[12] I. Tzypkin, "Adaptative optimization algorithms with *a priori* uncertainty," *Automi. Telem.*, vol. 40, no. 6, 1979 (In Russian).

[13] L. Ljung and E. Trulsson, "Adaptative control based on explicit criterion minimization," Linköping Univ. Sweden, Report Li TH-ISY-I-0401, 1980.

[14] B. Widrow *et al.*, "Stationary and non-stationary learning characteristics of the LMS adaptive filter," *Proc. IEEE*, vol. 64, Aug. 1976.

[15] R. Ortega, "On PID self-tuners," presented at 3rd IFAC/IFIP Symp. SOCOCO, Madrid, Oct. 5-8, 1982.

[16] A. C. Schilling, *Economics of Autopilot Steering Using an IBM System/7. Ship Operation Automation.* Amsterdam: North-Holland, 1976.

[17] Y. Nishikawa and N. Sannomija, "A method for autotuning of PID control parameters," presented at 8th IFAC Congress, Kyoto, Japan, 1981.

[18] K. J. Astrom, "Ziegler-Nichols autotuners," Dept. of Aut. Cont., Lund Inst. of Techn., Lund, Sweden, Rep. No. LUTGD2/... 7184), 1982.

[19] S. C. Shah, "Internal model adaptive control," in *Proc. 20t... CDC*, (San Diego, CA), Dec. 1981.

[20] R. Ortega, "Robustness enhancement of adaptive control... process *a priori* knowledge," *Syst. and Cont. Letters* vol. 4, p... 141, 1984.

[21] P. Belanger, "On type 1 systems and the Clarke-Gawthrop method," in *Proc. Appl. Adapt. Syst. Th.*, (Yale Univ.), May 1981.

[22] A. Y. Allidina and F. M. Hughes, "A general adaptive scheme ... *Jour. Cont.*, vol. 38, no. 6, Dec. 1983.

[23] A. H. Glattfelder *et al.*, "Microcomputer based self-tuning and ... selecting controllers," *Automatica*, vol. 16, no. 1, Jan. 1980.

[24] D. W. Clarke and B. A. Gawthrop, "Self-tuning control," *Proc ... Elec. Eng.*, vol. 126, no. 6, 1979.

A Design and Experiment of Speed Controller with PI-Plus Bang-Bang Action for a DC Servomotor with Transistorized PWM Drives

IL HONG SUH, SEUNG HO HWANG, AND ZEUNGNAM BIEN, MEMBER, IEEE

***Abstract*—A speed controller with proportional-integral (PI)-plus bang-bang action is proposed for dc servomotors with transistorized pulse width modulated (PWM) drives. The controller employs the PI-action when the magnitude of the error between the reference signal and the speed output signal is smaller than some precribed value. Otherwise, the controller produces the maximum allowable control signal with the integrator reset. Specifically, a mathematical analysis of the motor system with the proposed speed controller is presented and a rule of thumb for parameter design is provided.**

To show that the performance of the controller with PI-plus bang-bang action is superior to those of conventional controllers only with PI-action or with proportional (P) plus limited integral (I)-action in the sense of the classical figures of merit, such as overshoots, settling time, and rise time, a design example and some experimental results are illustrated.

NOMENCLATURE

B_m	Friction coefficient of motor.
B_l	Friction coefficient of load.
$B = B_m + B_l$	Friction coefficient.
J_m	Moment of inertia of motor.
J_l	Moment of inertia of load.
$J = J_m + J_l$	Moment of inertia of total system.
L_a	Armature inductance.
L_{ext}	Reactor inductance.
$L = L_a + L_{ext}$	Total inductance.
R_a	Armature resistance.
K_t	Torque constant.
K_b	Back EMF constant.
T	Output torque.
T_L	Load torque.
K_c	Proportional gain of current controller.
K_{cF}	Current feedback gain.
K_v	Proportional gain of speed controller.
K_{vF}	Speed feedback gain.
i_a	Armature current.
v_a	Armature voltage.
v_b	Back EMF voltage.
r	Speed reference input voltage.
y	Speed feedback voltage.
e_i	Current reference input voltage.
$e = r - y$	Error signal voltage.
q	Integration of error signal e.
α	Zero of speed PI controller.
β	Zero of current PI controller.
δ	Per unit overshoot $\triangleq \dfrac{\text{peak overshoot value} - \text{steady-state value}}{\text{steady-state value}}$
η	Voltage magnitude of switching.

Manuscript received May 9, 1983; revised January 12, 1984.

I. H. Suh and S. H. Hwang are with the Technical Center, Daewoo Heavy Industries, Ltd., Dong-Ku, Incheon, Korea.

Z. Bien is with the Department of Electrical Engineering, Korea Advanced Institute of Science and Technology, Chongyangni, Seoul 131, Korea.

ω	Output speed.
E_i	Laplace transform of e_i.
E	Laplace transform of e.
I_a	Laplace transform of i_a.
V_a	Laplace transform of v_a.
Ω	Laplace transform of.
E_{max}	Maximum allowable voltage of e_i.
I_{max}	Maximum allowable current.
Ω_{max}	Maximum allowable speed.
R	Speed reference input magnitude.
$\tilde{R}$	Incremental speed input magnitude.

I. INTRODUCTION

THERE HAVE BEEN a number of research works [1]-[4] for the analysis and synthesis of the speed controller for separately excited dc servomotors. Most of the proposed design methods are aimed at a control system with the properties of small overshoot and fast settling time to a step input change. Traditionally, current controllers of the P (proportional) or PI (proportional-integral) type are used [3] in cascade with a device for limiting the armature current of the motor to a prescribed maximum allowable value. Also, the current feedback is frequently used to suppress the armature current [5]. At any rate, the design, in these cases, means to establish a rule of determining the parameters such as PI gains for a simplified first-order open-loop speed control system [2].

However, the saturation-type nonlinearity is not considered in these conventional methods of analysis and design of the speed controller, though such nonlinearity practically exists in the current reference input node, as shown in Fig. 1. As a consequence, the so-called integral wind-up phenomenon is not effectively controlled, causing large overshoots and/or lengthy settling time [6], [7].

The integral wind-up phehomenon, causing an actuator saturation and large overshoots, was discussed in detail by Phelan [7], who further suggested a possible solution to the problem, to turn off the integral action when the integral term exceeds some prescribed value. More recently, Krikelis [8] proposed another type of 'intelligent' integrator (shown in Fig. 2) to avoid the integral wind-up.

In the Krikelis intelligent integrator in Fig. 2, the parameters μ and H are supposed to be the designer's choices. But, unfortunately, such freedom vanishes when the integrator is to be used for the purpose of designing a speed servo system for dc servomotors. To explain it specifically, first note that the steady-state value of output voltage of the speed controler should lie between zero and the maximum allowable command E_{max} to counteract a constant load torque. Also, the steady-state value u_{ss} of the intelligent integrator output for a step error signal E is given by

$$u_{ss} = \mu + E/H.$$

Thus, u_{ss} also should lie between zero and E_{max}. Since the slope H of the intelligent integrator determines the controller response time, H is chosen to be relatively large. Eventually, an optimum response may be obtained by letting μ be equal

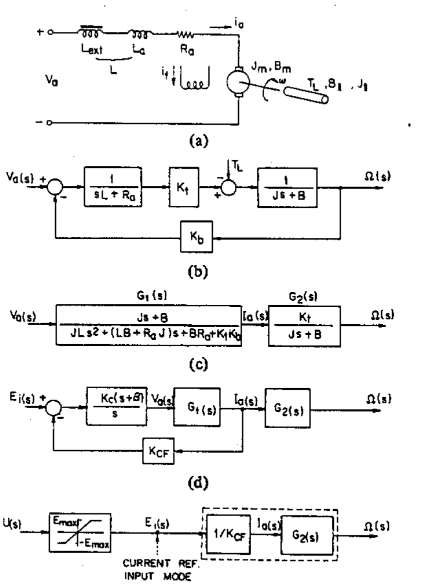


Fig. 1. Development of motor transfer function. (a) Seperately excited dc servomotor. (b) Complete transfer function. (c) Transfer function without load-torque disturbance. (d) Current control system. (e) Simplified open-loop speed control system.

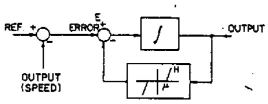


Fig. 2. Block diagram of Krikelis' intelligent integrator.

to E_{max} and letting H be infinite. In this way, the Krikelis intelligent integrator becomes the type of a limited integrator suggested by Phelan [7].

It turns out that the speed controller with a proportional plus limited I-action still shows undesirable large overshoots due to the limited integral action. This will be experimentally shown in a later section.

In this paper, to overcome the difficulties due to the integral wind-up of the speed controller with the PI-action [1]-[3], or with P- and limited I-action, a speed controller with the PI-plus bang-bang action (as shown in Fig. 3) is proposed. In this proposed configuration, the controller employs the PI-action when the magnitude of the error is smaller than a prescribed value. Otherwise, the controller generates the maximum permissible control signal with the integrator being forced to reset. A design method for the proposed type of controller based on a mathematical analysis will be given. The design, which is experimentally verified, proves that the speed controller with PI-plus bang-bang action renders a fast settling time property with small overshoots regardless of the magnitude of the reference input signal by not only preventing the integral wind-up, but also utilizing the maximum ratings of hardware components such as power

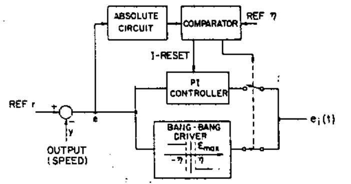


Fig. 3. Conceptual block diagram of a speed controller with PI-plus bang-bang drive action.

transistors and motor armature windings. Thus, the proposed speed controller seems better than either the speed controller with PI-action [1]-[3], or the one with P- and limited I-action.

The organization of this paper is as follows. In Section II, a design method of a current controller of the PI-type considering nonnegligible reactor inductance will be proposed. In Section III, a sufficient condition will be given for the output response of the proposed speed controller to track asymptotically the reference step input. Also, a rule of thumb for choosing the design parameters of the speed controller will be given on the basis of the sufficient condition. In Section IV, a numerical design example with some experimental results will be illustrated.

II. CURRENT CONTROL OF THE DC SERVOMOTOR

In the design of a current controller of the P- or PI-type [1]-[4], it is usual that the magnitude of the inherent motor armature inductance is sufficiently small to be neglected. However, in case of the transistorized PWM dc servomotor drives [4], the armature inductance is highly increased when the transisitor switching frequency and the magnitude of the current ripple is to be reduced. This is accomplished by connecting a reactor (external inductor) in series with the motor armature winding whose inductance cannot be neglected. Thus, the design method of the current controller proposed in [1]-[4] may not directly apply for the motor with transistorized PWM drives.

In this section, a new design method of the current controller of the PI-type is proposed. In this method, the increased armature inductance is not neglected, and further it will be shown that the motor system with the current controller of the PI-type can be approximated as a first-order system. For this, consider the separately excited dc servomotor with an armature voltage control system shown in Fig. 1(a), whose voltage loop and torque balance equations are given by

$$v_a = v_b + R_a i_a + L\frac{di_a}{dt}$$
$$v_b = K_b \omega$$
$$T = J\frac{d\omega}{dt} + B\omega + T_L$$
$$T = K_t i_a. \tag{1}$$

These relationships are shown in block diagram form in Fig. 1(b). As in [1], if the load torque term T_L is neglected, one can show that

$$\frac{\Omega(s)}{V_a(s)} = \frac{K_t}{JLs^2 + (LB + R_a J)s + (BR_a + K_t K_b)} \tag{2}$$

and

$$\frac{I_a(s)}{V_a(s)} = \frac{Js + B}{JLs^2 + (LB + R_a J)S + (BR_a + K_t K_b)}. \tag{3}$$

Thus, the motor can be represented as two blocks, as in Fig. 1(c). Now let the current controller be of the PI-type shown in Fig. 1(d). Then one can show that

$$\frac{I_a(s)}{E_i(s)} = \frac{K_c(s+\beta)(s+B/J)/L}{\Delta(s)} \tag{4}$$

where $\Delta(s)$ is given by

$$\Delta(s) = s^3 + [(R_a + K_c K_{cF})/L + B/J]s^2 + [K_c K_{cF}\beta/L + (R_a B + BK_c K_{cF} + K_c K_b K_t/JL)]s + BK_c K_{cF}\beta/JL. \tag{5}$$

It follows from (4) and (5) that, if the inequalities given by

$$(R_a + K_c K_{cF})^2 \gg L\beta K_c K_{cF} \tag{6}$$

and

$$K_c K_{cF}\beta \gg K_b K_t/J \tag{7}$$

hold, then the transfer function in (4) can be approximated as

$$\frac{I_a(s)}{E_i(s)} \cong \frac{K_c(s+\beta)}{(R_a + K_c K_{cF})(s + \beta K_{cF}/(R_a + K_c K_{cF}))}. \tag{8}$$

Further, if the inequality given by

$$K_c K_{cF} \gg R_a \tag{9}$$

holds, then the transfer function can be simply given by

$$\frac{I_a(s)}{E_i(s)} \cong 1/K_{cF}. \tag{10}$$

From (10), it can be concluded that the motor with a current controller of the PI-type can be approximated as a first-order system given by

$$\frac{\Omega(s)}{E(s)} = \frac{K}{s+p} \tag{11}$$

where p and K are constants defined by

$$p \triangleq B/J, \qquad K \triangleq K_t/JK_{cF}. \tag{12}$$

On the basis of the inequalities (6), (7), and of thumb of designing current controllers of the PI-type is now proposed in the following.

Step 1:

Choose K_{cF} such that, when the maximum current reference voltage E_{max} is applied, the armature current should be the maximum allowable current I_{max} of the motor.

This simply implies that

$$K_{cF} = E_{max}/I_{max}. \quad (13)$$

Step 2:

Choose K_c and β by solving

$$K_c K_{cF} = C_1 R_a$$

$$K_c K_{cF} = C_2 L \beta$$

and

$$K_c \beta = C_3 K_b K_t / K_{cF} J$$

where $C_i (i = 1, 2, 3)$ are the real constants greater than 5.

It should be noticed from the above rule of thumb that the maximum current reference voltage E_{max} should be limited to a certain value (which is often practically chosen as 6 ~ 10 V), since it is the output signal of the operational amplifier whose absolute magnitude of supply voltage is normally less than 15 V. Thus, the open-loop speed control system with a current controller of the PI-type in (1) should include the saturated-type nonlinearity in the current reference input node as shown in Fig. 1(e).

III. DESIGN OF SPEED CONTROLLER WITH PI-PLUS BANG-BANG ACTION

Consider the simplified open-loop speed control system shown in Fig. 1(e) whose dynamics are given by

$$\dot{\omega} = -p\omega + Ku, \qquad |u| < E_{max}. \quad (14)$$

For the system (14), let us apply the controller with PI-plus bang-bang action shown in Fig. 3. Let

$$y(t) \triangleq K_{vF}\,\omega(t) \quad (15)$$

$$e(t) \triangleq r(t) - y(t) \quad (16)$$

and

$$q(t) \triangleq \int_0^t e(t)\, dt. \quad (17)$$

Then the control law can be written as

$$u(t) = \begin{cases} K_v(e(t) + \alpha q(t)), & \text{for } |e(t)| \leq \eta \\ E_{max} \text{ and } q(t) = 0, & \text{for } e(t) > \eta \\ -E_{max} \text{ and } q(t) = 0, & \text{for } e(t) < -\eta. \end{cases} \quad (18)$$

In (15) and (18), K_{vF} is the positive real constant which represents the gain of the speed to voltage transducer, and K_v, α, and η are the real constants to be determined.

Recall that our design problem is to find K_v, α, and η such that the system response after controller switching not only remains in an η band but also has a small overshoot value. For this, first is given a sufficient condition that $y(t)$, after controller switching, remains in an η band. Then, on the basis of the sufficient condition, a rule of thumb to choose K_v, α, and β will be proposed.

Theorem 1. Let K_v and α be chosen such that the characteristic equation of the system given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -(p + KK_vK_{vF}) & KK_v\alpha \\ -K_{vF} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} KK_v \\ 1 \end{bmatrix} \tilde{R},$$

$$[x_1(0) \quad x_2(0)]^T = [0 \quad 0]^T \quad (19)$$

has two negative real roots $-a$ and $-b$ satisfying

$$b > 2a \quad (20)$$

and, further, the per unit overshoot δ of $x_1(t)$ in (19) is less than one. Then, if the inequality given by

$$pR/(KK_vK_{vF} + p) < \eta < E_{max}/K_v \quad (21)$$

holds, the output response $y(t)$ of the system (14) with the controller (18) asymptotically tracks the reference step input $r(t)$.

For the proof of Theorem 1, the following Lemma is needed.

Lemma 1. Consider the two systems given by

$$G_1(s) = \frac{(s + \alpha_1)}{(s + a)(s + b)} \quad (22)$$

and

$$G_2(s) = \frac{(s + \alpha_2)}{(s + a)(s + b)}. \quad (23)$$

Then, if $0 < \alpha_1 < a < b$ and $\alpha_1 < \alpha_2$, the per unit overshoot of the step response of the system in (23) is always less than that of the system in (22).

The proof of Lemma 1 is given in Appendix A.

Proof of Theorem 1: Let t_1 be the controller switching time. Then, if $|u(t)| < E_{max}$ for $t > t_1$, the system is linear for $t > t_1$. Thus, it is sufficient to consider only the case when $0 < y(t_0) < R$.

For the case when $0 < y(t_0) < R$, it can be easily shown that a sufficient condition for $y(t)$ of the system (14) with controller (19) to track the reference input $r(t)$ is given by

i) $\dot{\omega}(t_1) > 0$,
ii) $y(t)$, for $t > t_1$, has the peak overshoot value not exceeding $\eta + R$,
iii) $|u(t)| < E_{max}$ for $t > t_1$.

Thus, the proof will be completed if it is shown that, under the assumptions given in Theorem 1, conditions i), ii), and iii) are satisfied.

First, consider the $\omega(t)$ after controller switching. Then the system (14) with controller (18) can be written as

$$\dot{\omega} = -(p + KK_vK_{vF})\omega + KK_v\alpha q + KK_vR$$

$$\omega(t_1) = \frac{R-\eta}{K_{vF}}$$

$$\dot{q} = r - K_{vF}\omega, \qquad q(t_1) = 0. \tag{24}$$

In (24), we can show that

$$\dot{\omega}(t_1) = (p + KK_vK_{vF})\eta - PR. \tag{25}$$

Thus, from the left-hand inequality in (21), it follows that $\dot{\omega}(t_1) > 0$, and thus condition i) is satisfied.

Secondly, to see the output response $y(t)$ after $t = t_1$, without loss of generality, let $t_1 = 0$. Then, by taking Laplace transformation of the system in (24), we can show that

$$y(s) \triangleq K_{vF}\Omega(s) = \frac{\eta[(p + KK_vK_{vF} - pR/\eta)s + KK_vK_{vF}\alpha]}{s(s+a)(s+b)} + \frac{R-\eta}{s}. \tag{26}$$

From (26), observe that if the per unit overshoot of $y(t) - (R - \eta)$ for $t > t_1$ is less than one, condition ii) will be satisfied. Here, note that the per unit overshoot of the output response $\tilde{y}(t) \triangleq K_{vF}x_1(t)$ of the system in (19) is less than one, where $\tilde{y}(s)$ is given by

$$\tilde{y}(s) = \frac{\tilde{R}KK_v(s+\alpha)}{s(s+a)(s+b)} \tag{27}$$

and also note that the steady-state value of $y(t) - (R - \eta)$ and $\tilde{y}(t)$ are the same when $\tilde{R} = \eta$. Then, by Lemma 1 and by the fact that (21) implies

$$\frac{KK_{vF}K_v\alpha}{p + KK_vK_{vF} - pR/\eta} > \alpha$$

we can conclude that condition ii) is satisfied.

Finally, if the peak value of $|u(t)|$ of the controller in (18) is less than E_{max} for $t > t_1$, then condition iii) is satisfied. To show this, as in the proof of ii), let $t_1 = 0$. Then, the Laplace transform of $u(t)$ can be given by

$$u(s) = \frac{\eta[(p + KK_vK_{vF} - pR/\eta)s + KK_vK_{vF}\alpha](s+p)}{K_{vF}Ks(s+a)(s+b)}. \tag{28}$$

Let α' be defined by

$$\alpha' \triangleq KK_vK_{vF}\alpha/(p + KK_vK_{vF} - pR/\eta). \tag{29}$$

Then from (21), we can know that

$$\alpha < \alpha' < 2a \tag{30}$$

and $u(s)$ in (28) can be written as

$$u(s) = \frac{\eta(p + KK_vK_{vF} - pR/\eta)}{KK_{vF}} \frac{1}{s} \cdot \left[1 - \frac{(a+b-\alpha'-p)\left(s + \frac{ab-\alpha'p}{a+b-\alpha'-p}\right)}{(s+a)(s+b)}\right]. \tag{31}$$

Now it remains to show that the per unit overshoot of the unit step response of the transfer function defined by

$$\tilde{u}(s) \triangleq \frac{(a+b-\alpha'-p)\left(s + \frac{ab-\alpha'p}{a+b-\alpha'-p}\right)}{(s+a)(s+b)} \tag{32}$$

is less than one, and that the peak value of $|u(t)|$ is less than E_{max}. For this, let α'' be given by

$$\alpha'' \triangleq \frac{ab - \alpha'p}{a+b-\alpha'-p}$$

and note from (30) and (20) that

$$\alpha' < b. \tag{33}$$

Since the transfer function $x_1(s)/\tilde{R}(s)$ of the system (19) has the pole-zero pattern given by

$$\alpha < a < b$$

it can be easily shown that

(Case 1) if $\alpha < \alpha' \leq a$, then $\alpha' < \alpha'' \leq a$

(Case 2) if $a < \alpha' < b$, then $a < \alpha'' < b$.

Thus, it follows from Lemma 1 that the per unit overshoot of the unit step response of $\tilde{u}(s)$ in (32) is less than one.

Now, note that since the second part of (31) is the same as $\tilde{u}(s)$ in (32), and thus has per unit overshoot less than one, the peak value of $|u(t)|$ occurs at $t = t_1 = 0$. Here, the initial value of (30) is given by

$$u(t_1) = \frac{\eta(p + KK_vK_{vF} - pR/\eta)}{KK_{vF}}. \tag{34}$$

Thus, from the left-hand inequality in (21), it follows that

$$u(t_1) < \eta K_v.$$

Further, from the right-hand inequality in (21), it also follows that

$$\max_{t>t_1} |u(t)| = u(t_1) < E_{\max}.$$

From the proofs of i), ii), and iii), the proof of Theorem 1 is completed.

It is noted from Theorem 1 that, though the inequality condition in (21) seems very restrictive, there exists η satisfying the inequality condition in (21) for all practical systems. This can be shown as follows: if

$$pR/(KK_vK_{vF} + p) < E_{\max}/K_v \tag{35}$$

then there always exists η satisfying (21). Note that the inequality (35) can be rewritten as

$$R < \frac{K_{vF}KE_{\max}}{p} + \frac{E_{\max}}{K_v}. \tag{36}$$

Also note that in practical motor systems

$$\Omega_{\max} < \frac{KE_{\max}}{p} \tag{37}$$

since the maximum allowable current command $E_{\max}$ should be large enough for the steady-state value of the motor output speed response to be rated speed $\omega_{\max}$. Thus, from (37), it follows that

$$\begin{aligned} R &= K_{vF}\omega \\ &< K_{vF}\Omega_{\max} \\ &< K_{vF}\frac{K}{p}E_{\max} + \frac{E_{\max}}{K_v}. \end{aligned} \tag{38}$$

This implies that for all the practical systems, there exists η satisfying the condition (21) of Theorem 1.

To propose a rule of thumb for the design of parameters K_v, α, and η, it should be noted that as K_v and α get larger, the speed response becomes faster if we design the speed controller only for the idealized open-loop speed control system in (14). However, there are some practical limitations, such as the dynamics of the current controller and current feedback filter, which have been neglected. Hence, in designing the speed controller, we should choose K_v and α such that all those dynamics do not seriously affect the closed-loop system performance. This implies that the zero of the speed controller should be located far from the zero of the current control system in the right-hand side of the zero of the current control system. For this reason, it is proposed that α be in the range of

$$\frac{1}{10}\beta < \alpha < \frac{1}{5}\beta \tag{39}$$

and the gain parameter K_v is chosen such that the two negative real poles are located at $-a$ and $-b$ in the S-plane while satisfying the condition (20), and $-a$ is located nearby $-\alpha$ to have small overshoot. These are summarized in the following rule of thumb.

Step 1) Choose α such that

$$\frac{\beta}{10} < \alpha < \frac{\beta}{5}.$$

Step 2) Choose K_v such that two negative real poles $-a$ and $-b$ satisfy $a < 2b$ and $|a - \alpha|$ is sufficiently small.

Step 3) Choose η such that η is sufficiently small to obtain a fast settling time response, while satisfying

$$pR_{\max}/(KK_vK_{vF} + p) < \eta < E_{\max}/K_v. \tag{40}$$

Here, it is remarked that the maximum magnitude of reference signal should be limited as $R_{\max} \leqslant 10$ V, since we usually implement the controller by the Op Amp's for signal amplification.

IV. A NUMERICAL DESIGN EXAMPLE AND ITS EXPERIMENTAL RESULTS

To show validities of the results in Sections II and III, some experimental results are illustrated for a dc servomotor with parameters given in Appendix B. More specifically, three types of speed controllers, i.e.,

(A) controller with PI-action,
(B) controller with P- and limited I-action, and,
(C) controller with PI-plus bang-bang action

are compared and discussed. For this, the design parameters K_c, β, and K_{cF} of the current controller in Section II will be first determined. Then, the design parameters of the three types of speed controllers will be determined.

Let the maximum allowable current $I_{\max}$ and the maximum current reference signal $E_{\max}$ be 6 A and 6 V, respectively. Then it follows from (13) that K_{cF} becomes unity. Next, by applying Step 2) of the rule of thumb in Section II, we obtain that K_c and β are 26 and 256, respectively. In this case, the transfer function in (11) is given by

$$\frac{\Omega(s)}{E_i(s)} = \frac{185}{s + 0.2}. \tag{41}$$

It is noted that the parameters K_v, α, and η, of the controller type (C) is determined by applying the method in Section III. The K_v and α of the controller type (C) will be used as the gain parameters of the speed controller type (A) and (B). To proceed further, let K_{vF} be 0.05, which implies that the rated output speed $\omega(t)$ is 120 rad/s for 6-V maximum speed reference input voltage $R_{\max}$. Since the zero of the current control system is located at -256, let α be equal to 50 by the design Step 1) in Section III. Then, by choosing the per unit overshoot δ of $x_1(t)$ of the system in (19) to be 0.10, we can obtain that K_v is equal to 30. From these gain parameters, the characteristic equation of the system (19), which is given by

$$D(s) = s^2 + (0.2 + 9.25K_v)s + 9.25K_v\alpha \tag{42}$$

has two real roots, -65 and -212, which satisfy the condition in (20). Thus, by (38) of design Step 3) in Section III, η is chosen to be 0.2.

It is remarked that the current controller and speed controllers (A), (B), and (C) were implemented as the circuit diagrams shown in Figs. 4 and 5. It is further remarked that in the case of the current controller shown in Fig. 4, the ratio of the resistor values implying the proportional gain is chosen to be 3.25. The rationale is that $K_c = 26$ is the product of the proportional gain of the circuit in Fig. 4 and the gain of the transistorized PWM power amplifier whose value is about 8. The gain parameter values of the three types of speed controllers (A), (B), and (C) are summarized in Table I.

To evaluate the performances of the three types of speed controllers (A), (B), and (C), we apply a 50-mV squarewave signal as the 'small' reference speed input signal, and apply a 1.6-V squarewave signal as the 'large' reference speed input signal. The speed output responses of the system with controllers (A), (B), and (C) with respect to the small-signal input and large-signal input are shown in Figs. 6 and 7, respectively. When the input signal is small, controllers (A), (B), and (C) are of identical PI-type. The speed response of the system is depicted in Fig. 6(a), with the controller gain being determined by the rule of thumb in Section III. It is interesting to note from Fig. 6(b) that, even when K_v and α are chosen such that the damping ratio of $D(s)$ in (42) is 0.707 as suggested in [1]-[3], and that the natural frequency is the same as the above case, i.e., $K_v\alpha = 1500$, the speed response shown in Fig. 6(a) is better than that of the system with a controller as designed in [1]-[3]. We can also observe from Fig. 7 that the speed response of the system with controller (C) is better in the sense that the speed response shown in Fig. 7(c) has smaller overshoots and faster settling time than those of the speed responses of the system with controller (A) or (B) in Fig. 7(a) and (b).

Here, it should be noted that with the same K_v and α, the speed response of the system with controller (C) oscillates as shown in Fig. 8, when η reduces to zero, which violates the condition in (21). The damping effect of load torque disturbances shown in Fig. 9 is very similar to that of a step change in speed reference. In our experiment, as in [1], a dc generator was coupled to the motor and loaded using a resistance bank to load the motor.

V. CONCLUDING REMARKS

In this paper, a design method is proposed for the PI current controller for dc servo systems with the transistorized PWM driver applied to a large inductive load. A novel type of speed controller using PI-action incorporated with bang-bang action is suggested. It was further shown via comparative experimental study that the proposed speed controller was better than the speed controller only with PI-action, or with P- and limited I-action in the sense of classical figures of merit.

Fig. 4. Circuit diagram of current controller.

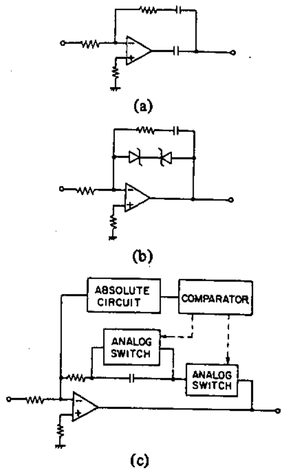
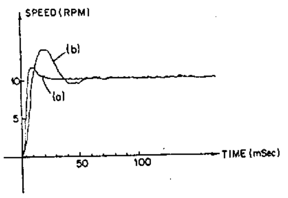


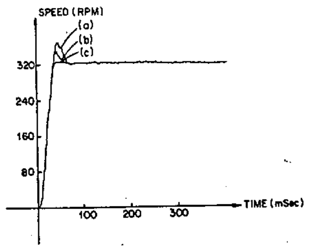
Fig. 5. Circuit diagram of speed controller. (a) PI-type. (b) P- and limited I-type. (c) PI-plus bang-bang type.

TABLE I
PARAMETER VALUES OF THE THREE TYPES OF SPEED CONTROLLER

Controller type / Description (symbol)	(A) PI-action	(B) P- and limited I-action	(C) PI-plus Bang-bang action
Proportional gain (K_v)	≅ 30	≅ 30	≅ 30
Integrator gain (α)	≅ 50	≅ 50	≅ 50
Switching reference value (ω_{gap})	None	None	≅ 0.2 Volt
Slope of the Krikelis's Integrator (H)	None	≅ ∞	None
Limiting value of the Krikelis's Integrator (μ)	Inherently ≅ 13 Volt (13 Amp)	6 Volt (6 Amp)	None



. 6. Small-signal speed response. (a) Response with gain determined by the proposed method. (b) Response with gain determined by the conventional method.



. 7. **Large-signal speed response. (a) Response for PI-type controller (A). (b) Response for P- and limited I-type controller (B). (c) Response for PI-plus bang-bang type controller (C).**

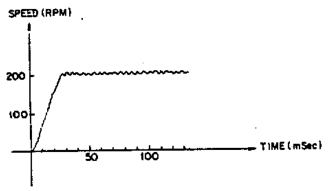


Fig. 8. **Oscillating response of large-signal input with $\eta = 0$.**

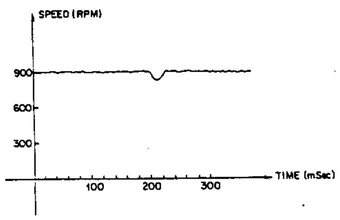


Fig. 9. **Speed response with load torque disturbance.**

It is remarked that the proposed controller may be utilized)r many industrial process control systems which are often iodeled as first-order systems with saturated nonlinearity. : is also remarked that the proposed control law can be easily nplemented by a microprocessor for digital control.

APPENDIX A

Proof of Lemma 1:

For a unit step response given by

$$\frac{1}{s}G(s) = \frac{s + \tilde{\alpha}}{s(s+a)(s+b)} \tag{A1}$$

where $0 < \tilde{\alpha}$ and $0 < a < b$, it can be easily shown [9] by inverse Laplace transformation that there can be overshoot only when $\tilde{\alpha} < a$. Hence, for the proof, it is sufficient to show that the per unit overshoot of (A1) is a monotonically decreasing function of $\tilde{\alpha}$ for $0 < \tilde{\alpha} < a < b$. To show this, let us take the inverse Laplace transformation of (A1) and find the per unit overshoot δ. After some tedious calculation we can obtain the per unit overshoot δ as follows:

$$\delta = [(a - \tilde{\alpha})^b/(b - \tilde{\alpha})^a]^{\frac{1}{b-a}}/\tilde{\alpha}$$

at

$$t = \frac{1}{b-a} \ln\left[\frac{b - \tilde{\alpha}}{a - \tilde{\alpha}}\right]. \tag{A2}$$

Now differentiate (A2) with respect to $\tilde{\alpha}$ and obtain

$$\frac{\partial \delta}{\partial \alpha} = [(a - \tilde{\alpha})^b/(b - \tilde{\alpha})^a]^{\frac{1}{b-a}} \cdot \left[\frac{-\tilde{\alpha}}{(b - \tilde{\alpha})(a - \tilde{\alpha})} - 1\right]. \tag{A3}$$

Here, we know that (A3) is negative for $0 < \tilde{\alpha} < a < b$; hence, this completes the proof.

APPENDIX B

SPECIFICATIONS OF DC SERVOMOTOR (0.4 Kw)

Item	*Value*	*Unit*
Rated Power	400	Watt
B	7.2	$Kg \cdot cm^2/rad \cdot s$
J	36	$Kg \cdot cm^2$
La	1.38	mH
L_{ext}	10	mH
L	11.4	mH
R_a	2	Ω
K_t	6664	$Kg \cdot cm^2/s^2 \cdot A$
K_b	0.7	V/rad/s
V_{amax}	90	V
I_{max}	6	A
Ω_{max}	120	rad/s

REFERENCES

[1] P. C. Sen and M. L. MacDonald, "Thyristorized dc drives with regenerative braking and speed reversal," *IEEE Trans. Indust. Electron. Contr. Instrum.*, vol. IECI-25, no. 4, 1978.
[2] T. Krishnan and B. Ramaswami, "Speed control of dc motor using thryristor dual converter," *IEEE Trans. Indust. Electron. Contr. Instrum.*, vol. IECI-23, no. 4, 1976.
[3] T. Krishnan and B. Ramaswami, "A fast-response dc motor speed control system," *IEEE Trans. Ind. Appl.*, vol. IA-10, 1974.
[4] C. K. Taft and E. V. Slate, "Pulsewidth modulated dc control: a parameter variation study with current loop analysis," *IEEE Trans. Indust. Electron. Contr. Instrum.*, vol. IECI-26, 1979.
[5] S. J. Bailey, "Servomotor-amplifier choice is key to better position loop response," *Contr. Eng.*, vol. 28, no. 7, pp. 63–68, 1981.
[6] Y. Takahashi, M. J. Rabins, and D. M. Auslander, *Control and Dynamic Systems.* London: Addison-Wesley, 1972.
[7] R. M. Phelan, *Automatic Control Systems.* London: Cornell University Press, 1977.
[8] N. J. Krikelis, "State feedback integral control with 'intelligent' integrators," *Int. J. Contr.*, vol. 32, no. 3, pp. 465–473, 1980.
[9] J. H. D'azzo and C. H. Houpis, *Linear Control System Analysis and Design.* New York: McGraw-Hill, 1975.