for all t_1 and t_2 ($\geqslant t_1$), where

$$\|x_t\| \triangleq \sup_{t-h_\alpha \leqslant \tau \leqslant t} |x(\tau)|$$

$$|x(\tau)| \triangleq (x^T(\tau)x(\tau))^{1/2}.$$

III. Stabilizability Condition

The result of this note is summarized as follows.

Theorem: If 1) $(A_0(\cdot), B(\cdot))$ is a uniformly completely controllable pair [11], and 2) the columns of $A_1(t)$ can be represented as linear combinations of the columns of $B(t)$ at every t, then for any specified stability degree, there exists a feedback gain $K(\cdot)$ which actualizes the degree in the closed loop system (4).

Remark: The proof of the theorem implies that the feedback gain can be constructed without the precise knowledge of the delay $h(\cdot)$; it is enough to know the upper bounds h_α of the delay and h_β of the derivative.

The following lemmas are necessary to show the theorem.

Lemma 1 [12]: If $(A_0(\cdot), B(\cdot))$ is a uniformly completely controllable pair, then $(A_0(\cdot)+\lambda I, B(\cdot))$, where λ is any real number and I is the identity matrix, is also a uniformly completely controllable pair.

Lemma 2 [12]: If $(A_0(\cdot)+\lambda I, B(\cdot))$ is a uniformly completely controllable pair, then the matrix Riccati equation

$$\dot{P}(t) + (A_0^T(t)+\lambda I)P(t) + P(t)(A_0(t)+\lambda I) - P(t)B(t)B^T(t)P(t) = -I \quad (6)$$

has a solution which satisfies

$$a_1 I \leqslant P(t) \leqslant a_2 I \quad (7)$$

for all t, where a_1 and a_2 are positive numbers.

Proof of Theorem: Assume that the conditions 1) and 2) of the theorem are satisfied. Then there exists a matrix $Q(\cdot)$ such that

$$A_1(t) = B(t)Q(t) \quad (8)$$

and, from Lemmas 1 and 2, a matrix $P(\cdot)$ exists which satisfies (6) and (7).

Define the feedback gain $K(\cdot)$ for any specified stability degree λ by

$$K(t) = -\frac{1}{2}\left(I + \frac{\exp(2\lambda h_\alpha)}{1-h_\beta} Q(t)Q^T(t)\right)B^T(t)P(t). \quad (9)$$

Then the closed loop system (4) is described by

$$\dot{x}(t) = \left(A_0(t) - \frac{1}{2}B(t)\left(I + \frac{\exp(2\lambda h_\alpha)}{1-h_\beta} Q(t)Q^T(t)\right)B^T(t)P(t)\right)x(t) + B(t)Q(t)x(t-h(t)). \quad (10)$$

Let a scalar function be defined by

$$V(t) = \exp(2\lambda t)x^T(t)P(t)x(t) + \int_{t-h(t)}^{t} \exp(2\lambda s)x^T(s)x(s)\,ds. \quad (11)$$

Then $V(\cdot)$ satisfies

$$a_1|x(t)|^2\exp(2\lambda t) \leqslant V(t) \leqslant (a_2+h_\alpha)\|x_t\|^2\exp(2\lambda t) \quad (12)$$

and the time derivative along any solution of (10) satisfies

$$\begin{aligned}\dot{V}(t) = &-\exp(2\lambda t)\frac{\exp(2\lambda h_\alpha)}{1-h_\beta} \\ &\cdot|Q^T(t)B^T(t)P(t)x(t) - (1-h_\beta)\exp(-2\lambda h_\alpha)x(t-h(t))|^2 \\ &-\exp(2\lambda t)\big((1-\dot{h}(t))\exp(-2\lambda h(t)) - (1-h_\beta)\exp(-2\lambda h_\alpha)\big) \\ &\cdot|x(t-h(t))|^2 \\ \leqslant\ & 0. \end{aligned} \quad (13)$$

From these inequalities, it can be shown that the solution of (10) satisfies

$$|x(\tau)| \leqslant \tilde{c}\|x_{t_1}\|\exp(-\lambda(\tau-t_1)) \quad (14)$$

for all t_1 and τ ($\geqslant t_1$), where $\tilde{c} \triangleq ((a_2+h_\alpha)/a_1)^{1/2}$. Hence

$$\begin{aligned}\|x_{t_2}\| &= \sup_{t_2-h_\alpha \leqslant \tau \leqslant t_2} |x(\tau)| \\ &\leqslant \sup_{t_2-h_\alpha \leqslant \tau \leqslant t_2} \tilde{c}\|x_{t_1}\|\exp(-\lambda(\tau-t_1)) \\ &= \tilde{c}\exp(\lambda h_\alpha)\|x_{t_1}\|\exp(-\lambda(t_2-t_1)) \end{aligned} \quad (15)$$

for all t_1 and t_2 ($\geqslant t_1$).

Thus the feedback gain $K(\cdot)$ defined by (9) actualizes the specified stability degree λ in the closed loop system (10). Q.E.D.

IV. Conclusion

We have obtained a sufficient condition for the possibility of actualizing an arbitrarily specified stability degree in a linear time-varying delay system by means of linear feedback without delay. The stabilizing feedback stated in this note has the merit that it can be realized even if the time-varying delay is not known precisely.

References

[1] L. A. Zadeh, "Operational analysis of variable-delay systems," *Proc. IRE*, vol. 40, pp. 564–568, May 1952.
[2] S. G. Margolis and J. J. O'Donnell, "Rigorous treatments of variable time delays," *IEEE Trans. Electron. Comput.*, vol. EC-12, pp. 307–309, June 1963.
[3] J. P. Seddon and R. A. Johnson, "The simulation of variable delay," *IEEE Trans. Comput.* (Short Notes), vol. C-17, pp. 89–94, Jan. 1968.
[4] R. A. Johnson, "Functional equations, approximations, and dynamic response of systems with variable time delay," *IEEE Trans. Automat. Contr.* (Tech. Notes and Corresp.), vol. AC-17, pp. 398–401, June 1972.
[5] Y. Sato, "A study on stability and stabilization of a time or state-variable time lag system," M. Eng. thesis (in Japanese), Dep. Syst. Eng., Kobe Univ., Kobe, Japan, 1978.
[6] Y. S. Osipov, "Stabilization of controlled systems with delays," *Differential Equations*, vol. 1, pp. 463–473, May 1965.
[7] D. W. Ross and I. Flugge-Lotz, "An optimal control problem for systems with differential-difference equation dynamics," *SIAM J. Contr.*, vol. 7, pp. 609–623, Nov. 1969.
[8] G. J. Nazaroff, "Stability and stabilization of linear differential delay systems," *IEEE Trans. Automat. Contr.* (Tech. Notes and Corresp.), vol. AC-18, pp. 317–318, June 1973.
[9] G. J. Nazaroff and G. A. Hewer, "Stabilization of linear, autonomous differential delay systems," *IEEE Trans. Automat. Contr.* (Tech. Notes and Corresp.), vol. AC-18, pp. 673–674, Dec. 1973.
[10] K. Ryo, M. Ikeda and S. Kitamura, "Stabilization of linear systems with delay: An extended stabilizability condition" (in Japanese), *Trans. SICE*, vol. 13, pp. 221–226, June 1977.
[11] R. E. Kalman, "Contributions to the theory of optimal control," *Bol. Soc. Mat. Mexicana*, vol. 5, pp. 102–119, 1960.
[12] M. Ikeda, H. Maeda, and S. Kodama, "Stabilization of linear systems," *SIAM J. Contr.*, vol. 10, pp. 716–729, Nov. 1972.

Proportional Minus Delay Controller

I. H. SUH AND Z. BIEN

***Abstract*—A new type of controller, which utilizes the time-delay effect, is proposed. It is shown that the conventional P-controller equipped with an appropriate time-delay performs an averaged derivative action and thus can replace the PD-controller, showing quick responses to input changes but being insensitive to high-frequency noise.**

I. Introduction

In many industrial processes, the presence of time delay often causes various practical difficulties in the system control and a great deal of research efforts have been devoted to solving the problem of controlling

Manuscript received October 3, 1978.
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0018-9286/79/0400-0370$00.75

the system with inherent time delays [1], [2]. On the other hand, however, it has also been proposed to use the time delay in the controller design to improve the overall system performances. For example, it was suggested in [3] that the time delay be used to stabilize an unstable system, and an example was illustrated for such compensating time-delay elements. Other examples may be found in [4] and [5], where Smith and Rubin had shown that in the control of lightly damped oscillatory systems, the time delay when used in the controller may cancel the effect of oscillatory complex poles and produce a deadbeat response.

In this note, it is further shown that the time delay, incorporated in conventional controllers, can improve the system by its averaged derivative action.

II. Averaging Effect of Proportional Minus Delay Action

Consider a closed-loop control system shown in Fig. 1 in which the controller has the following transfer function:

$$U(s)/E(s)=\hat{K}_p-\hat{K}_d e^{-T_d s}, \qquad \hat{K}_p \geq \hat{K}_d. \tag{1}$$

Here $\hat{K}_p$ and $\hat{K}_d$ are positive parameters to be adjusted and T_d is the intentional time delay introduced for the effect of averaged derivative action. We call the controller of (1) as the proportional minus delay controller or PMD-controller for short. To find out the features of such a controller, it is first observed that (1) can be rewritten as

$$\begin{aligned} u(t)&=\hat{K}_p e(t)-\hat{K}_d e(t-T_d)\\ &=(\hat{K}_p-\hat{K}_d)e(t)+T_d\hat{K}_d[(e(t)-e(t-T_d))/T_d]. \end{aligned} \tag{2}$$

On the other hand

$$\frac{1}{T_d}[e(t)-e(t-T_d)]=\frac{1}{T_d}\int_{t-T_d}^{t}\dot{e}(t)\,dt \tag{3}$$

where $\dot{e}(t)=(de(t)/dt)$. Combining (2) and (3), and letting $K_p=\hat{K}_p-\hat{K}_d$, and $K_d=T_d\hat{K}_d$, one may write (2) as

$$u(t)=K_p e(t)+K_d\left[\frac{1}{T_d}\int_{t-T_d}^{t}\dot{e}(t)\,dt\right]. \tag{4}$$

Thus the proportional minus delay controller performs the proportional action plus the derivative action averaged over a period of T_d. Compare this controller with the conventional PD (proportional plus derivative)-controller whose control action may be represented as

$$u(t)=K_p' e(t)+K_d'\dot{e}(t). \tag{5}$$

If the two controllers are adjusted so that $K_p=K_p'$ and $K_d=K_d'$, and further, if the time delay T_d is chosen to be relatively small compared with the time constant of the controlled system but relatively large compared with the periods of high frequency components of noise, then one may conclude that: 1) the two controllers exhibit almost the same control actions for the input changes; but 2) the derivatives of the noise components in the PMD controller are averaged (to zero, possibly) but those in the PD controller remain active. These features are illustrated by the following example.

III. Simulation

For simplicity, the controlled process is chosen as a first-order system

$$G(s)=\frac{1}{s+1},$$

and the input change $r(t)$ and the noise $w(t)$ are arbitrarily chosen as

$$r(t)=1.5(1-e^{-6t}), \qquad t\geq 0$$
$$w(t)=0.2\sin 100t, \qquad 0\leq t\leq 5.$$

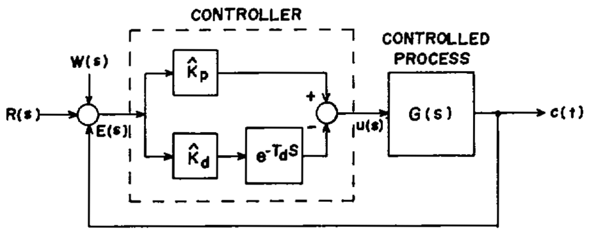

Fig. 1. Closed-loop system with PMD-controller.

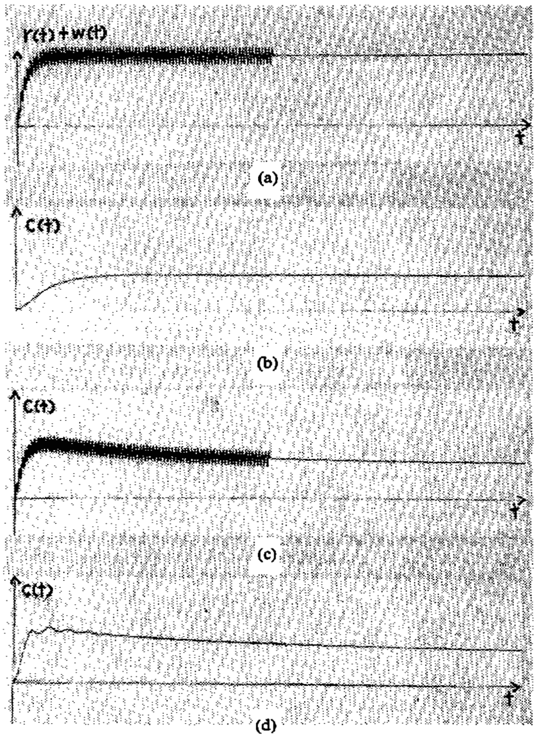

Fig. 2. Output response for various controller types. (a) Input with noise. (b) Output for P-controller. (c) Output for PD-controller. (d) Output for PMD-controller.

For the parameters $K_p=K_p'=1$, $K_d=K_d'=4$, and $T_d=0.314$, the responses $c(t)$ of the system for the P-, PD-, and PMD-controllers are sketched in Fig. 2. These graphs are obtained via a digital computer simulation. Also, the frequency responses of the system for PD- and PMD-controllers are given in Fig. 3. One may observe that the PMD-action shows the advantages of the PD-controller, that is, it yields fast response (compared to P-controller action), but it is insensitive to noise signal and thus eliminates the disadvantage of PD-controller action. It would be instructive to compare the control actions $u(t)$ themselves responding to the same noisy input signal as shown in Fig. 4.

It is remarked that the steady-state error of the system when the PMD-controller is used is the same as that for PD-control action. Also, it can be easily shown (see [7]) that the closed-loop system using the PMD-controller is stable as is the one with PD action.

IV. Concluding Remarks

In designing a control system, the derivative action of the controller is desirable for obtaining a fast response and/or smoothing an oscillatory response. When the control system is subject to noisy inputs, however, the function of the controller with derivative action may be jeopardized due to its sensitivity to high frequency noise [6]. In this case, an appropriate proportional minus delay controller seems to be a good replacement as the example of the PMD-controller shows. The conventional PID-controller may be modified in a similar manner. It seems that

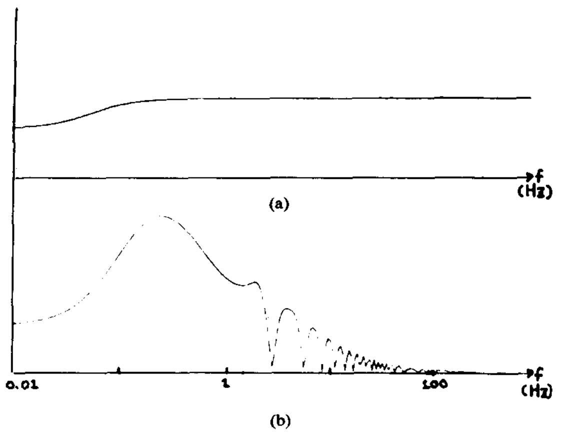

Fig. 3. Frequency response. (a) $\left|\frac{C(jw)}{R(jw)}\right|$ for PD-controller. (b) $\left|\frac{C(jw)}{R(jw)}\right|$ for PMD-controller.

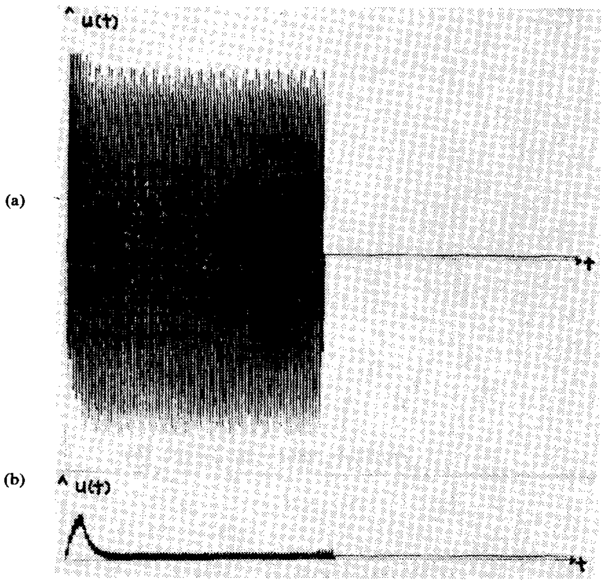

Fig. 4. Control actions. (a) PD-controller action. (b) PMD-controller action.

the advent of programmable digital controllers using microprocessors makes such a controller with time delay plausible and attractive.

The magnitude of the time delay T_d should be determined according to various factors such as the following: 1) time-constant of the system; 2) frequencies of undesired noises; and 3) the stability of the closed-loop system. It is remarked, in particular, that the overall system using the delay action may be unstable while the same system with derivative action is not, and hence the designer needs to examine the stability of the resultant delay-differential equations.

References

[1] Y. Alekal, P. Brunovsky, D. H. Chyung, and E. B. Lee, "The quadratic problem for systems with time delays," *IEEE Trans. Automat. Contr.*, vol. AC-16, pp. 673–87, 1971.
[2] H. T. Banks and A. Manitius, "Application of abstract variational theory to hereditary systems—A survey," *IEEE Trans. Automat. Contr.*, vol. AC-19, pp. 524–33, 1974.
[3] N. H. Chosky, "Time lag systems," in *Progress in Control Engineering*, vol. 1, R. H. MacMillan *et al.*, Eds. New York: Academic, 1962.
[4] O. J. M. Smith, "Analog study of deadbeat posicast control," *IRE Trans. Automat. Contr.*, vol. AC-3, 1958.
[5] O. Rubin, "Delay lines in analog computer studies," *IEEE Trans. Ind. Electron. Contr. Instrum.*, vol. IECI-11, pp. 39–46, 1976.
[6] J. M. Douglas, *Process Dynamics and Control*, vol. 2. Englewood Cliffs, NJ: Prentice-Hall, 1972.
[7] R. D. Driver, *Ordinary and Delay Differential Equations*. Berlin: Springer, 1977.

Optimal Feedback Control Via Block-Pulse Functions

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***Abstract*—Using block-pulse functions, a method is presented to determine the piecewise constant feedback controls for a finite linear optimal control problem of a power system. The method is simple and computationally advantageous.**

Introduction

It is known that the optimal control of a linear system with respect to a quadratic performance criteria over a finite interval of time often involves the determination of time-varying feedback gains. Walsh functions have been used for determining piecewise constant feedback gains [1], [2]. Recently, a method based on block-pulse functions for numerical integration of a system of differential equations were presented [3]. The block-pulse functions can be used for determining the piecewise constant gains for dynamic systems with certain computational advantages.

This paper presents the results of piecewise constant feedback gains for a simple power system using the method of block-pulse functions. Taking the average for each pair of consecutive values of the piecewise constant gains obtained for m-segments for a finite time and continuing this process of taking averages till we obtain for two subintervals, the set of gains corresponding to the first subinterval were found to be same as the gains obtained for an infinite-time linear optimal control problem.

Problem Formulation

A simple model of a power system, consisting of a single generator connected to an infinite bus through a transmission line is considered. A simplified third-order model, for studying dynamic performance with optimal feedback controls for excitation, can be obtained as in [4], [5].

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= a_1 - a_2\sin x_1\cdot x_3 - a_3\sin 2x_1\\ \dot{x}_3 &= u - a_4x_3 + a_5\cos x_1\end{aligned}\tag{1}$$

where $x_1 = \delta$, $x_2 = p\delta$, $x_3 = E_q'$ and

$$a_1 = \frac{p_i}{M_i},\qquad a_2 = \frac{V}{M_i(x_d'+x_e)},\qquad a_3 = \frac{(x_d'-x_q)V^2}{2M_i(x_d'+x_e)(x_q+x_e)},$$

$$a_4 = \frac{(x_d+x_e)}{T_{d0}'(x_d'+x_e)},\qquad a_5 = \frac{(x_d-x_d')V}{T_{d0}'(x_d'+x_e)},\qquad u = \frac{E_{ex}}{T_{d0}'}.$$

x_e is the external reactance to the synchronous machine and p_i is the prime-mover input. u is the control variable corresponding to excitation.

For the study of transients involving small disturbances, the original nonlinear system (1) can be linearized about the operating point, giving rise to a linear vector matrix equation

$$\Delta\dot{x} = A\Delta x + B\Delta u \tag{2}$$

where $\Delta x = [\Delta\delta, \Delta\omega, \Delta E_q']^t$ and A and B are matrices of appropriate dimensions.

It is assumed that the transients in the system states Δx's are to be minimized within a finite-time interval t_f s. The quadratic cost function

$$J = \int_0^{t_f} (\Delta x^t Q \Delta x + \Delta u^t R \Delta u)\, dt \tag{3}$$

is minimized using the optimal control given by

$$\Delta u^* = R^{-1}B^tP(t) \tag{4}$$

Manuscript received September 18, 1978.
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