Abstract—In manipulation tasks, skills are usually modeled using the continuous motion trajectories acquired in the task space. The motion trajectories obtained from a human’s multiple demonstrations can be broadly divided into four portions, according to the spatial variations between the demonstrations and the time spent in the demonstrations: the portions in which a long/short time is spent, and those in which the spatial variations are large/small. In these four portions, the portions in which a long time is spent and the spatial variation is small (e.g., passing a thread through the eye of a needle) are usually modeled using a small number of parameters, even if such portions represent the movement that is essential for achieving the task. The reason for this is that these portions are slightly changed in the task space as compared with the other portions. In fact, such portions should be densely modeled using more parameters (i.e., overfitting) to improve the performance of the skill because the movements of those portions must be accurately executed to achieve the task. In this paper, we propose a method for adaptively fitting these skills based on the temporal and the spatial entropies calculated by a Gaussian mixture model. We found that it is possible to retrieve accurate motion trajectories as compared with those of well-fitted models, whereas the estimation performance is generally higher than that of an overfitted model. To validate our proposed method, we present the experimental results and evaluations when using a robot arm that performed two tasks.

I. INTRODUCTION

In manipulation tasks, skills are usually learned using continuous motion trajectories. An intelligent robot should therefore be able to learn such skills, using a set of the continuous motion trajectories obtained from a human’s multiple demonstrations. In such a set of motion trajectories, the trajectories can be categorized into four portions, according to the spatial variations between the demonstrations and the duration (i.e., time spent) of the demonstrations. Let us consider an example to intuitively understand the portions that constitute a manipulation task. A robot learns a skill for painting an assembly part based on a human’s multiple demonstrations. The procedure is as follows: the robot first lifts up a brush to the part. Next, it uses the brush to paint the part along a zigzag path. Finally, the brush is withdrawn from the part. In this painting example, Fig. 1 shows the set of the motion trajectories obtained from a human’s multiple demonstrations. In the figure, portion (1) involves the painting of the assembly part, and the other portions involve the lifting of the brush and withdrawal of the brush from the part. Portion (1) takes up the longest time, although the movement is slightly changed in the demonstrations when the part is fixed in a specific location, as shown in Fig. 1.

Next, let us consider modeling a Gaussian Mixture Model (GMM) using the motion trajectories in Fig. 1. The GMM is estimated using Bayesian Information Criterion (BIC) and Expectation-Maximization (EM) algorithms, as shown in Fig. 2-(a). In the GMM, portion (1) in Fig. 1 is sparsely modeled, even though the GMM is well fitted by the BIC and EM algorithms. The problem can be formulated in terms of the differences between Fig. 1 and Fig. 2-(b). There is no zigzag path in the motion trajectories retrieved by a Gaussian Mixture Regression (GMR) process. The GMM should be modeled to use more parameters to retrieve the zigzag path because this path is essential in achieving the task. When the number of parameters (i.e., the number of Gaussians) is forcefully increased, in this context, the rest of the portions are more densely overfitted than portion (1), as shown in Fig. 3. The reason for this is that the changes of the movements in the rest of the portions are larger than they are in portion (1). Although in portion (1), slightly accurate motion trajectories can be retrieved by the GMM in Fig. 3, the estimation performance is lower than that of the GMM in Fig. 2. As shown in Fig. 3-(b), the motion trajectories are overfitted in all portions. The model should be able to remodeled using more parameters only in portion (1), while maintaining the rest of the portions.

In this context, we propose a method for learning skills that consider the spatial variations between multiple demonstrations and the time spent in these demonstrations, based on the entropies involved. Fig. 4 shows the entire process of the proposed method. The set of the continuous motion trajectories obtained from multiple demonstrations is temporally aligned using a Dynamic Time Warping (DTW) algorithm, as shown in Fig. 4-(a). The motion trajectories are projected onto the reduced dimensional subspace by Principal Component Analysis (PCA), as shown in Fig. 4-(b). A GMM is estimated to contain the temporal and the spatial information based on the BIC and EM algorithms for the calculation of the entropies of each portion, as shown in Fig. 4-(c). The temporal and the spatial entropies are calculated as per the Gaussians from the estimated GMM,
Fig. 1. Motion trajectories acquired by a human’s two demonstrations in painting example: (a) motion trajectories observed on x-z axes and (b) motion trajectories observed on time axis. Here, portion (1) indicates the motion trajectories of the painting of the assembly part.

Fig. 2. GMM well fitted by BIC, and motion trajectories retrieved by GMR in painting example: (a) GMM estimated by four Gaussians using BIC and EM algorithms and (b) motion trajectories retrieved by GMR. Here, the red-dotted boxes are portion (1) from Fig. 1.

Fig. 3. Overfitted GMM, and motion trajectories retrieved by GMR in painting example: (a) GMM estimated by eight Gaussians and (b) motion trajectories retrieved by GMR. Here, the red-dotted boxes are portion (1) from Fig. 2.

as shown in Fig. 4-(d). Based on the entropies, the portions that should be remodeled are segmented by the Gaussians of the GMM, as shown in Fig. 4-(e). Based on the BIC and EM algorithms, only the segmented portions are remodeled using more parameters.

Many researchers have proposed various approaches for learning manipulation skills. Calinon et al. and Hersch et al. proposed the methods for learning skills and retrieving motion trajectories based on GMM and GMR [1], [2]. Kruger et al., Kulic et al., and Lee et al. used the Hidden Markov Model (HMM) for recognizing and reproducing skills [3]–[5]. Pastor et al., Ude et al., and Kormushev et al. proposed the methods for learning the external force in a set of differential equations (i.e., dynamic movement primitives) [6]–[8]. Using these approaches, the learning of manipulation skills has been verified through many experiments to date. These approaches, however, did not explicitly consider adaptively modeling the skills by considering the entropies obtained from multiple demonstrations. The problem we noted earlier is frequently encountered in the process of learning skills, in which various manipulation tasks consist of portions with different values and levels of significance. In this context, we propose a method for modeling skills by adopting an explicitly different model fitting strategy that uses the entropies obtained from multiple demonstrations. Vigorito et al. proposed a method for hierarchically and incrementally modeling skills in the form of a Markov decision process, by selecting the training examples based on
The rest of this paper is organized as follows: Section II describes the details of learning a skill based on the temporal and the spatial entropies. Section III presents the experimental results of using a robot arm to perform two tasks: painting an assembly part, and assembling the part. In this section, we describe the evaluation results, which were obtained using two criteria: the root mean square error and the log-likelihood. Section IV discusses the proposed method. Finally, in Section V we present our conclusions and plans for future research.

II. SKILL LEARNING USING TEMPORAL AND SPATIAL ENTROPIES

A Gaussian of the GMM competitively partitions the input space and learns a linear regression segment in the hyper-sphere [12]. Representing the continuous motion trajectories as a GMM provides a way of encoding the local directions and the local relations among the variables taking part in the trajectories. As a result, a GMM expresses such motion trajectories as a number of sub-portions, using the set of the Gaussians. In this paper, the temporal and the spatial entropies are therefore obtained as per the Gaussians of the GMM to detect the portions that should be remodeled using more parameters. Here, the temporal and the spatial entropies indicate the amount of uncertainties (i.e., spatial variations and time durations) in the temporal and the spatial variables.

So the reader can intuitively understand our entropy-based skill learning method, we present the entire process, using the products of each step obtained from the manipulation task of painting an assembly part, which was introduced in Section I. The end-effector’s continuous motion trajectories (i.e., the (x, y, z) positions of the end-effector) of a robot arm developed by Neurosensics (i.e., Katana) are recorded at 50 Hz using a kinesthetic teaching method.

The continuous motion trajectories, \( \mathbf{X} \in \mathbb{R}^{(D+1)\times N} \), of the robot arm are extracted from the human’s multiple demonstrations. Here, \( (D+1) \) denotes a \( D \)-dimensional spatial variable and a one-dimensional temporal variable, and \( N \) is the sum of the lengths of the multiple trajectories. In the painting task, the motion trajectories, \( \mathbf{X} \in \mathbb{R}^{(3+1)\times 2114} \), which are extracted from five demonstrations are shown in Fig. 5-(a). Before modeling a GMM, the motion trajectories should be temporally aligned by measuring the similarity between the multiple demonstrations, which may vary in time or speed, as shown in Fig. 5-(b), based on the DTW algorithm [13]. The temporally aligned motion trajectories, \( \mathbf{X}_{\text{draw}} \in \mathbb{R}^{(3+1)\times 2165} \), are then projected into the low-dimensional space by the PCA algorithm. The reasoning behind this is as follows: the determinants of the covariance matrices contained in the GMM should be calculated first to obtain the temporal and the spatial entropies. There may be singularities when calculating the determinants from the GMM estimated in the original space. The correlated variables of the original space should therefore be transformed into linearly uncorrelated variables in the subspace; when the GMM is estimated using the EM algorithm, it becomes increasingly difficult
to converge to appropriate local optima when increasing the dimension of the variable [14].

In this context, the original motion trajectories, except for the temporal variable, are transformed as

\[ \Psi' = A^{-1}(X_{\text{org}} - \bar{X}_{\text{org}}) \]

where \( X_{\text{org}} \in \mathbb{R}^{D \times N} \), \( \bar{X}_{\text{org}} \in \mathbb{R}^{D' \times N} \), and \( A \in \mathbb{R}^{D' \times D} \). Here, \( \Psi' \in \mathbb{R}^{D' \times N} \) refer to the original trajectories, the transformed motion trajectories, the means of the original trajectories, the transformation matrix of PCA, and the motion trajectories in the reduced dimensional space, respectively. Here, \( D' \) denotes the \( D' \)-dimensional spatial variable transformed by PCA. In the painting task, the three-dimensional motion trajectories are projected into two-dimensional trajectories, \( \Psi' \in \mathbb{R}^{(2+1) \times 2165} \), by PCA (Here, the dimensionality is determined by the eigenvectors, in which the value of the largest eigenvalue is 0.99).

A GMM is then estimated based on the BIC and EM algorithms. The GMM is defined as

\[ P(\Psi) = \sum_{i=1}^{K} w_i \cdot N(\Psi_i | \mu_i, \Sigma_i) \]

where \( w_i \), \( \mu_i \), and \( \Sigma_i \) refer to the priors, the means, and the covariances of the \( i \)-th Gaussian, respectively. The GMM is initialized by a k-means clustering algorithm, and the number of Gaussian \( K \) is determined by the BIC algorithm [15]. The BIC algorithm is a method for resolving the overfitting problem based on the criterion of minimum description length. The GMM can be well fitted, without overfitting, by the BIC algorithm. In the painting task, the GMM consists of five Gaussians based on the BIC and EM algorithms, as shown in Fig. 6-(a).

Before calculating the temporal and the spatial entropies, the means and the covariances of the GMM are divided into two parts, the temporal and the spatial components. The mean and the covariance matrices of the \( i \)-th Gaussian are represented as

\[ \mu_i = \{ \mu_i,t, \mu_i,s \} \]

\[ \Sigma_i = \begin{pmatrix} \Sigma_{i,t} & \Sigma_{i,t,s} \\ \Sigma_{i,s,t} & \Sigma_{i,s} \end{pmatrix} \]

where \( t \) and \( \Psi' \) refer to the one-dimensional temporal variable and the \( D' \)-dimensional spatial variable in \((D' + 1)\)-dimensional variable \( \Psi \).

In the GMM, each Gaussian is represented as a multivariate normal distribution. The temporal and the spatial entropies are therefore calculated as

\[ H_i = \frac{1}{2} \ln((2\pi e)|\Sigma_{i,t}|) \]

\[ H_i^{\Psi'} = \frac{1}{2} \ln((2\pi e)^{D'}|\Sigma_i \Psi'|) \]

where \( \Sigma_{i,t} \) and \( \Sigma_i \Psi' \) refer to the covariances of the temporal and the spatial components represented in (4), and \( D' \) denotes the dimensionality of the spatial variable. In the painting
As noted earlier, we focus on remodeling those portions in which a long time is spent and the spatial variation is small as shown in Fig. 8. Based on the certain thresholds, which are indicated by dotted lines in Fig. 8, the portions associated with the upper and the lower entropies of the temporal and the spatial thresholds should be remodeled. Here, the detection of the portions depends on the specific thresholds. In this paper, the thresholds are determined by the mean values of the entropies, though a human expert can adaptively adjust the thresholds.

To remodel a portion, the motion trajectories contained in that portion should then be segmented from the complete motion trajectories. In the GMM, the intersections between two consecutive Gaussians can be estimated from the weights along the time component of the GMM. The intersections are therefore used as the segmentation points of the motion trajectories [16]. The weights estimated along the time component are calculated as

$$h_i(t) = \frac{w_i N(t; \mu_i, \Sigma_i)}{\sum_{k=1}^{K} w_k N(t; \mu_k, \Sigma_k)}, \quad (7)$$

where $i$ and $K$ refer to the index of Gaussians and the total number of Gaussians, respectively. $\mu_i$ and $\Sigma_i$ denote the parameters of the time component in (3) and (4), respectively.

Fig. 9 shows the weight $h(t)$ estimated along the time component of the GMM and all intersections in the painting task. The segmentation points are at 40, 97, 179, and 364 in the entire 433 steps of the motion trajectories.

The portions selected by the entropies are re-estimated as individual GMMs by the BIC and EM algorithms. It is of course possible for them to be remodeled using more parameters, because the GMMs are independently re-estimated using only the portions, even though the portions are nearly unchanged in the task space as compared with the other portions. In the painting task, the parameters obtained from the remodeled GMM are combined into the parameters of the original GMM after the segmented portion of the fourth Gaussian is remodeled by the BIC and EM algorithms. The combined GMM is finally estimated as shown in Fig. 10-(a).

These remodeling processes are repeated until the temporal and the spatial entropies of all portions are satisfied within the thresholds. Fig. 11 shows the temporal and the spatial entropies obtained from the remodeled GMM. In Fig. 11, the remodeling processes are complete because the entropies of all portions are satisfied within the threshold of the temporal entropy. The motion trajectories retrieved from the remodeled GMM can be qualitatively identified as having been improved because the zigzag paths were retrieved from the remodeled GMM, as shown in Fig. 10-(b).
III. EXPERIMENTAL RESULTS AND EVALUATION

To validate the entropy-based skill learning method, the task of assembling the part was additionally tested using the Katana arm. The three-dimensional motion trajectories of the assembling task were acquired using the following procedures: the robot picks up the part, and the part is then delivered a bar, after which it is inserted into the bar. Finally, the robot arm is withdrawn from the bar. The human’s ten demonstrations were acquired without changing the position of the objects (i.e., the part and the bar). The motion trajectories were also recorded at 50Hz using a kinesthetic teaching method. In the ten demonstrations, five motion trajectories were used for learning the skills and the rest were used for evaluating the learned skills.

Figs. 12-15 show the observed data obtained from the procedures for learning the skill, based on the entropies in the assembling task. The motion trajectories, \( \mathbf{X} \in \mathbb{R}^{(3+1) \times 1017} \), which were extracted from five demonstrations are temporally aligned by the DTW algorithm. Here, the four dimensions indicate the \((x, y, z)\) of the Katana’s end-effector and one-dimensional time step. To calculate the temporal and the spatial entropies, the temporally aligned motion trajectories, \( \mathbf{X}_{dtw} \in \mathbb{R}^{(3+1) \times 1195} \), are then projected onto the \((2+1)\)-dimensional motion trajectories (using eigenvectors in which the sum of the eigenvalues is 0.99) using PCA, as shown in Fig. 12. Based on the BIC and EM algorithms, the GMM is estimated as containing six Gaussians, as shown in Fig. 13-(a). The temporal and the spatial entropies are calculated as per the Gaussians of the learned GMM, as shown in Fig. 14. The thresholds are determined by the mean values of the temporal and the spatial entropies, as in the painting task. According to the thresholds, in the assembling task, the portions associated with the second and fifth Gaussians should be remodeled using more parameters, as shown in Fig. 14. Here, the second and fifth Gaussians physically indicate the portions of grasping the part and inserting the part into the bar in the complete trajectories.

To remodel the portions, the segmentation points are then extracted from the weights estimated along the time component of the GMM. The segmentation points are 49, 95, 126, 160, and 209 in the entire 239 lengths of the motion trajectories. The two portions (i.e., the portion between 49 and 95, and the portion between 160 and 209) divided by the segmentation points are re-estimated using the BIC and EM algorithms. The two GMMs are then remodeled so as to contain each of the two Gaussians and three Gaussians in the portions. Fig. 15 shows the remodeled GMM, based on the entropies and the motion trajectories retrieved by the GMR process.

To quantitatively analyze the proposed method, we evaluate the learned GMMs in terms of two criteria: the root mean square errors (RMSE) and the log-likelihood estimation. The GMM estimated by our method is obviously overfitted as compared to the GMM that is well fitted by BIC be-
because some portions are re-estimated using more parameters. Therefore, the GMM should be evaluated using the criterion of the retrieval of accurate motion trajectories (i.e., the RMSE between the motion trajectories of the training data and the motion trajectories retrieved from the learned GMM), as well as the criterion of the degree of overfitting (i.e., the log-likelihood estimated using the set of the test data).

Based on these two criteria, we evaluate our method by comparing the three GMMs as follows: 1) the GMM well fitted without overfitting by the BIC, based on the principle of the minimum description length; 2) the GMM in which the specific portions are overfitted, using the entropies calculated by our method; and 3) the GMM overfitted using the same number of parameters as used with the GMM of 2).

Fig. 16-(a) shows the results of the RMSE obtained by comparing each of the training data with the motion trajectories retrieved from the three GMMs by GMR process in the painting task. When the RMSE is small, the GMM retrieves the accurate motion trajectories. In Fig. 16-(a), our focus is on the portion of the fourth segment because that portion was remodeled using the greater number of parameters in the GMM of 2) than in the GMM of 1). In this portion, the motion trajectories retrieved from the GMM of 2) are the most accurate because the RMSE obtained by the GMM of 2) is the smallest. The RMSE is not important except for the portion of the fourth segment, although the GMM of 3) retrieves more accurate motion trajectories in some portions. It can be worse for the portions to be overfitted, because these portions represent the motion trajectories in which the spatial variation is large such as the physical acts of lifting the brush and withdrawing the brush.

As noted earlier, the GMMs of 2) and 3) were overfitted as compared to the GMM of 1). The degree of overfitting should be evaluated using the set of test data. Fig. 16-(b) shows the log-likelihood estimated by the three GMMs, using the test data of the painting task. Although the GMM of 2) is also overfitted using the same number of parameters as are used in the GMM of 3), its estimation performance is better than that of the GMM of 3). The reason for this is that the GMM of 2) is overfitted only in that portion in which the spatial variation is small. Moreover, the estimation performance in the GMM of 2) is better than in the GMM of 1) even though the GMM of 2) is overfitted due to the use of more parameters. The rationale for this is that the likelihood is very high in the portion of the fourth segment because the overfitted portion allows for very small spatial variation, even in the set of test data.

Fig. 17-(a) shows the results of the RMSE obtained from the three GMMs in the assembling task. Here, we also focus the two portions of the second and the fifth segments because these portions were remodeled using more parameters. In these portions, more accurate motion trajectories are retrieved from the GMMs of 2) and 3) than from the GMM of 1). Although the GMM of 3) can retrieve more accurate motion trajectories than the GMM of 2) in the portion of the second segment, the estimation performance in the GMM of 3) is the worst among the GMMs when using the set of test data, as shown in Fig. 17-(b). The estimation performance in

![Fig. 15. GMM remodeled by temporal and spatial entropies, and motion trajectories retrieved by GMR in assembling task: (a) GMM remodeled by entropies. Here, the GMM consists of nine Gaussians. (b) Motion trajectories retrieved by GMR. Here, the red-dotted boxes indicate the Gaussians remodeled by the segmented motion trajectories.](image)

![Fig. 16. Results of RMSE and log-likelihood estimation in painting task: (a) RMSE obtained by comparing training data with motion trajectories retrieved by three GMMs and (b) log-likelihood estimated by three GMMs, using the set of test data.](image)

![Fig. 17. Results of RMSE and log-likelihood estimation in assembling task: (a) RMSE obtained by comparing training data with motion trajectories retrieved by three GMMs and (b) log-likelihood estimated by three GMMs, using the set of test data.](image)
As a result, the entropy-based skill learning method, which considering the spatial variations and the time spent, makes it possible for more accurate motion trajectories to be retrieved as compared with well-fitted models, whereas the estimation performance is generally higher than it is for overfitted models.

IV. DISCUSSION

When modeling skills using multiple demonstrations, many approaches used to date have well fitted the models using an adequate number of parameters. In manipulation tasks, there are a number of motion trajectories in which the changes in the movement are small, even though long time is spent on them. Common examples of this are passing a thread through the eye of a needle or the portion of time spent inserting one object into another object. Such portions tend not to be modeled using sufficient parameters, even though these portions are of great importance in achieving the task. Such portions should be overfitted by using sufficient parameters to improve the performance of the models. When overfitting the models, however, the portions in which the spatial variation is large tend to be densely modeled. Therefore, these portions should individually be handled by being segmented from the portions in which the spatial variation is small. Here, the models should moreover be considered from the viewpoint of the estimation because the models are overfitted for improving the accuracy of the models. In the entropy-based skill learning method, the models can retrieve more accurate motion trajectories by comparing the well-fitted models, while their performance of estimation is maintained.

In this paper, the entropy-based skill learning method is verified using the GMMs. This core idea can be also applied to well-known skill learning approaches such as HMMs and Dynamic Movement Primitives (DMPs). Moreover, our skill learning method can be exploited as a preprocessing stage. In the learning process of the HMMs, the parameters obtained by the GMMs can be used as the initial parameters for Baum-Welch algorithm. The HMMs can be well fitted by the initial parameters (i.e., the number of states, the means, and the covariances) of the GMMs considering the accuracy and the estimation performance. Learning the external forcing terms is important in the DMPs. Our skill learning method can provide information for adjusting the arrangement of the Gaussian basis functions because the forcing terms are fitted by the Gaussian basis functions. This method can also be used to facilitate approaches to incremental learning because the incremental learning process can be efficiently executed by focusing on the portions that should be remodeled.

V. CONCLUSIONS AND FUTURE WORKS

In this paper, we have proposed a method for learning skills based on the temporal and the spatial entropies involved. This learning method focuses on those portions in which a long-time is spent, even though the movement slightly changes because those portions are of great importance in achieving the task. In this context, we have modeled skills by adopting an explicitly different model fitting strategy that is based on the entropies obtained from multiple demonstrations. There is a certain advantage to the use of such an entropy-based skill learning method. The skill can retrieve accurate motion trajectories by adaptively overfitting only those portions that need to be remodeled using more parameters, while maintaining the performance of the estimation.

In our future work, we intend to apply our scheme to various types of motion trajectories such as force/torque. Furthermore, we aim to apply this learning method to efficient skill learning approaches such as the HMMs and DMPs.

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