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Hierarchical optimal control of oversaturated urban traffic networks

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A simple discrete-time model of oversaturated urban traffic networks is suggested, and an optimal control problem of traffic signals during the rush hour is formulated. To solve this optimal control problem, an hierarchical optimization method is also suggested which is based upon the idea that the overall system can be decomposed into subsystems with only one input and one state by means of additional controls and equality constraints.

1. Introduction

As the number of vehicles in urban areas is ever increasing, it has been a major concern of city authorities to facilitate effective control of traffic flows in restricted urban areas. Especially in rush hours, even a short period of poor control at traffic signals may result in a long time traffic jam causing a chain of delays in traffic flows. The total amount of accumulated delay time in a city due to waiting at signal stops is enormous if it is counted on an annual basis. To reduce the waiting time of vehicles at traffic signals is to reduce consumption of fuel and man-hours, and thus it is important to control the traffic signals in an optimal manner.

The problem of systematically controlling signals at traffic stops especially during rush hours has been considered by several investigators (Gordon 1969, Longley 1968, Burhardt and Kulikowski 1970). Among these, it was Singh and Tamura (1974) who first handled the control problem of oversaturated urban traffic networks by suggesting a deterministic discrete-time dynamic model and applying an hierarchical computational method of Tamura (1975). In their problem formulation, however, certain inequality constraints seem unrealistically incorporated. As a consequence, a contradictory situation may occur in the sense that an optimal control scheme for a given problem is not correctly obtained though it exists.

In this paper, we first propose a new model of oversaturated urban traffic networks to remedy the shortcomings of Singh and Tamura's model. Secondly, in order to solve a class of control problem involving the suggested model, we propose an hierarchical optimization technique slightly different from that of Tamura (1975). This is devised to handle the linear equality constraints at high level so that the computation at low level is very simple and efficient.

2. Discussion on Singh and Tamura's model of oversaturated urban traffic networks

2.1. *Singh and Tamura's model*

An oversaturated intersection is defined to be one where queues remain at the end of the green interval (Singh and Tamura 1974). Consider the

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problem of traffic signal control for networks of the types shown in Figs. 1 and 2. Note that these are the basic elements of the traffic networks. For these networks, Singh and Tamura's model and their problem formulation are briefly described in the following.

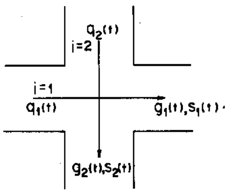

Figure 1. An oversaturated one-way no-turn intersection.

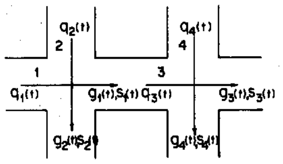

Figure 2. A model for the intersecting road.

In Fig. 1, let $i=1$ denote the horizontal traffic flow direction and $i=2$ the vertical traffic flow direction. Let C denote the duration of the cycle time. Let $q_i(t)$ denote the arrival rates of vehicles in the direction i. Let s_i denote the saturation flow rates of vehicles in the direction i. In other words, s_i is the maximum number of vehicles which can pass through the intersection per cycle in the direction i if all the signals in this direction are green. Let $\bar{g}_i$ be the averaged departure rate over the duration of the cycle time C. If we let l be the loss time due to the amber phase, and G_i the duration of the green signal in the direction i, the following relation holds

$$C = G_1 + G_2 + l \tag{1}$$

Define the control variables $u_i(k)$ to be the percentage of 'green' over C in the direction i, and also define the state variables $x_i(k)$ to be the number of vehicles which wait at time k to pass the intersection in the direction i. Then, for a given constant C, it is easily shown that

$$u_i(k) \triangleq \frac{G_i}{C} = \frac{\bar{g}_i(k)}{s_i}, \quad i = 1, 2 \tag{2}$$

and the evolution process of the states can be described by

$$\begin{aligned} x_i(k+1) &= x_i(k) + q_i(k) - \bar{g}_i(k) \\ &= x_i(k) + q_i(k) - s_i u_i(k) \quad i=1,2, \quad k=0,1,\ldots,k_t-1 \end{aligned} \tag{3}$$

where k_t is the final time of control duration. Now from eqns. (1) and (2) we obtain

$$u_1(k) + u_2(k) = (G_1 + G_2)/C = 1 - l/C = G_e \tag{4}$$

where G_e is the effective green. As we have assumed that C and l are known constants, there is in effect only one control variable per intersection. If $u_1(k)$ is replaced by $u(k)$, then eqn. (3) can be written as

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} -s_1 \\ +s_2 \end{bmatrix} u(k) + \begin{bmatrix} q_1(k) \\ q_2(k) - s_2 G_e \end{bmatrix}$$

$$k=0,1,\ldots,k_t-1 \tag{5}$$

Following Singh and Tamura (1974), we require that the state and control variables be subject to the inequality constraints given below

$$0 \leqslant x_i(k) \leqslant x_{i,\max}, \quad k=0,1,\ldots,k_t, \quad i=1,2 \tag{6}$$

and

$$u_{\min} \leqslant u(k) \leqslant u_{\max}, \quad k=0,1,\ldots,k_t-1 \tag{7}$$

Singh and Tamura considered the problem of finding an optimal control sequence $u(k)$, $k=0,1,\ldots,k_t-1$ for the system described by eqn. (5), under the constraints of eqns. (6) and (7), and minimizing the cost function J given by

$$J = \sum_{k=0}^{k_t-1} \{\|x(k+1)\|_Q^2 + \|u(k) - u^d(k)\|_R^2\} \tag{8}$$

Here $x^T = (x_1, x_2)$, and $\|\cdot\|$ denotes a euclidean norm ; Q and R are diagonal weighting matrices, and $u^d(k)$ is a desired value for the control.

It is remarked that for the interconnected road as shown in Fig. 2, the model is easily extended by incorporating in eqn. (5) a pure delay element such as

$$q_3(k) = s_1 u_1(k-\theta) \tag{9}$$

Here, it is assumed that the delay time d between two intersections is θC where θ is an integer. Then, in general, the traffic flow process in the overall network can be described by a linear difference equation with pure delays in the controls

$$x(k+1) = E_n x(k) + \sum_{j=0}^{\theta} B_j u(k-j) + d(k), \quad k=0,1,\ldots,k_t-1 \tag{10}$$

where $x(k)$ is the $n \times 1$ vector of queues, E_n is an $n \times n$ identity matrix, B_j $(j=0,1,\ldots,\theta)$ are $n \times n$ matrices, $u(k)$ is the $m \times 1$ control vector and $u(k-j)$ $(j=1,2,\ldots,\theta)$ are the delayed controls used to account for the pure delays on the interconnected roads, $d(k)$ is the $n \times 1$ vector of inputs which come from the outside of the network. In this general case, the state and control variables must satisfy the constraints as in eqns. (6) and (7).

The formulation of an optimal control problem in the spirit of Singh and Tamura can easily be done as before.

2.2 *Discussiou of Singh and Tamura's model*

In Singh and Tamura's formulation, the state variable constraints in eqn. (6) seem inappropriate. To be specific, note that for any optimal solution pair $x(k)$ and $u(k)$ to exist, it is necessary that $x(k)$ and $u(k)$ satisfying eqns. (5), (6) and (7) for all k must exist. In Singh and Tamura's model, a solution pair $x(k)$ and $u(k)$ which satisfies the state equation and constraints (eqns. (5), (6) and (7)) may not exist since the modelling assumption of 'oversaturatedness' is ignored. Hence, an optimal solution may not exist, contrary to the practical situation.

Let us further discuss this difficulty with the following simple example. Consider the first component of eqn. (5), i.e.

$$x_1(k+1) = x_1(k) - s_1 u_1(k) + q_1(k) \tag{11}$$

Here, $x_1(k)$ and $u(k)$ must satisfy

$$0 \leqslant x_1(k) \leqslant x_{1,\max} \quad \text{for } k = 0, 1, \ldots, k_f \tag{12}$$

and

$$u_{\min} \leqslant u(k) \leqslant u_{\max} \quad \text{for } k = 0, 1, \ldots, k_f - 1 \tag{13}$$

For a given $x_1(k)$ at $k = k^*$ $(0 \leqslant k^* \leqslant k_f - 1)$, $q_1(k^*)$ must satisfy

$$s_1 u_{\min} - x_1(k^*) \leqslant q_1(k^*) \leqslant x_{1,\max} + s_1 u_{\max} - x_1(k^*) \tag{14}$$

for the existence of $x_1(k^*+1)$ satisfying eqns. (11), (12) and (13). Now assume that without loss of generality, $x(k^*)$ is zero for some control $u(k^*-1)$. Then, the traffic input queues $q_1(k^*)$ must be at least $s_1 u_{\min}$ for the existence of $x_1(k^*+1)$ satisfying eqns. (11), (12) and (13). If, at this time, the traffic input queue $q_1(k^*)$ is less than $s_1 u_{\min}$, $x_1(k^*+1)$ satisfying eqns. (11), (12) and (13) never exists though this $q_1(k^*)$ can be eliminated using the control $u_{\min}$. That is, though $x_1(k) = 0$ is the desired value (i.e. complete elimination of queues), no optimal solution exists due to the infeasibility of eqn. (12), which does not meet the requirements of physical phenomena.

Actually, in the simulation example taken from Singh and Tamura (1974), this inaccuracy occurs. The optimal solutions of all the examples do not satisfy the system equation.

In addition to the non-existence of an optimal solution, it is remarked that in Singh and Tamura's model, the delay time is assumed to be an integer multiple of the cycle period. Generally, this assumption is not always valid in a practical sense.

3. A new formulation of the optimal control problem for oversaturated urban traffic networks

To remedy some flaws of Singh and Tamura's model as pointed out in § 2, we propose in this section a new formulation of the control problem for oversaturated traffic networks.

In § 2, it was noted that non-existence of an optimal solution results from the fact that the state inequality constraints in eqn. (6) do not coincide with the modelling assumption that the controlled output traffic flow (i.e. $s_i u_i(k)$) must be less than or equal to the current queues (i.e. $x_i(k)$). This can be mathematically described as

$$s_i u_i(k) \leqslant x_i(k) \leqslant x_{i,\max} \tag{15}$$

This assumption arises from the fact that an oversaturated intersection is the one where queues remain at the end of the 'green' interval, and most intersections in urban traffic networks would be oversaturated during peak periods.

From eqns. (5), (7) and (15), the oversaturated urban traffic network in Fig. 1, can be described as

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} -s_1 & 0 \\ 0 & -s_2 \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix} + \begin{bmatrix} q_1(k) \\ q_2(k) \end{bmatrix} \tag{16}$$

$$s_i u_i(k) \leqslant x_i(k) \leqslant x_{i,\max}, \quad k = 0, 1, \ldots, k_f - 1 \tag{17}$$

$$u_{i,\min} \leqslant u_i(k) \leqslant u_{i,\max}, \quad k = 0, 1, \ldots, k_f - 1 \tag{18}$$

for $i = 1, 2$, and

$$u_1(k) + u_2(k) = G_e, \quad k = 0, 1, \ldots, k_f - 1 \tag{19}$$

It is remarked that the same modelling assumption was applied to the traffic assignment problem by Merchant and Nemhauser (1978).

Note that in eqn. (19), $u_2(k) = G_e - u_1(k)$ can be substituted in eqns. (16), (17) and (18) to reduce the dimension of control variables as in eqns. (5), (6) and (7). However, in eqns. (16), (17) and (18), the dimension of control variables is made equal to that of states with additional constraint (eqn. (19)), for ease of computation, as will be explained in § 4.

Now consider the case where the time delay in control exists, which cannot be handled effectively by Singh and Tamura's modelling methods as in eqn. (9). To begin with, consider the simple case where the time delay d is given by

$$d < C \tag{20}$$

In this case, since the macro-behaviour is our concern, the current inflow may well be regarded as some percentage $((c-d)/C)$ of currently controlled traffic out-flow plus another percentage (d/c) of the previous (one cycle-time before) controlled traffic out-flow. Hence, $q_3(k)$ may be expressed as

$$q_3(k) = \frac{c-d}{C} s_1 u_1(k) + \frac{d}{C} s_1 u_1(k-1) \tag{21}$$

By extending this idea, we can treat the case where the delay time d is given by

$$d = (\theta + a)C \tag{22}$$

where θ is an integer and $0 \leqslant a < 1$. In the case of time delay given in eqn. (22), $q_3(k)$ may be given by

$$q_3(k) = (1-a)s_1u_1(k-\theta) + as_1u_1(k-\theta-1) \tag{23}$$

in a similar way as in eqn. (21).

Let the cost function be

$$J = \tfrac{1}{2} \sum_{k=0}^{k_t-1} \{\|x(k+1)\|_Q^2 + \|u(k) - u^d(k)\|_R^2\} \tag{24}$$

where $x(\cdot) = (x_1(\cdot), x_2(\cdot), \ldots, x_n(\cdot))^T$, $u(\cdot) = (u_1(\cdot), u_2(\cdot), \ldots, u_n(\cdot))^T$, Q and R are $n \times n$ diagonal weighting matrices, and $\|\cdot\|$ denotes a euclidean norm. Then the optimal control problem for oversaturated traffic networks is formulated as follows.

Optimal control problem (P)

Find an optimal control sequence $u(0), u(1), \ldots, u(k_t - 1)$, while minimizing the cost function J in eqn. (24) subject to the following constraints

(i) $x(k+1) = E_n x(k) + \sum_{j=0}^{m} B_j u(k-j) + d(k)$ (25)

(ii) $s_i u_i(k) \leqslant x_i(k) \leqslant x_{i,\max}$ (26)

(iii) $u_{i,\min} \leqslant u_i(k) \leqslant u_{i,\max}$ for $i = 1, 2, \ldots, n$ and $k = 0, 1, \ldots, k_t - 1$ (27)

(iv) $0 \leqslant x_i(k_t) \leqslant x_{i,\max}$ for $i = 1, 2, \ldots, n$ (28)

(v) $u_s(k) = u_t(k)$ for some s, t (29)

(vi) $\sum_{m_j} u_{m_j}(k) = G_e$ for $k = 0, 1, \ldots, k_t - 1$ (30)

In problem (P), $x(k)$ is an $n \times 1$ state vector whose ith component is $x_i(k)$, and $u(k)$ is an $n \times 1$ control vector whose ith component $u_i(k)$ is associated only with $x_i(k)$; E_n is an $n \times n$ identity matrix, and B_j $(j = 0, 1, \ldots, m)$ are $n \times n$ matrices. And m is the largest unit of time delay in the overall networks. Equations (29) and (30) are due to the fact that we define control $u_i(k)$ to every $x_i(k)$, though the dimension of control can be reduced. In eqn. (30), the $u_{m_j}(k)$ are controls associated with one traffic signal only.

4. An hierarchical optimization method

In this section, an hierarchical optimization method is suggested as a means of solving the problem (P) of § 3. This method is based upon the dual decentralization method of Tamura (1975) (see also Singh and Titli 1978). We use the idea of decomposing the problem with respect to time to handle the time delay. Furthermore, to treat the inequality constraints of the form in eqn. (26) effectively, the dimension of the control variables is made to be equal to that of the state variables as in problem (P). By doing this in addition to time decomposition, the sub-problems can be further decomposed into simple quadratic problems which have only two variables $x_i(k)$, $u_i(k)$ and linear constraints in eqns. (26), (27) and (28). In this way, lower-level sub-problems are made very easy to handle.

Let us write the lagrangian L of the optimal control problem (P) as

$$\mathrm{L}=\sum_{k=0}^{k_t-1}\Bigg[\tfrac{1}{2}\|x(k+1)\|_Q^2+\tfrac{1}{2}\|u(k)\|_R^2 -p^{\mathrm{T}}(k)\left\{x(k+1)-E_n x(k)-\sum_{j=0}^{m}B_j u(k-j)-d(k)\right\} -\beta^{\mathrm{T}}(k)\{u_s(k)-u_t(k)\}-\gamma^{\mathrm{T}}(k)\left\{\sum_{m_j}u_{m_j}(k)-G_e\right\}\Bigg] \quad (31)$$

Then, as in Tamura (1975), it can be shown that

$$\min_{u} J=\max_{p,\,\beta,\,\gamma}\phi(p,\beta,\gamma) \quad (32)$$

where

$$\phi(p,\beta,\gamma)=\min_{x,\,u} L \quad (33)$$

subject to eqns. (26), (27) and (28).

Note that the lagrangian in eqn. (31) is additively separable with respect to $x_i(k)$ and $u_i(k)$ for given p, β and γ. That is, for given p^*, β^* and γ^*, the lagrangian L can be first decomposed with respect to time k into the sum of L_k

$$L=\sum_{k=0}^{k_t}L_k(x(k),u(k),p^*,\beta^*,\gamma^*) \quad (34)$$

where, for $k=0$

$$L_0=\tfrac{1}{2}\|u(0)\|_R^2-p^{*\mathrm{T}}(0)\{E_n x(0)+B_0 u(0)-d(0)\} -\beta^{*\mathrm{T}}(0)\{u_s(0)-u_t(0)\}-\gamma^{*\mathrm{T}}(0)\left\{\sum_{m_j}u_{m_j}(0)-G_e\right\}$$

for $k=1,2,\ldots,k_t-1$

$$L_k=\tfrac{1}{2}\|x(k)\|_Q^2+\tfrac{1}{2}\|u(k)\|_R^2-p^{*\mathrm{T}}(k-1)x(k)-p^{*\mathrm{T}}(k)x(k) +\sum_{j=0}^{m}p^{*\mathrm{T}}(k+j)B_j u(k)+p^{*\mathrm{T}}(k)d(k)-\beta^{*\mathrm{T}}(k)\{u_s(k)-u_t(k)\} -\gamma^{*\mathrm{T}}(k)\left\{\sum_{m_j}u_{m_j}(k)-G_e\right\} \quad (35)$$

and for $k=k_t$

$$L_{k_t}=\tfrac{1}{2}\|x(k_t)\|^2-p^{*\mathrm{T}}(k_t-1)x(k_t)$$

Let us rewrite all the vectors in L_k into their components. Then, since Q and R are diagonal matrices, it is easily shown that L_k can be decomposed into the form of

$$L_k=\sum_{i=1}^{n}L_{ik}(x_i(k),u_i(k),p^*,\beta^*,\gamma^*) \quad (36)$$

Consequently, L can be decomposed as

$$L=\sum_{k=0}^{k_f}\sum_{i=1}^{n}L_{ik}(x_i(k),u_i(k),p^*,\beta^*,\gamma^*) \tag{37}$$

Hence, to minimize L under the constraints of eqns. (26), (27) and (28) is equivalent to minimizing L_{ik} subject to the constraints corresponding to each $x_i(k)$, $u_i(k)$. There are $(k_f+1)\times n$ sub-problems at lower level which are a simple quadratic minimization problem of only two variables $(x_i(k), u_i(k))$ with linear constraints.

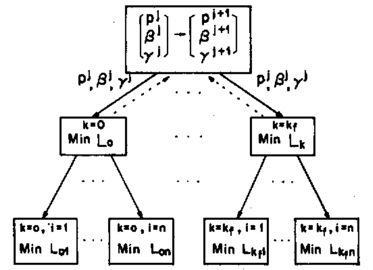

Figure 3. Two-level computation algorithm.

The hierarchical optimization method explained so far is summed up as follows. First choose an initial guess of coordinates p, β, γ, and send these values to the lower level as shown in Fig. 3. At the higher level, the p, β, γ trajectories are improved using some gradient techniques. This can be done since it is easily shown that gradients of ϕ with respect to p, β, γ are

$$\nabla_p\phi(p,\beta,\gamma)=x(k+1)-E_nx(k)-\sum_{j=0}^{m}B_ju(k-j)-d(k) \tag{38}$$

$$\nabla_\beta\phi(p,\beta,\gamma)=u_s(k)-u_i(k) \tag{39}$$

$$\nabla_\gamma\phi(p,\beta,\gamma)=\sum_{m_j}u_{m_j}(k)-G_e \tag{40}$$

5. Simulation result

As a simulation example, consider a simple one-way no-turn intersection as shown in Fig. 4.

The state equations can be written as

$$x_1(k+1)=x_1(k)-s_1u_1(k)+q_1(k)$$

$$x_2(k+1)=x_2(k)-s_2u_2(k)+q_2(k)$$

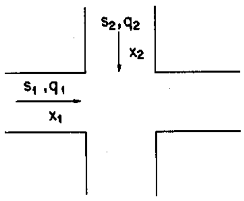

Figure 4. One-way no-turn intersection.

Here it is assumed that $u_1^d(k) = u_2^d(k) = 0{\cdot}5$, and the control constraints are $0{\cdot}2 \leqslant u_1(k) \leqslant 0{\cdot}7$ and $0{\cdot}2 \leqslant u_2(k) \leqslant 0{\cdot}7$. Hence, if we let $\Delta u_i = u_i - u_i^d$, for $i = 1, 2$, then the control constraints become

$$-0{\cdot}3 \leqslant \Delta u_1(k) \leqslant 0{\cdot}2 \quad \text{and} \quad -0{\cdot}3 \leqslant \Delta u_2(k) \leqslant 0{\cdot}2$$

The data chosen are

$$s_1 = s_2 = 50, \quad q_1 = 35 \quad \text{and} \quad q_2 = 25$$

Here the cycle period C is 60 s, and the loss time is assumed to be zero. Hence, the equality constraint for the control variables is

$$u_1(k) + u_2(k) = 1$$

i.e.

$$\Delta u_1(k) + \Delta u_2(k) = 0$$

The state constraints are chosen to be

$$s_1 u_1(k) \leqslant x_1(k) \leqslant 80 \quad \text{and} \quad s_2 u_2(k) \leqslant x_2(k) \leqslant 100$$

Hence

$$50\Delta u_1(k) + 25 \leqslant x_1(k) \leqslant 80 \quad \text{and} \quad 50\Delta u_2(k) + 25 \leqslant x_2(k) \leqslant 100$$

Suppose the initial conditions are

$$x_1(0) = x_2(0) = 50$$

The weighting matrices Q, R are chosen to be

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad R = \begin{bmatrix} 200 & 0 \\ 0 & 200 \end{bmatrix}$$

The time interval k_f is taken as $k_f = 3$.

The optimal control problem is the following. Minimize J given by

$$J = \sum_{k=0}^{2} \tfrac{1}{2}\{\|x(k+1)\|_Q^2 + \|\Delta u(k)\|_R^2\}$$

$$= \sum_{k=0}^{2} \sum_{i=1}^{2} \{\tfrac{1}{2}x_i^2(k+1) + 100\Delta u_i^2(k)\}$$

under the constraints of

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} -50 & 0 \\ 0 & -50 \end{bmatrix} \begin{bmatrix} \Delta u_1(k) \\ \Delta u_2(k) \end{bmatrix} + \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 50 \\ 50 \end{bmatrix}, \quad \Delta u_1(k) + \Delta u_2(k) = 0$$

$$50\Delta u_1(k) + 25 \leqslant x_1(k) \leqslant 80, \quad 50\Delta u_2(k) + 25 \leqslant x_2(k) \leqslant 100$$

$$0 \leqslant x_1(3) \leqslant 80, \quad 0 \leqslant x_2(3) \leqslant 100$$

$$-0{\cdot}3 \leqslant \Delta u_1(k) \leqslant 0{\cdot}2, \quad -0{\cdot}3 \leqslant \Delta u_2(k) \leqslant 0{\cdot}2, \quad k = 0, 1, 2$$

The optimal control sequence was calculated for this problem using the two-level algorithm as explained in § 4. The dual cost ϕ and primal cost J were

$$J = 0{\cdot}109 \times 10^5 \quad \text{and} \quad \phi = 0{\cdot}109 \times 10^5$$

The optimal state trajectories are shown in Fig. 5, and the optimal control sequences in the Table.

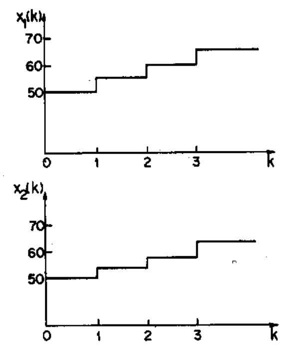

Figure 5. The optimal state trajectories.

Convergence to the optimum was obtained in a computation time of 0·333 s using a CYBER CDC computer.

k	0	1	2
$u_1(k)$	0·59	0·60	0·59
$u_2(k)$	0·41	0·40	0·41

The optimal control sequences.

6. Concluding remarks

The control problem of oversaturated urban traffic networks was studied by suggesting a new model for traffic networks and a new hierarchical computational method was used to solve the problem. The flaw in Singh and Tamura's model was resolved in our model. Further it was shown that the case where the delay time between adjacent intersections was a non-integer multiple of the cycle period could be handled effectively in our model.

One of the advantages of the proposed hierarchical optimization method is that, at lower level, sub-problems are made very simple by means of additional controls and equality constraints. And also the inequality constraints of special form of eqn. (26) can be handled easily.

Using the proposed modelling method and the computational technique, the optimal control problem of the complex traffic networks around the City Hall in Seoul, Korea, one of the trouble spots in Seoul, is being studied.

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