Probabilistic Temporal Prediction for Proactive Action Selection

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I. Introduction

Prediction of a future situation is an important factor contributing to the intelligent interaction of a robot with its environment. Predictive abilities facilitate anticipation and smart decision-making by allowing the robot to decide which action to perform to obtain or avoid a predicted situation. In general, to predict future situation, robots are required to learn complex relationships among many stimuli in real-time; moreover, these relationships can change dynamically and non-deterministically. Therefore, a probabilistic method is required for the prediction of a future situation in real robotic applications.

Many researchers have proposed probabilistic approaches to predict future events from both temporal and causal viewpoints [1] [2]. Among many approaches for prediction of future events, the Dynamic Bayesian Network (DBN) proposed by Dean and Kanazawa [3] has been one of the most widely known methods to predict sequential events. DBN provides a method to represent and infer temporal sequences of events by discretizing time and creating an instance of each random variable at each point in time [4]. Although DBN approaches are widely accepted for the prediction of stochastic events, DBN approaches may lead to difficulties when they are applied to practical applications. Firstly, DBN can only work in the discrete time domain with an uniform time interval; however, many real applications needs various time intervals. Therefore, continuous time-based approaches would be suitable for practical consideration. Secondly, time is not represented explicitly in DBN. Because all random variables are temporally discretized within the same interval in DBN, two similar temporal sequences may be regarded as different. For example, the sequence of ordering a cup of coffee after passing 10 s while sitting at the table is almost the same as the sequence of ordering a cup of coffee after passing 9 s while sitting at the table. Although these two sequences are similar, they may be encoded as different sequences when modeling DBN with an interval of 1 s.

In this paper, we will represent explicitly time as a temporal node in a Bayesian network. By using temporal Bayesian network, both temporal and causal information can be inferred simultaneously within one framework using an explicit representation of time. By predicting both time and causality of an action-triggering event, moreover, there can be also inferred the best proactive action to take and what time is the best to take the proactive action minimizing the waiting time.

II. Probabilistic temporal representation of events

To represent the temporal probability of an event in a specific time interval, we have assumed that the probability of the occurrence of an event is independent of the start time of the event. Thus, the time interval probability of an event can be modeled as the joint probability of those random variables: a random variable corresponding to the occurrence of the event and a random variable corresponding to the start time of the event. The time interval probability of an event $A = true$ from t_1 to t_2 is defined as

$$P(t_1 < a < t_2) \equiv P(s_A)P(t_1 < T_A < t_2) = P(s_A)\int_{t_1}^{t_2} f_{T_A}(t_A)\,dt_A, \quad (1)$$

where S_A is a random variable corresponding to the occurrence of event A, T_A is a temporal random variable corresponding to the start time of event A, $f_{T_A}(t_A)$ is the Probability Density Function (PDF) of the temporal random variable of T_A, and t_A is a value of T_A.

Next, the time interval probability of an event $A = false$ from t_1 to t_2 can be defined as

$$P(t_1 < \neg a < t_2) \equiv P(\neg s_A)P(t_1 < T_{A^*} < t_2) = P(\neg s_A)\int_{t_1}^{t_2} f_{T_{A^*}}(t_{A^*})\,dt_{A^*}, \quad (2)$$

where T_{A^*} is a complementary temporal random variable of an event not occurring. It may be strange to represent a temporal random variable of an event not occurring, because it has no physical meaning in temporal domain. However, the causal probability of $P(\neg a)$ can has a meaningful probability value; $P(\neg a)$ can be obtained by integrating T_{A^*} with respect to all time range. Although T_{A^*} has no physical meaning, it is used to be consistent with a temporal random variable of starting time of an event, T_A.

Because T_A and T_{A^*} are independent, $P(t_1 < a < t_2) + P(t_1 < \neg a < t_2)$ is not 1. Instead, $P(a) + P(\neg a) = 1$. Thus, we can obtain

$$P(-\infty < a < t_1) + P(t_1 < a < t_2) + P(t_2 < a < \infty) + P(-\infty < \neg a < t_1) + P(t_1 < \neg a < t_2) + P(t_2 < a < \infty) = 1. \quad (3)$$

To predict future events, it is necessary to consider time interval between two events rather than a single event. These time intervals can be derived from the conditional probability of events. Given an event A, the temporal probability of B from t_1 to t_2 can be represented as

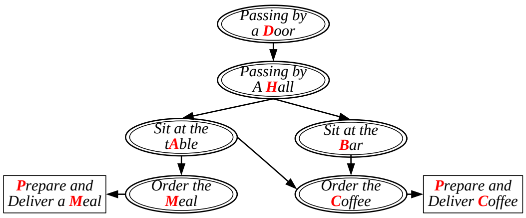


Fig. 1. An example of a temporal Bayesian network of a serving robot scenario.

$$P(t_1 < b|a < t_2) = P(s_B|s_A)\int_{t_1}^{t_2} f_{T_B|T_A}(t_B|t_A)dt_B = P(s_B|s_A)\int_{t_1}^{t_2} f_{T_{BA}}(t_B - t_A)dt_B, \quad (4)$$

where T_{BA} is a temporal random variable of the interval between T_A and T_B. Other time interval probabilities including $P(\neg b|a)$, $P(b|\neg a)$, and $P(b|\neg a)$ can be defined by using the same way as above.

III. Temporal Bayesian Network and Inference

To predict both the temporal and causal probability of an event in a complex situation, we designed a temporal Bayesian network using conditional temporal probabilities as shown in section II. The probability of both when and whether an event occurs can be inferred from the temporal Bayesian network in the same way as the ordinary Bayesian network. Fig. 1 shows a temporal Bayesian network for a robot-serving scenario in a restaurant. A robot has to serve a cup of coffee or a meal when a person orders it. Before ordering a cup of coffee or a meal, the person passes by some site such as a door and a hall and sits at a table or bar. When the person passes these sites or sits at the table or bar, the robot can detect the event; however, these events are not action-triggering events. Only events such as ordering a meal or a cup of coffee are action-triggering events. Non-action-triggering events are called neutral events. By temporally predicting an action-triggering event from observed neutral events, a robot can predict what and when will happen far ahead into the future. The rectangle notation such as ***P**repare-and-Deliver-**M**eal* called as *PM* and ***P**repare-and-Deliver-**C**offee* called as *PC* represents the conditional probability of an action that is triggered by the action-triggering event.

When a person by a door, a serving robot can predict when and whether the person will order a meal by using the temporal Bayesian network as shown in Fig. 1. The temporal probability of ordering a meal given a door-event between time t_1 and t_2 is given by $P(t_1 < m|d < t_2) = P(s_M|s_D)\int_{t_1}^{t_2} f_{T_M|T_D}(t_M|t_D)dt_M$.

By using conditional independence and ancestral sets in the temporal Bayesian network, the nodes B, C, and PC can be eliminated. By marginalization of the random variables A, H, and by using definition the temporal conditional probability, the temporal conditional probability $P(t_1 < m|d < t_2)$ can be obtained.

Marginalization should be performed both causally and temporally. Marginalization with respect to causal random variable is given by

$$P(s_M|s_D) = \sum_{S_A}\sum_{S_H} P(s_M, S_A, S_H|s_D). \quad (5)$$

Next, temporal marginalization is done by integrating the PDF over the entire time range as follows:

$$f_{T_M|T_D}(t_M|t_D) = \sum_{\mathbf{T_A}}\sum_{\mathbf{T_H}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{T_M,\mathbf{T_A},\mathbf{T_H}|T_D}(t_M, \mathbf{t_A}, \mathbf{t_H}|t_D)d\mathbf{t_A}d\mathbf{t_H}, \quad (6)$$

where $\mathbf{T_A} = \{T_A, T_A^*\}$ and $\mathbf{T_H} = \{T_H, T_H^*\}$. From conditional independence from the Bayesian network, (5) and (6) can be simplified. Because (5) can be solved by using well known inference algorithms in conventional Bayesian network, we should pay attention into (6). There are four number of cases in the combination of $\mathbf{T_A}$ and $\mathbf{T_H}$. We will show only one case T_A and T_H; other cases can be given as the same way in the first case. By using conditional independence, (6) can be rewritten as $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{T_M|T_A}(t_M|t_A) f_{T_A|T_H}(t_A|t_H) f_{T_H|T_D}(t_H|t_D)\, dt_B dt_H$.

As in (4), the conditional PDF for two events can be rewritten by using a new temporal random variable of the interval between two events. Temporal terms can be simplified using convolution theory. If two PDFs are sequential, the product of their integrals is given by the integral of their convolutions[1]. Therefore, we can obtain

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{T_M|T_A}(t_M|t_A) f_{T_A|T_H}(t_A|t_H) f_{T_H|T_D}(t_H|t_D)\, dt_A dt_H = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{T_{MA}}(t_M - t_A) f_{T_{AH}}(t_A - t_H) f_{T_{HD}}(t_H - t_D)\, dt_A dt_H = f_{T_{MA}} * f_{T_{AH}} * f_{T_{HD}}(t_M - t_D). \quad (7)$$

Finally, temporal probability of the conditional event is give by

$$P(t_1 < m|d < t_2) = \sum_{S_A}\sum_{S_H} P(s_M, S_A, S_H|s_D) \times \int_{t_1}^{t_2} f_{T_{MA}} * f_{T_{AH}} * f_{T_{HD}}(t_M - t_D)dt_M. \quad (8)$$

Here, the causal random variable can be computed using the same method as in traditional Bayesian network, and all combinations of PDFs for temporal probabilities can be combined into several PDFs using convolution. If PDFs are modeled as well known forms such as Gaussian, uniform, and Dirac delta, the convolution can be computed by using only means and variances of the PDFs.

IV. Proactive Action Selection

It is recalled that important issues in proactive action selection are the best proactive action to take and the best time to take proactive action in order to minimize the waiting time by temporal prediction of an action-triggering events among many neutral events. To decide the best action

[1]The convolution of f and g is $(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)\cdot g(t-\tau)\,d\tau$

depending on the predicted future situation, it is necessary to make a comparison among probabilities of action-triggering events given evidences. Therefore, an index of the best proactive action, pa, is given by

$$pa = \arg\max_{i} P\left(s_i|\mathbf{e}\right), \tag{9}$$

where s_i is the action-triggering event for the i-th action and $\mathbf{e}$ is evidences.

After the best proactive action is selected, a robot should know the best time for minimizing the waiting time of both human and robot. The waiting time is defined by the difference between the start time of an action-triggering event of $t_{s_i|\mathbf{e}}$ and the end time of an action t_{PAE_i} as:

$$t_W = t_{S_i|\mathbf{e}} - t_{PAE_i} = t_{S_i|\mathbf{e}} - t_{PAS_i} - t_{PAD_i}. \tag{10}$$

Where, T_{PAS_i} is a temporal random variable corresponding to the starting time distribution of the proactive action that is related to the action-triggering event, T_{PAE_i} is a random variable corresponding to the end time of the proactive action, and T_{PAD_i} is a random variable corresponding to the duration of the action. In this situation, the person would wait for the robot to serve. The waiting time of the person is given by $T_{Hw} = T_{PAE_i} - T_S$. Using only a Gaussian function and a Dirac delta function for the PDFs, the expectation of the waiting time of the human is given by

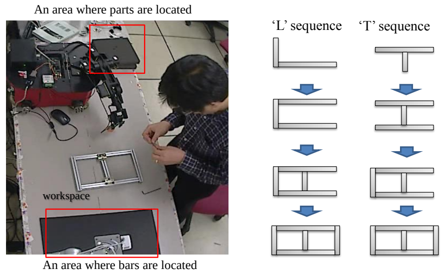
$$\begin{aligned} E[T_{Hw}] &= \int_{-\infty}^{\infty} t_{Hw} f_{T_{Hw}}\left(t_{Hw}\right) dt_{Hw} \\ &= \int_{-\infty}^{\infty} t_{Hw} f_{T_{S_i}|\mathbf{e}} \star f_{T_{PAE}}\left(t_{Hw}\right) dt_{Hw}, \end{aligned} \tag{11}$$

where μ_{PAE_i} is the mean of the PDF corresponding to the end time of the proactive action and $\star$ is the cross-correlation operator[2]. By minimizing the expected waiting time, a robot can select the best time of a proactive action.

V. Experiments

To evaluate our proactive action selection method, we have performed experiments involving a human-robot cooperative scenario. In the scenario, the human being has to make a θ-shaped assembly by using three types of bars and two types of parts. As shown in Fig 2(a), a human pick up a bar on the left side of the workspace and makes an assembly. When the person requests an L-part or T-part, a robot with a 5-DOF arm hands the part to the person. In the human-robot cooperative assembling scenario, the robot does not know how these assemblies are made because there is no visual recognition module for the workspace. Instead, there is a camera to recognize remaining bars in the area where the bars are. By observing the remaining bars, a robot can infer the shape of the assembly. There can be many possible sequences to make a θ-shaped assembly by using three types of bar and two types of part. Fig 2(b) shows two typical sequences to make the assembly: the L-sequence and T-sequence. In the human-robot cooperative scenario, the main objective of the robot is to hand a T-part or L-part just in time for when the person is predicted to order the part. In other words, the robot has to predict both the time and a type of future action-triggering event given evidences, and the robot has to hand over the appropriate part depending on the predicted action-triggering event before the person orders it. Using the temporal prediction of action-triggering events given by the PDFs, the best proactive action and the best time for the selected proactive action is obtained. The task execution time and improvement ratio are shown in Table. I by comparing proactive task execution with on-demand task execution. In the table, it is seen that there is a 33% improvement in task execution time when the proposed proactive action selection method is used.

[2]The cross-correlation of f and g is $(f \star g)(t) = \int_{-\infty}^{\infty} f(\tau) \cdot g(t+\tau)\, d\tau$



(a) Experimental setup. (b) Two typical assembling sequences.

Fig. 2. Illustration of a human-robot cooperation task and the experimental setup.

Assembly method	On-demand	Proactive assistance	improvement
$T-sequence$	6' 20"	4' 10"	34%
$L-sequence$	4' 40"	3' 25"	32%
Averaged	5' 30"	3' 50"	33%

TABLE I
TIME ELAPSED FOR THE ASSEMBLY.

VI. Conclusion

The key contribution of our proposed model is to enable the model to infer both causal and temporal probabilities using a single framework. For this, we have proposed a new type of random variable, which hosts both temporal and causal probabilities. Using new random variables, we have modeled a Bayesian network for both causal and temporal inference; the network has the same structure as the classical Bayesian network for causality. We have showed also that the temporal inference can be given by the convolution of temporal probability density functions associated with causal relations by using the same structure for causal probability, but computational complexity is similar to a classical Bayesian network.

References

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