-\Delta y)\sin(\Delta\theta),\\
y_{robot_{new}} &= (-\Delta x)\sin(\Delta\theta)+(-\Delta y)\cos(\Delta\theta),\\
\theta_{robot_{new}} &= \Delta\theta,
\end{aligned} \tag{6}$$

where $\Delta\theta = -\arctan\big((y_{goal}-y_{box})/(x_{goal}-x_{box})\big)$, $\Delta x = -x_{box}$, and $\Delta y = -y_{box}$. Here, x_{goal} and y_{goal} imply x and y



(a) Description of 36 states of $\boldsymbol{S}_{pos}$



(b) Description of 12 states of $\boldsymbol{S}_{heading}$
$\boldsymbol{S} \Longleftarrow \boldsymbol{S}_{pos} \times \boldsymbol{S}_{heading}$

Fig. 8. Definition of state space

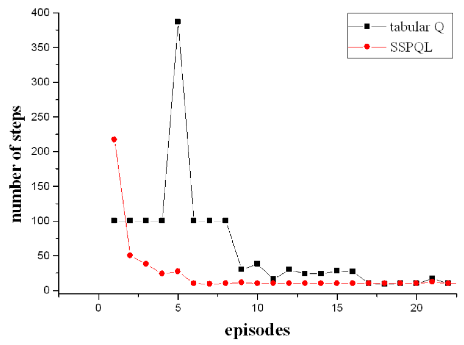


Fig. 9. Number of steps in 'approach' sub-task of Fig. 5-(a)

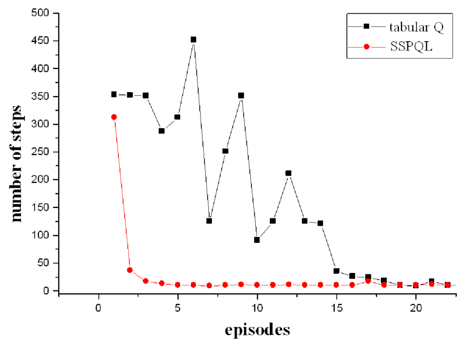


Fig. 10. Number of steps in 'approach' sub-task of Fig. 5-(b)

coordinate of the goal in world cartesian space, respectively. And, x_{box} and y_{box} imply x and y coordinate of the box in world cartesian space, respectively.

$\boldsymbol{S}_{pos}$ is a space of pose-dependent states. $\boldsymbol{S}_{pos}$ is designed to have 36 different states determined by 12 robot orientations as well as 3 distances from a box to a goal as shown in Fig. 8-(a). $\boldsymbol{S}_{heading}$ is a state space of robot heading angle, and is here designed to have 12 states as shown in Fig. 8-(b). The $x_{robot_{new}}$ and $y_{robot_{new}}$ is used to determine a state in $\boldsymbol{S}_{pos}$, and $\theta_{robot_{new}}$ is used to determine a state in $\boldsymbol{S}_{heading}$. Whole state space is defined as the product space of $\boldsymbol{S}_{pos}$ and $\boldsymbol{S}_{heading}$.

Next, four primitive actions are defined: 'move forward by 0.3m', 'move backward by 0.2m', 'turn 30^o', 'turn -25^o'. A reward signals is generated whenever the robot arrives at the state of $\boldsymbol{S}_{pos}$ marked "A" and $\boldsymbol{S}_{heading}$ $=0^o$ as shown in Fig. 8-(a).

D. Experimental and Simulation Results

We applied our SSPQL method to the box-pushing task with two initial positions as shown in Fig. 7. For comparison, Q-learning approach was also experimented in simulations. During all experiments, parameters for Q-function are chosen as $\gamma = 0.8$, $\alpha = 0.8$ and $\varepsilon = 0.1$. And the weight values of SSP is given as $w_{ssp} = 1.5$.

1) Simulational results: To simulate environment and a real robot, we use an open source simulator, Player and Stage [16]. Fig. 9 shows the number of steps per episode of Q-learning and our SSPQL method with initial configuration as in Fig. 7-(a). The number of steps of Q-learning is decreased after the ninth episode, because propagation of a reward signal by Q-value-update is slow. This is not because the state-space is large, but because state-transitions are stochastic so that the optimal path is not clear. However, the number of steps of SSPQL is decreased more quickly than that of Q-learning. Fig. 10 shows the number of steps per episode with the initial configuration set as shown in Fig. 7-(b). This figure also illustrates that our SSPQL method makes the learning process much faster than Q-learning. From the above results, it is apparent that the speed of propagation of reward signals in SSPQL is much faster than that of Q-learning. This is because the shortest path finding

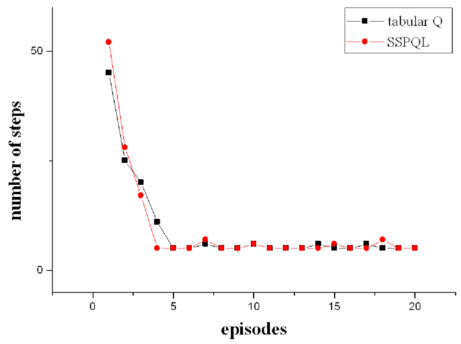


Fig. 11. Number of steps in 'turn' sub-task

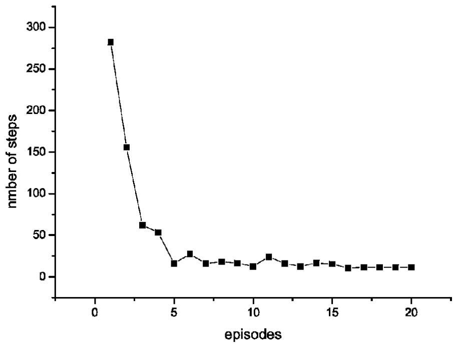


Fig. 12. ‘approach’ sub-task of real robot

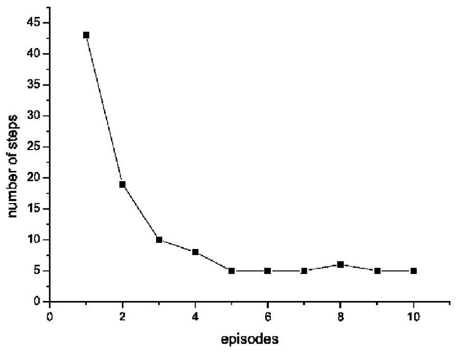


Fig. 13. ‘turn’ sub-task of real robot

method is a batch process with a given state-transition model. Fig. 11 shows the result of ‘turn’ sub-tasks. In this result, the convergence speed of both Q-learning and SSPQL are almost the same, since the state space is small and state-transitions is simple. The state space of ‘turn’ sub-task is restricted in region “A” as shown in Fig. 8-(a) such that the number of state-space is 24 (2×12, 2 for positions and 12 for directions). Finally, Fig. 14 shows the comparison of trajectory of a robot made by SSPQL and hand-coded optimal rules in case of ‘approach’ sub-task. From this result, it seems that there is a negligible difference between learned rules by SSPQL and hand-coded rule.

2) Experimental results for a real-robot: Fig. 12 and Fig. 13 show the results of ‘approach’ and ‘turn’ sub-tasks for a real robot. Experimental results are similar to those of simulations, because simulational environement was prepared to be close to the real environment. It is observed that there was a little increase of steps per episode for the experiment due to more noisy sensor signals and/or more non-deterministic floor characteristics than we expected in simulations.

IV. Conclusion

We proposed a Stochastic Shortest Path-based Q-Learning (SSPQL) method. One advantage of SSPQL method is the improvement of convergence speed when compared to model-free approaches such as Q-learning. The experimental results show that SSPQL is much faster than Q-learning. Another advantage of SSPQL is that it works well even

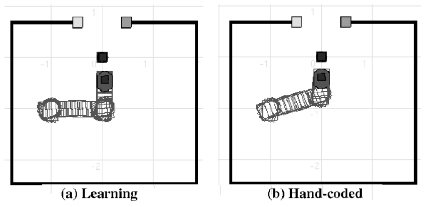


Fig. 14. Trajectory of a robot

when state transitions are stochastic. However, our SSPQL needs to be more improved by considering the followings; Firstly, it requires a lot of memory to store state-transition probabilities and stochastic shortest paths. Secondly, it needs more computations to find shortest path.

As a conclusion, SSPQL would be suitable for fast learning of a task in real environment where state-transitions are non-deterministic and a robot has to be operating under some restricted resources. Here, time is considered as a restricted resource, since a lot of computational time is not usually allowed in real time applications

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