be the period of the swinging object while O is accelerating. Start with the system at rest at time 0. O accelerates at rate a_x^O. Then at time $t = \tau_a$ the object has completed one full swing and is at the same position and velocity relative to O as at time 0. Relative to a fixed reference frame the suspended object is vertical and has velocity equal to the velocity of O. If O stops accelerating at time $t = \tau_a$ then the suspended object has no relative movement and no x forces and therefore will travel beneath O in a swing-free path. O is moving at speed $a_x^O \tau_a$ and accelerated for a distance $d = 1/2 a_x^O \tau_a^2$. To come to rest at some point, merely reverse the process, accelerating at rate $-a_x^O$ starting at a point d from the goal.

An alternative is a two-part acceleration strategy similar to Starr [1]. Starting as before with the system at rest, point O accelerates for some amount of time t_a. Let t_b be the smallest $t > t_a$ such that $\theta(t_a) = \theta(t_b)$ and $\dot{\theta}(t_a) = -\dot{\theta}(t_b)$. Let O travel at constant velocity $t_a a_x^O$ for $t_b - t_a$, then accelerate at rate a_x^O starting at time t_b. Since $\theta'(t_a) = \theta'(t_b)$ and $\dot{\theta}'(t_a) = -\dot{\theta}'(t_b)$, then the position and velocity of the suspended object at time $t_a + t_b$ will be the same as for $t = 0$. Therefore, relative to the fixed reference frame $\theta(t_a + t_b) = \theta(0) = 0$ and the x component of the velocity of G is equal to the x component of the velocity of O. If O stops accelerating at $t_a + t_b$, then G will move in swing-free transport below O.

To solve for t_b let ω_a be the frequency while O is accelerating and ω_c be the frequency while O is at constant velocity. For $0 \le t \le t_a$ we observe the motion from the reference frame of O. In this frame we have $\theta'(0) = \psi$ and $\dot{\theta}'(0) = 0$. We solve for A and B of (13) under these initial conditions and get $A = \psi$ and $B = 0$. Then at time t_a when acceleration ends

$$\theta'(t_a) = \theta(t_a) = \psi \cos \omega_a t_a \tag{15}$$

$$\dot{\theta}'(t_a) = \dot{\theta}(t_a) = -\psi \omega_a \sin \omega_a t_a. \tag{16}$$

Solving (13) from the reference frame of constant velocity we get

$$A = \psi \left(\cos \omega_c t_a \cos \omega_a t_a + \frac{1}{\cos \psi} \sin \omega_c t_a \sin \omega_a t_a \right) \tag{17}$$

$$B = \psi \left(\sin \omega_c t_a \cos \omega_a t_a - \frac{1}{\cos \psi} \cos \omega_c t_a \sin \omega_a t_a \right). \tag{18}$$

In order to simplify finding t_b we observe that one can write (13) as

$$\theta(t) = C \sin \omega (t + \alpha) \tag{19}$$

where

$$C = \sqrt{A^2 + B^2} \tag{20}$$

$$\alpha = \frac{1}{\omega} \arctan \frac{A}{B}. \tag{21}$$

Substituting (17) and (18) into (20) and (21) we get

$$C = \psi \sqrt{\left(\frac{\sin^2 \omega_a t_a}{\cos^2 \psi} + \cos^2 \omega_a t_a \right)} \tag{22}$$

$$\alpha = \frac{1}{\omega_c} \arctan \left(\frac{\cos \omega_c t_a \cos \omega_a t_a + \frac{1}{\cos \psi} \sin \omega_c t_a \sin \omega_a t_a}{\sin \omega_c t_a \cos \omega_a t_a - \frac{1}{\cos \psi} \cos \omega_c t_a \sin \omega_a t_a} \right). \tag{23}$$

From our definition of t_b we know that

$$C \sin \omega_c (t_b + \alpha) = C \sin \omega_c (t_a + \alpha) \tag{24}$$

$$C \omega_c \cos \omega_c (t_b + \alpha) = -C \omega_c \cos \omega_c (t_a + \alpha). \tag{25}$$

Let $\xi = \omega_c(t_a + \alpha) \bmod 2\pi$, $-\pi \le \xi \le \pi$. Then we know that $\omega_c(t_b + \alpha) = \pi - \xi \bmod 2\pi$.

Since $\alpha \to 0$ as $t_a \to 0$, our result matches that of Starr [1], which is to accelerate for some short period t_a, wait for half the period of the free swinging object, and then accelerate again for length t_a.

IV. Summary

Our result shows that there is a family of control strategies which will result in swing-free movement of objects suspended from a movable point. While it is possible to control the swing with acceleration periods of arbitrary length, it is necessary in that case to know the period of the suspended object, which requires knowledge of the location of the object's center of mass. In a practical application of this result, the best strategy is to build a sensing system into the bridge and crane hook that allows the crane to recognize when the hook is directly below its support point. The control strategy is then to accelerate for one full period, which is recognized when the hook returns to the point directly below its support point. If the bridge continues to move at this velocity, the suspended object will not swing. To stop in a swing-free state, the crane begins to decelerate at the same rate as it accelerated, starting at a distance from its goal equal to the distance traveled while accelerating. In typical applications it is expected that the rate of acceleration will be sufficiently smaller than g and that the masses moved by the crane will be sufficiently uniform in composition that the period of the system can be approximated adequately to allow the controller to select a desired velocity of travel and compute the necessary rate of acceleration.

References

[1] G. P. Starr, "Swing-free transport of suspended object with a path controlled robot manipulator," *J. Dyn. Syst., Meas., Contr.*, vol. 107, no. 1, pp. 97–100, Mar. 1985.

[2] I. H. Shames, *Engineering Mechanics*, Vol. II, *Dynamics*, 2nd ed. Englewood Cliffs, NJ: Prentice-Hall, 1966.

[3] F. P. Beer and E. R. Johnston, Jr., *Mechanics for Engineers: Statics and Dynamics*, 2nd ed. New York, NY: McGraw-Hill, 1962, pp. 705–706.

Coordination of Dual Robot Arms Using Kinematic Redundancy

IL HONG SUH AND KANG G. SHIN

Abstract—**A new method is developed to coordinate the motion of dual robot arms carrying a solid object, where the first robot (leader) grasps one end of the object rigidly and the second robot (follower) is allowed to change its grasping position at the other end of the object along the object surface while supporting the object. It is shown that this flexible grasping**

Manuscript received October 8, 1987; revised June 3, 1988. This work was supported in part by the NSF under Grants ECS-8409938 and DMC-8721492, and by the Post Doctoral Program of the Korean Science and Engineering Foundation. An earlier version of this communication was presented at the 1988 International IEEE Conference on Robotics and Automation. All correspondence regarding this communication should be addressed to K. G. Shin.

I. H. Suh is with the Department of Electronics Engineering, Hanyang University, Seoul, Korea.

K. G. Shin is with the Department of Electrical Engineering and Computer Science, The University of Michigan, Ann Arbor, MI 48109-2122.

IEEE Log Number 8823931.

1042-296X/89/0400-0236$01.00

is equivalent to the addition of one more degree of freedom (DOF), giving the follower more maneuvering capabilities. Especially, motion commands for the follower are generated by using kinematic redundancy.

To show the utility and power of the method, an example system with two PUMA 560 robots carrying a beam is analyzed in detail.

I. Introduction

Recently, several methods have been proposed to coordinate the control of two robot arms carrying a solid object [1]-[4], with the relative position and orientation of the two robot arms fixed during the entire execution of the robots' task. Under this setting, however, some tasks such as carrying an object along a prespecified path with two robots may not be accomplished due to the insufficient number of degrees of freedom (DOF's) available to them. Let us call one of the two robots the *leader* and the other the *follower*. Consider the motion planning for these two robots, where motion commands for the follower are to be generated based on the leader's motion. When the leader's motion leads the follower to a singular region, or when the motion command generated from the leader's motion requires the follower to violate its joint limits and/or to collide with obstacles, the task cannot be accomplished with the invariant grasping position unless the desired path is modified or the number of the follower's DOF's is increased.

In this communication, a new method to overcome the above difficulty is proposed by relaxing the assumption of invariant grasping position of the follower. With this relaxation, the follower will first be shown to be considered as a redundant manipulator without physically adding joints to it. Then, the follower's motion commands are generated by employing the kinematic control techniques commonly used for redundant manipulators [5]-[8]. The kinematic control of a redundant manipulator is known to find joint angles and/or velocities such that its end-effector attains desired positions and orientations while minimizing some cost function. The object surface is required to be smooth enough for the follower to change its grasping position while keeping its (supporting) orientation and position in order not to drop the object. This requirement may be satisfied by placing ball bearings on the follower's palm such that the sliding contact friction between the follower and the object can be made sufficiently small for the follower's hand to move along the object surface smoothly. It is remarked that our method may be considered as a generalization of the Zheng and Luh's method [1] in the sense that if the smoothness assumption of the object does not hold, then our method reduces to theirs.

It is also worth pointing out the fact that our proposed method is totally different from the one described in [3] for the following reason.[1] The work in [3] considered only passive damping effects when the two robot arms grasp an object loosely. Once a robot's grasping position is determined, the approaches in [3] attempted to regulate the position against unwanted interaction forces/torques. By contrast, we propose a method to actively use the looseness, which results in kinematic redundancy.

The organization of the communication is as follows. In Section II, a problem formulation is presented along with the definition of "supporting orientation." In Section III, a solution approach is proposed by employing the control techniques for redundant manipulators. In Section IV, a numerical example is presented to show the utility and power of our proposed method, where two PUMA 560 manipulators are considered. The paper concludes with Section V.

II. Problem Formulation

Consider two robots each with n joints carrying a rigid object as shown in Fig. 1, which is too large and too long for a single robot to handle. The task given to the two robots is to move the object from one location to another along a prescribed path, while not exceeding joint limits and/or avoiding obstacles. Under the smoothness assumption mentioned earlier, the problem is to determine the joint trajectories of two robots as well as the trajectory of the follower's grasping position to accomplish the task.

[1] This fact was not stated explicitly in the first version of this communication and thus appears to have led to one reviewer's confusion.

Let (x^o, y^o, z^o), (x_n^l, y_n^l, z_n^l), and (x_n^f, y_n^f, z_n^f) be the coordinate frames of object, end-effectors of the leader and the follower, respectively (see Fig. 1). Let r_1 and r_2 be the position vectors of the origins of (x^o, y^o, z^o) and (x_n^f, y_n^f, z_n^f) with reference to (x_n^l, y_n^l, z_n^l) and (x^o, y^o, z^o), respectively. Let H_1 and H_2 be the 4×4 homogeneous matrices representing the coordinate frame of the object with reference to (x_n^l, y_n^l, z_n^l) and that of the follower's end-effector with reference to (x^o, y^o, z^o), respectively. Also, let q be an n-dimensional vector representing joint positions, and $n(q)$, $s(q)$, $a(q)$, and $p(q)$ be the normal, sliding, approaching, and position vectors of the leader with reference to the base coordinate (x_B, y_B, z_B). Define $T_B^n(q)$ and $R_B^n(q)$ as

$$T_B^n(q) \triangleq \begin{bmatrix} n(q) & s(q) & a(q) & p(q) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

and

$$R_B^n(q) \triangleq [n(q) \quad s(q) \quad a(q)]. \tag{2}$$

Let $D(t)$ be the 4×4 homogeneous matrix representing the desired motion trajectory of the object with reference to the base coordinate.

To obtain the holonomic constraints for the two robots, define a "supporting orientation" as a relative orientation between the two robots or between the object and the follower in order not to drop the object. Such a supporting orientation may vary with the types of object as shown in Figs. 2 and 3. In Fig. 2, the follower's hand has to grasp the object in order to support it, while in Fig. 3 the follower's hand needs to support the object without grasping it. Supporting orientations for these examples could be obtained by the analysis of object surface and/or static force equilibrium.

Suppose the equation of object surface represented in the object coordinate (x^o, y^o, z^o) is given by

$$\phi(r_2) = 0, \qquad r^{\min} \le r_2 \le r^{\max} \tag{3}$$

where $\phi: R^3 \to R^1$ is twice continuously differentiable and $r^{\min}$ and $r^{\max}$ are the 3×1 vectors representing the positional limits within which the object must lie. Let $r^* \triangleq [r_x^* \; r_y^* \; r_z^*]^T$ be the current supporting position vector of the follower's hand with reference to the object coordinate. Also, let $s^*(r^*)$ and $a^*(r^*)$ be the unit sliding (tangential) and normal vectors of the object at r^*, respectively, and $n^*(r^*) \triangleq a^*(r^*) \times s^*(r^*)$. Then, align z_n^f and x_n^f of the follower's hand coordinate with $a^*(r^*)$ and $s^*(r^*)$, respectively.

For the example in Fig. 3, a supporting orientation can be obtained by a simplified analysis of the static force equilibrium. Assume that a) the leader's hand is capable of generating forces along the x, y, and z axes of the object coordinate by grasping the one side of object firmly, b) the object's mass is concentrated at its mass center, and c) the follower's hand can exert force only to the normal direction of its contact point. Also, let g be the gravitational acceleration vector and be represented as $[g_x, g_y, g_z]^T$ in the object coordinate, and let $\tilde{g} \triangleq [g_x, 0, 0]^T$. Further, let f^l, f^f, M, and r_1^0 be force vectors of the leader and the follower, the mass of object, and the position vector representing the origin of (x_n^l, y_n^l, z_n^l) with respect to the origin of (x_o, y_o, z_o), respectively. Then we can obtain the following static force equilibrium equation in the object coordinate:

$$r_1^0 \times f^l + r_2 \times f^f = 0 \tag{4}$$

$$f^l + f^f + M\tilde{g} = 0. \tag{5}$$

Note that two other components Mg_y and Mg_z of Mg in the object coordinate can be compensated by the corresponding counter-force component of the leader's hand force. By substituting (5) for (4), we obtain

$$r_1^0 \times M\tilde{g} = (-r_1^0 + r_2) \times f^f. \tag{6}$$

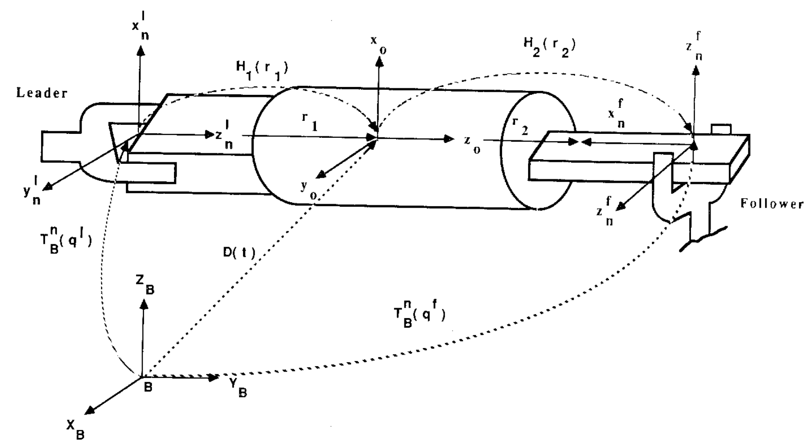

Fig. 1. Two robots handling a rigid body object.

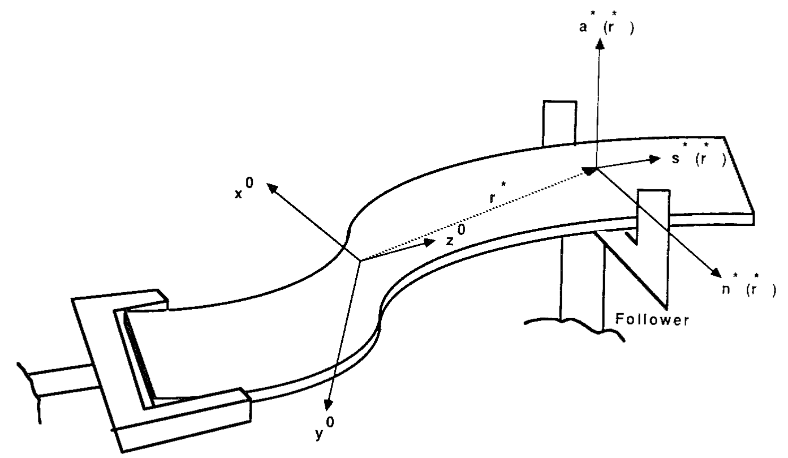

Fig. 2. Supporting orientation when a robot grasps an object.

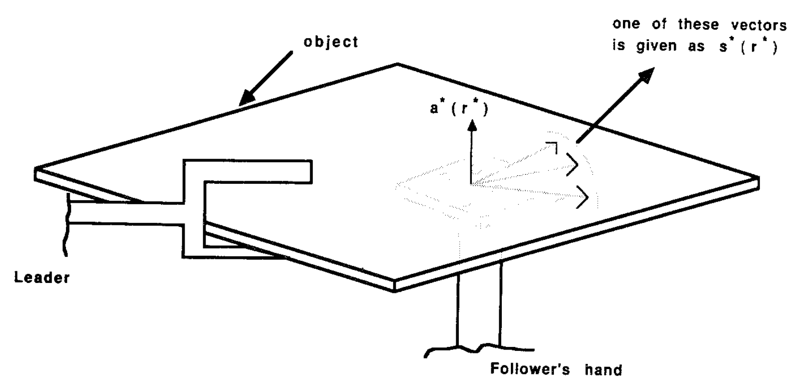

Fig. 3. Supporting orientation when the robot does not have to grasp the object.

Since we assume that the follower's hand can exert force only to the normal direction of its contact point, the force vector f^f of the follower can be written as

$$f^f = ha^*(r_2), \qquad h>0 \tag{7}$$

where h is a constant representing the magnitude of the force vector. It is noted that after substituting (7) into (6), (6) becomes a nonlinear equation with an unknown vector r_2 and an unknown constant h. If solutions to (6) exist at $r_2 = r^*$ and $h = h^*$, and if the follower's hand can be continuously positioned at such an r^*, then we can compute the sliding vector $s^*(r^*)$. To see this, consider an example shown in Fig. 3, where x_0 is aligned with z_B, making $g_y = g_z = 0$. The object surface equation is given as $x = 0$, $-y_{\max} \le y \le y_{\max}$, and $-z_{\max} \le z \le z_{\max}$. Then, $a^*(r^*) = [1, 0, 0]^T$ for any r^*. If r_1^0 is given as $[0, r_{1y}^0, -z_{\max}]^T$, then one can obtain from (6) and (7)

$$r^* = \begin{bmatrix} r_x^* \\ r_y^* \\ r_z^* \end{bmatrix} = \begin{bmatrix} 0 \\ \dfrac{-g_x r_{1y}^0 M}{h} + r_{1y}^0 \\ \dfrac{g_x z_{\max} M}{h} - z_{\max} \end{bmatrix} \tag{8}$$

and thus

$$r_y^* = -\left(\frac{r_{1y}^0}{z_{\max}}\right) r_z^*. \tag{9}$$

In this case, $s^*(r^*)$ is given by

$$s^*(r^*) = \frac{\left[0, \dfrac{-r_{1y}^0}{z_{\max}}, 1\right]^T}{\sqrt{1+\left(\dfrac{r_{1y}^0}{z_{\max}}\right)^2}}. \tag{10}$$

Now, we can obtain the following holonomic constraints; for all $t \in [t_0, t_f]$:

$$D(t) = T_B^n(q^l) H_1(r_1) \tag{11}$$

and

$$T_B^n(q^l) H_1(r_1) H_2(r_2) = T_B^n(q^f) \tag{12}$$

where t_0 and t_f are the initial and final time of the desired trajectory $D(t)$, and $H_1(r_1)$ is a constant 4×4 matrix resulting from the prespecified distance vector r_1, and $H_2(r_2)$ is given by

$$H_2(r_2) = \begin{bmatrix} s^*(r_2) & n^*(r_2) & a^*(r_2) & r_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{13}$$

In (11) and (12), q^l and q^f are bounded as

$$q_{\min}^l \le q^l \le q_{\max}^l \tag{14}$$

and

$$q_{\min}^f \le q^f \le q_{\max}^f. \tag{15}$$

Now, the problem becomes: For given $D(t)$, $t_o \le t \le t_f$ and $H_1(r_1)$, find $q^l(t)$, $q^f(t)$, and $r_2(t)$ such that (11)–(15) and (3) are satisfied for all $t \in [t_o, t_f]$.

It is here assumed that the leader has a sufficient number of DOF's such that (11) and (14) can be completely solved for all $t \in [t_o, t_f]$. Thus we will concentrate on solving (3), (12), (13), and (15).

In (12), the maximum number of independent constraint equations is six as discussed in [9]. Thus if r_2 is prespecified as in [1], and each of the two robots has six DOF's, then the follower's position q^f can be uniquely determined by the leader's position q^l from the six constraint equations. However, in our case, r_2 is a vector to be determined and, thus, the number of unknown variables could be greater than that of constraint equations. This implies that the follower be considered as a redundant manipulator with more than six DOF's due to the presence of the variable r_2.

III. Solution Approaches

A solution to the problem formulated in Section II will be developed using the following steps:

- Derive the linear relationship among joint velocities and the time derivative of the supporting position vector r_2 of the follower. This relation is necessary to solve the holonomic constraint equations (3) and (12). The supporting orientation is not updated here to get a simple rotational velocity relationship between the two robots.
- Find the supporting position vector r_2 and joint vector q^f of the follower by integrating $\dot{r}_2 \triangleq dr_2/dt$ and $\dot{q}_l^f \triangleq dq^f/dt$ subject to the follower's joint limits and the bounds of r_2. The local optimization technique developed for the control of redundant manipulators [5]–[8] is used for this.
- Recalculate the follower's joint vector q^f to get the desired supporting orientation with r_2 fixed.

Note that the above solution approach does not simultaneously calculate the desired supporting orientation and r_2 in order to avoid the computational complexity. Instead, we first find r_2 and then calculate the supporting orientation. This is realistic if the object surface is smooth and not severely curved; otherwise, the grasping position cannot be changed while two robots are carrying an object.

A. Derivation of Linear Relationships Among Joint Velocities

Consider the holonomic constraint equation (12) which can be rewritten as

$$R_B^n(q^l) R_1(r_1) R_2(r_2) = R_B^n(q^f) \tag{16}$$

and

$$p(q^l) + R_B^n(q^l) r_1 = -R_B^n(q^l) R_1(r_1) r_2 + p(q^f). \tag{17}$$

Equations (16) and (17) represent the holonomic constraints for orientation and position, respectively.

To obtain a simple rotational velocity relationship between the two robots, let

$$R_2(r_2) = [s^*(r^*) \quad n^*(r^*) \quad a^*(r^*)]. \tag{18}$$

Then, from (16), we know that there is no relative motion between the two robots' end-effectors. Consequently, we can establish a linear relationship between the angular velocities of the leader and the follower given by

$$J_a^l(q^l)\dot{q}^l = J_a^f(q^f)\dot{q}^f \tag{19}$$

where $J_a^l(q^l)$ and $J_a^f(q^f)$ are the $3 \times n$ Jacobian matrices relating the angular velocity of the leader's and follower's end-effectors with reference to base coordinate to $\dot{q}^l$ and $\dot{q}^f$, respectively.

To calculate the follower's velocity, differentiate (17) with respect to time. Then we can obtain

$$[J_p^l(q^l) + L^1(q^l)]\dot{q}^l = J_p^f(q^f)\dot{q}^f - R_B^n(q^l)R_1(r_1)\dot{r}_2 - L^2(q^l, r^*)\dot{q}^l \tag{20}$$

where $J_p^l(q^l)$ and $J^f(q^f)$ are some other $3 \times n$ Jacobian matrices relating the positional velocities of the leader's and follower's end-effectors with reference to base coordinate to $\dot{q}^l$ and $\dot{q}^f$, respectively, and $L^1(q^l)$ and $L^2(q^l, r^*)$ are, respectively, given by

$$L^1(q^l) = \frac{\partial[R_B^n(q^l)r_1]}{\partial q_l}$$

and

$$L^2(q^l, r^*) = \left.\frac{\partial[R_B^n(q^l)R_1(r_1)r_2]}{\partial q^l}\right|_{r_2=r^*}.$$

To impose the constraints on r_2 as in (3) along with (19) and (20), differentiate (3) with respect to time. Then, $\dot{r}_2$ should satisfy

$$\frac{\partial \phi}{\partial t} = \left(\frac{\partial \phi}{\partial r_2}\right)^T \dot{r}_2 = 0, \qquad \text{for all } t \in [t_0, t_f]. \tag{21}$$

Let the moving velocity of the follower's end-effector $\dot{r}_2 = \eta s^*(r^*)$, where η is the magnitude of $\dot{r}_2$. Recall that $s^*(r^*)$ has been chosen to be tangential to the object surface that the follower's hand touches. Then, (21) becomes an inactive constraint, implying that the magnitude of moving velocity η be a design variable.

By plugging (21) into (20) and combining (19) with (20), we obtain the desired linear relationship between joint velocities

$$J_f\dot{\hat{q}}=J_l\dot{q}^l \quad (22)$$

where the $6 \times (n+1)$ matrix J_f, $6 \times n$ matrix J_l, and $(n+1)$-dimensional vector $\dot{\hat{q}}$ are, respectively, given by

$$J_f=\begin{bmatrix} J_p^f(q^f) & -R_B^n(q^l)R_1(r_1)s^*(r^*) \\ J_a^f(q^f) & 0 \end{bmatrix} \quad (23)$$

$$J_l=\begin{bmatrix} J_p^l(q^l) & +L^1(q^l) & +L^2(q^l, r^*) \\ & J_a^l(q^l) & \end{bmatrix} \quad (24)$$

and

$$\dot{\hat{q}}=[(\dot{q}^f)^T, \eta]^T. \quad (25)$$

Note from (22) that the number of DOF's is increased by one. Thus if the follower, like most industrial robots, has six joints, (22) plays a role of the Jacobian equation for a redundant manipulator with seven DOF's. It is also noted that if r_2 is prespecified, then $\eta = 0$ and, thus, (22) reduces to the Jacobian equation developed by Zheng and Luh [1].

B. Use of Kinematic Redundancy

In this section, the number of the follower's DOF's is assumed to be six. By employing the local optimization techniques commonly used for the control of redundant manipulators, we will derive a solution for the linear equation (22) subject to the inequality constraints on r_2 and q^f in (3) and (15), respectively. Define $J_f^\dagger$ as the generalized inverse of J_f, i.e., $J_f^\dagger \triangleq J_f^T(J_fJ_f^T)^{-1}$. Then a general solution of (22) can be given by [5]

$$\dot{\hat{q}}=J_f^\dagger J_l\dot{q}^l+(I-J_f^\dagger J_f)\hat{z} \quad (26)$$

where $\hat{z}$ is an arbitrary vector lying in the null space of J_f. Note that (26) implies the relationship among joint velocities, but (3) and (15) are expressed in terms of position vectors. Thus it is necessary to change (3) and (15) to the inequalities imposed on velocities.

To do so, let ΔT be the sampling interval and the subscript i denote the ith component of the corresponding vector. Also let

$$\rho_{\min}=\max_i\left\{\rho_i : \rho_i=\min\left(\frac{r_i^{\min}-r_i^*}{s_i^*(r^*)\Delta T}, \frac{r_i^{\max}-r_i^*}{s_i^*(r^*)\Delta T}\right)\right.$$
$$\left. s_i^*(r^*)\neq 0,\ i=1, 2, 3\right\} \quad (27)$$

and

$$\rho_{\max}=\min_i\left\{\rho_i : \rho_i=\max\left(\frac{r_i^{\min}-r_i^*}{s_i^*(r^*)\Delta T}, \frac{r_i^{\max}-r_i^*}{s_i^*(r^*)\Delta T}\right)\right.$$
$$\left. s_i^*(r^*)\neq 0,\ i=1, 2, 3\right\}. \quad (28)$$

Here, if $s_i^*(r^*) = 0$, then no more motion along the ith axis is generated. Noting that r_2 can be approximated as

$$r_2 \cong r^*+\eta\Delta Ts^*(r^*) \quad (29)$$

(3) is modified to

$$\rho_{\min}\leq\eta\leq\rho_{\max}. \quad (30)$$

Similarly, using the first-order approximation of $\dot{q}^f$, the inequality constraints on q^f can be modified to

$$\dot{\bar{q}}_{\min}\leq\dot{q}^f\leq\dot{\bar{q}}_{\max} \quad (31)$$

where $\dot{\bar{q}}_{\min}$ and $\dot{\bar{q}}_{\max}$ are given by

$$\dot{\bar{q}}_{\min} \triangleq \frac{q^f_{\min}-q^f}{\Delta T} \quad (32)$$

and

$$\dot{\bar{q}}_{\max} \triangleq \frac{q^f_{\max}-q^f}{\Delta T}. \quad (33)$$

To find $\hat{z}$ such that the inequality constraints (30) and (31) are satisfied, let the cost function $I(\dot{\hat{q}})$ be chosen as

$$I(\dot{\hat{q}})=\|\dot{\hat{q}}-\dot{\hat{q}}_c\|_S^2. \quad (34)$$

S is a 7×7 positive-definite weighting matrix and $\dot{\hat{q}}_c$ is a 7×1 vector defined by

$$\dot{\hat{q}}_c \triangleq [\dot{\bar{q}}_{1c}\ \dot{\bar{q}}_{2c} \cdots \dot{\bar{q}}_{6c}\ \rho_c] \quad (35)$$

where $\dot{\bar{q}}_{ic}$, $i = 1, 2, \cdots, 6$ and ρ_c are given by

$$\dot{\bar{q}}_{ic}=\frac{\dot{\bar{q}}_{i,\max}+\dot{\bar{q}}_{i,\min}}{2}, \qquad i=1, 2, \cdots, 6. \quad (36)$$

S is a 7×7 positive-definite weighting and

$$\rho_c=\frac{\rho_{\max}+\rho_{\min}}{2}. \quad (37)$$

Such a $\hat{z}$ can be found by minimizing $I(\dot{\hat{q}})$ in (34) subject to (26), since minimization of $I(\dot{\hat{q}})$ implies that all the joint variables and r_2 be located at the centers of their motion ranges. The solution vector $\hat{z}$ to the minimization of $I(\dot{\hat{q}})$ subject to (26) is given by

$$\hat{z}=[(I-J_f^\dagger J_f)S(I-J_f^\dagger J_f)]^\dagger(I-J_f^\dagger J_f)S(\dot{\hat{q}}_c-J_f^\dagger J_l\dot{q}^l). \quad (38)$$

C. Resolved Motion for Desired Supporting Orientation

It may be necessary to resolve the follower's joint vector q^f into the desired supporting orientation when the supporting position vector r_2 is updated. This can be done easily by solving the follower's Jacobian equation as follows: first, obtain the required differential change δw_x, δw_y, and δw_z of the hand orientation by employing two rotation matrices $R_B^n(q^l)R_1(r_1)R_2(r_2^*)$ and $R_B^n(q^l)R_1(r_1)R_2(r_2)$ as in [9]. Then, find the corresponding differential change of the joint vector δq^f by

$$\delta q^f=\begin{bmatrix} J_p(q_f) \\ J_a(q_f) \end{bmatrix}^{-1}[0\ 0\ 0\ \delta w_x\ \delta w_y\ \delta w_z]^T.$$

Finally, obtain the desired q^f by adding δq^f to the old q^f.

It is remarked that the updated q^f may not satisfy the constraints (15). In such a case, the weighting matrix S in (34) may have to be adjusted such that heavier weights are placed on those joints violating the limits in this updating procedure.

IV. A Numerical Example

Two PUMA 560 manipulators, each with six rotational joints, are employed to demonstrate the utility and power of the proposed coordination method. The task given to the two arms is to transfer an 850-mm-long rigid bar shown in Fig. 4 along a straight line path. Numerical values used in our simulation are as follows:

1) Link parameters and joint ranges of the PUMA 560 manipulator are listed in Table I.

2) Initial joint values and initial grasping position of the follower are tabulated in Table II.

3) $r_{\min}$ and $r_{\max}$ are chosen to be $[0, 0, 0]^T$ and $[0\ 0\ 400]^T$, respectively.

4) The sampling interval ΔT is chosen to be 31 ms.

5) The desired trajectory $D(t)$ with reference to the 0th coordinate

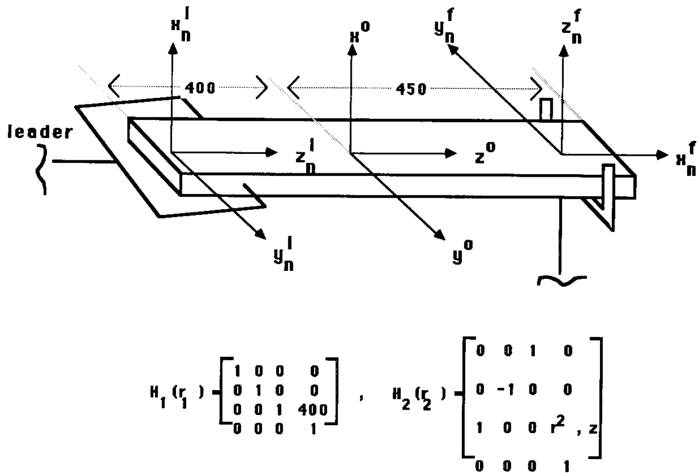

Fig. 4. A 850-mm-long rigid bar and hand orientations of the leader and the follower.

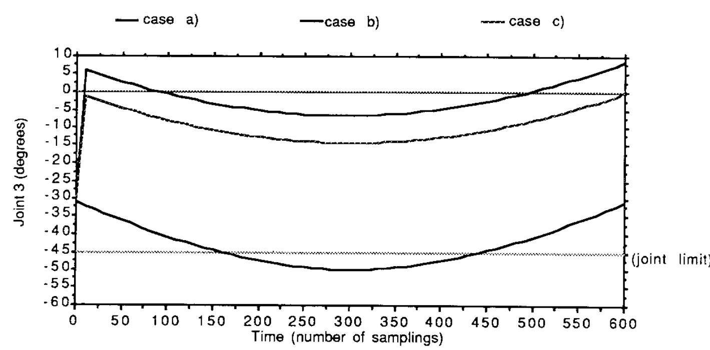

Fig. 5. Position trajectories of joint 3 of the follower.

TABLE I
LINK PARAMETERS AND JOINT RANGE OF PUMA ROBOT

Joint i	α_i (degree)	a_i (mm)	d_i (mm)	Joint Range (degree)
1	−90	0	0	−160 to +160
2	0	431.8	149.09	−225 to 45
3	90	−20.32	0	−45 to 225
4	−90	0	433.07	−110 to 170
5	90	0	0	−100 to 100
6	0	0	56.25	−266 to 266

TABLE II
INITIAL JOINT VALUES AND GRIPPING POSITION OF THE FOLLOWER

q_1	q_2	q_3	q_4	q_5	q_6	$r_{2,z}$
−164.1774	32.065	−30.9446	0	−1.1204	−105.8256	450

frame of the leader is given by

$$D(t)=\begin{bmatrix} 0 & 1 & 0 & d_x(t) \\ 0 & 0 & 1 & 690 \\ 1 & 0 & 0 & 260.4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $d_x(t) = (t/31) - 0.3$.

6) The 7×7 weighting matrix S is chosen to be a diagonal matrix given by

$$S = \text{diag}\,(1, 1, 1, 1, 1, 1, s_7)$$

where s_7 is a constant representing the weighting for the grasping position.

Note that these numerical values are chosen arbitrarily and the capability of our method is independent of the choice of these values.

It is remarked that the sliding vector $s^*(r^*)$ is easily given as $[0\ 0\ 1]^T$ and fixed during the entire motion for the task. Thus the supporting orientation in this example does not change with time during the entire task.

Case a): When the grasping position vector r_2 of the follower is fixed as in [1] and chosen as $[0\ 0\ 450]^T$.

Case b): When the proposed method is applied with s_7 equal to 100.

Case c): When the proposed method is applied with s_7 equal to 1000.

From Fig. 5, it is observed that in case a), a portion of joint trajectory violated the joint limit, while in cases b) and c), joint trajectories lie within the specified joint ranges as a result of moving the follower's hand toward the 3/4 position of the bar. Changes in the grasping position of cases b) and c) are shown in Fig. 6. It is noted from Figs. 5 and 6 that the initial joint motion in case c) is smaller than that in case b). Thus we may expect that if the initial movement of the follower's hand is large enough to cause the violation of the limits of joint velocity $\dot{q}^f_{\min}$ and $\dot{q}^f_{\max}$, then such violations can be avoided by adjusting the weighting matrix S together with $r_{\min}$ and $r_{\max}$ and/or by

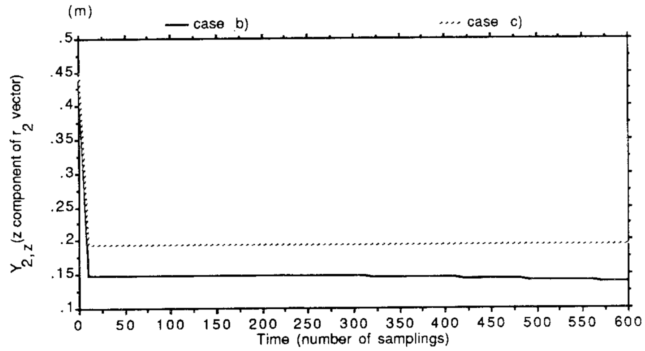

Fig. 6. Change in the gripping position.

modifying (31) as

$$\max(\dot{\bar{q}}_{\min}, \dot{q}^f_{\min}) \le \dot{q}^f \le \min(\dot{\bar{q}}_{\max}, \dot{q}_{\max}).$$

This example has shown that our coordination method is more powerful and flexible than those methods in [1]–[4].

V. Concluding Remarks

A new method for coordinating dual robot arms was proposed, where allowing the follower to change its grasping position was equivalent to adding one DOF to the follower without actual installation of an additional joint motor. The method was then mathematically and numerically shown to be effective for the task of carrying a solid object along a prespecified path while avoiding the joint limits. It is worth mentioning that our method can be considered as a generalization of Zheng and Luh's work [1] and can be further generalized by allowing not only the follower but also the leader to change their grasping position and orientation.

Note that a weighting matrix S has to be selected so as to avoid the violation of joint limits and/or singular regions before implementing our method for the actual coordination of two robot arms. This need could limit the method to preprogrammed motions in some cases. Thus it is necessary to develop a means of adjusting S in real time based on the past motion history. The dynamics of dual arms holding and/or supporting a common object also need to be investigated to deal with the constraints of joint torques. These two subjects are matters of our future research.

Acknowledgment

The authors would like to thank the anonymous reviewers for their insightful comments on an early version of this communication.

References

[1] Y. H. Zheng and J. Y. S. Luh, "Joint torques for control of two coordinated moving robots," in *Proc. IEEE Conf. on Robotics and Automation* (San Francisco, CA, Apr. 1986), pp. 1375–1380.

[2] J. Lim and D. H. Chyung, "On a control scheme for two cooperating robot arms," in *Proc. IEEE Conf. on Decision and Control* (Fort Lauderdale, FL, Dec. 1985).

[3] T. J. Tarn, A. K. Bejczy, and X. Yun, "Coordinated control of two robot arms," in *Proc. IEEE Conf. on Robotics and Automation* (San Francisco, CA, Apr. 1986).

[4] M. Uchiyama, N. Iwasawa, and K. Hakomori, "Hybrid position/force control for coordination of a two-arm robot," in *Proc. IEEE Conf. on Robotics and Automation* (Raleigh, NC, Apr. 1987), pp. 1242–1247.

[5] A. Liegeois, "Automatic supervisory control of the configuration and behavior of multi-body mechanisms," *IEEE Trans. Syst. Man, Cybern.*, vol. SMC-7, no. 12, pp. 868–871, Dec. 1977.

[6] C. A. Klein and C. H. Huang, "Review of pseudoinverse control for use with kinematically redundant manipulators," *IEEE Trans. Syst. Man, Cybern.*, vol. SMC-13, no. 2, pp. 245–250, Mar./Apr. 1983.

[7] W. D. Fisher, "The kinematic control of redundant manipulators," Ph.D. dissertation, School of Electrical Engineering, Purdue University, W. Lafayette, IN, 1984.

[8] P. H. Chang, "A closed-form solution for the control of manipulators with kinematic redundancy," in *Proc. IEEE Conf. on Robotics and Automation* (San Francisco, CA, Apr. 1986), pp. 9–14.

[9] K. S. Fu, R. C. Gonzalez, and C. S. G. Lee, *Robotics: Control, Sensing, Vision and Intelligence.* New York, NY: McGraw-Hill, 1987.

Evaluation of Dynamic Models for PUMA Robot Control

M. B. LEAHY, JR., K. P. VALAVANIS, AND G. N. SARIDIS

Abstract—**The first step in creation of a robotic manipulator controller performance database has been completed. Experimental evaluation of the trajectory tracking performance of a PUMA-600 under computed-torque control has provided valuable engineering insight into the role of dynamics compensation for manipulators with high torque amplification drive systems. Experimental results validate the assumptions that Coriolis and centrifugal forces are negligible and actuator dynamics play a central role in manipulator dynamics. Without complete modeling of the drive system dynamics, computed torque trajectory tracking accuracy was inadequate for gross motion control. More accurate representation of the real robot system dynamics is required for realistic simulation of modern control algorithms and improved real-time performance.**

I. Introduction

A large class of manipulator control techniques utilize some form of dynamical modeling in their control laws [16]. Those techniques assume a degree of modeling accuracy sufficient for cancellation of

Manuscript received April 8, 1986; revised November 23, 1987, and July 13, 1988. The material in this communication was presented at the 1986 IEEE International Conference on Robotics and Automation, San Francisco, CA, 1986. This work was partially supported by the National Science Foundation under Grant ECS83-12179. M. B. Leahy, Jr. attended the Rensselaer Polytechnic Institute, Troy, NY, under the Air Force Institute of Technology's civilian institute program.

M. B. Leahy, Jr., is with the Department of Electrical and Computer Engineering, Air Force Institute of Technology, WPAFB, OH 45433.

K. P. Valavanis is with the Department of Electrical and Computer Engineering, Northeastern University, Boston, MA 02115.

G. N. Saridis is with the Department of Electrical, Computer and Systems Engineering, Rensselaer Polytechnic Institute, Troy, NY 12180.

IEEE Log Number 8825223.