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A motion planning strategy for multifingered hands considering sliding and rolling contacts*

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SUMMARY

An algorithm for the motion planning of the multifingered hand is proposed to generate finite displacements and changes in orientation of objects by considering sliding contacts as well as rolling contacts between the fingertip and the object at the contact point. Specifically, a nonlinear optimization problem is firstly formulated and solved to find the minimum joint velocity and the minimum contact force to impart a desired motion to the object at each time step. Then, the relative velocity at the contact point is found by calculating the velocity of the fingertip and the object at the contact point. Finally, time derivatives of the surface variables and the contact angle of the fingertip and the object at the current time step is computed using the Montana's contact equation to find the contact parameters of the fingertip and the object at the next time step. To show the validity of the proposed algorithm, a numerical example is illustrated by employing the robotic hand manipulating a sphere with three fingers each of which has four joints.

1. INTRODUCTION

In recent years, dexterous multifingered robotic hands have become of interest as fine manipulations are required for more sophisticated tasks in robot applications. Various multifingered robotic hands have been designed and manufactured and many research works including basic analysis of kinematics and control for stable grasping have also been performed. Another important problem arising from the study of multifingered hands is how to impart finite displacements and changes in orientation to a grasped object. Several research works on such issues have been proposed,[1–10] where most of them consider only rolling contact between the fingertips and the object due to the difficulties in finding the evolution of contact points, even though the object could be manipulated more efficiently by allowing sliding contacts at the contact point. Pure rolling contacts can be applied only to the case that the friction coefficient between the fingertips and the object is sufficiently high and initial contact points are well chosen. Therefore, a new motion planning technique considering general contact motions in which rolling and sliding contacts simultaneously occur should be developed to effectively handle even the case that the friction coefficient is moderate or low and/or initial contact points are inappropriate.

Kerr[1] discussed how to move each finger in order to execute a finite displacement of the object. Kinematic equations are derived from the rolling constraint that the fingertip and object velocities are equal at the contact point. Montana[3] and Cai and Roth[4] independently studied the kinematic relations of rigid bodies that maintain contact while in relative motion. The kinematic equations for the contact point evolution were derived. They did not, however, consider the effects of the kinematics of a finger attached to the fingertip. Cole et al.[5] derived the kinematics of rolling contact for two arbitrary shaped surfaces rolling on each other and presented a scheme for the control of these hands. Cole et al.[6] also considered the problem of dynamic control for the planar case of a multifingered hand and presented a new control law that applies specifically to the situation of a hand manipulating a grasped object while certain prespecified fingers slide along the object surface. But their work may be impractical in the sense that they should impose the specific contact motions (fixed or slide) on the object surface. Brock[8] derived a kinematic relation among the object motion, the constraints of motion, and the grasp forces. Based on this relation, a method of reorienting a grasped object was proposed. He did not, however, consider the geometry of the fingertip and the kinematics of fingers. Fearing[7] considered slip from a quasi-static viewpoint to achieve grasp stability. But his work was mainly focuses on gross motion analysis for rigid objects of polygonal cross-section with breaking and remaking contacts. For given joint motions, Trinkle[9] developed the forward object motion problem to find out the velocity of the object, contact forces and the type of contact motions under the assumption that the manipulation system obeys Peshkin's minimum power principle.[10] He did not, however, consider the inverse problem. To the authors' knowledge, no previous work has been reported to positively utilize sliding contacts in the manipulation of the object by multifingered hands.

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In this paper, we propose a motion planning algorithm for multifingered hands manipulating an object of arbitrary shape considering general contact motions between the fingertip and the object. For given contact parameters, which are defined as the position vector and the rotation matrix of the coordinate frame attached to the contact point with respect to the body coordinate frame, the minimum joint velocity and the minimum contact force at the current time step are obtained by solving a nonlinear optimization problem. The relative velocity can then be determined by calculating the object and fingertip velocities at the contact point. And for obtained relative velocity, the contact parameters at the next time step are determined by utilizing the Montana's contact equation.[3] Simulation results are finally illustrated by employing a three fingered robotic hand manipulating a sphere to evaluate the validity of the proposed algorithm.

In the following section, motion planning problems for multifingered robotic hands are formulated. In Section 3, kinematics of multifingered hands grasping an object is described and the kinematics of contact is also described in Section 4. In Section 5, the motion planning is shown to be equivalent to finding the minimum joint velocity and the minimum contact force for each finger. Simulation results are summarized in Section 6 and conclusions are drawn in the final section.

2. PROBLEM STATEMENT

The motion planning for multifingered hands manipulating an object can be achieved by hierarchically solving the PROBLEM I and II at each time step. The analysis of dexterous manipulation in this paper involves the following simplifications: 1) the contact between the fingers and the grasped object occurs only one contact at the fingertip, 2) the links and fingertip of the fingers and the grasped object are rigid, 3) a point contact with friction and a Coulomb friction model are assumed, 4) the analysis is quasi-static, ignoring the dynamics of the fingers.

(PROBLEM I). For given contact parameters at the current time step, find the minimum joint velocity and the minimum contact force of the fingers to generate the desired motions of the object satisfying the force/moment equilibrium equation, the compatibility equation for the contact forces and the relative motions at the contact point, and the Coulomb's law of friction as well as some physical constraints.

It is remarked that the manipulability of fingertip may be implicitly maximized and energy consumption may be minimized when minimizing the joint velocities. It is also remarked that large contact forces might result in low grasp stability, because even a small position error may cause a large disturbing moment at the mass center of the object and the excessive contact forces are not proper for grasping fragile objects. Thus, in PROBLEM I, the minimum joint velocity and the minimum contact force are simultaneously to be obtained.

In general, a motion of the object will cause the contact point to move across the surfaces of the object

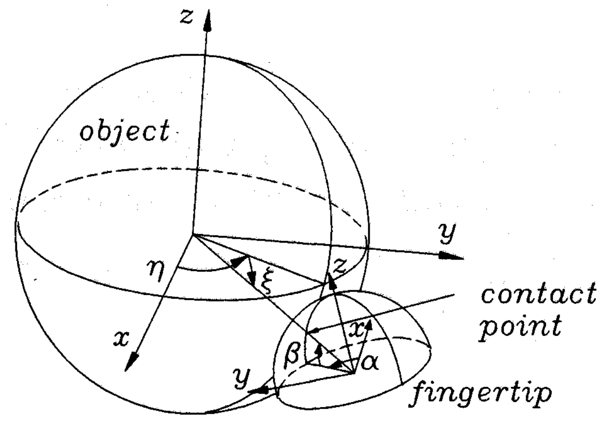

Fig. 1. The surface variables of the fingertip and the object.

and the fingertip through some combination of sliding and rolling. Thus, the contact parameters are changed according to the changes in the contact geometries of the object and the fingertip. But, most studies just examine the case when there is only pure rolling of the object on the fingertip[1,3,6] and/or when convex polygonal objects are held by a hand for planar case.[5,7] Let the surface variables u (α and β for the fingertip and η and ξ for the object, respectively, in this paper) be defined as the parameters which indicate the contact point on two contacting surfaces (Figure 1). Also let the contact angle be defined as the angle between the corresponding axes of two coordinate frames in the common tangent plane attached to respective contact points of two contacting surfaces. Assume that the surface geometry of a body can be described by the surface variables. Then, the evolution of contact parameters of arbitrary shaped contacting surfaces in response to sliding and rolling contacts can be found by solving the following problem at each time step.

(PROBLEM II). Find the time derivatives of the surface variables and the contact angle of the object and the fingertip in response to sliding and rolling contacts at the current time step to predict the contact parameters at the next time step.

Thus, PROBLEM I is iteratively solved at each time step under the contact parameters to be found from PROBLEM II. To the authors' knowledge, this strategy is definitely the first one to date for planning the motions of multifingered hands manipulating a 3-dimensional object with sliding and rolling contacts are simultaneously considered.

3. KINEMATICS OF MULTIFINGERED HAND

In this section, kinematics of multifingered robotic hands is briefly described as in Li et al.'s work.[2] A k-fingered hand grasping an object is shown in Figure 2. Let the number of joints and the joint variables of finger i, $i = 1, \ldots, k$, be denoted as m_i and $q_i \in R^{m}$, respectively. To describe the relative motions between a fingertip and an object, a set of coordinate frames are defined as follows: The reference frame, $\{C_p\}$, is fixed to the palm of the hand; the body coordinate frame, $\{C_b\}$, is fixed to

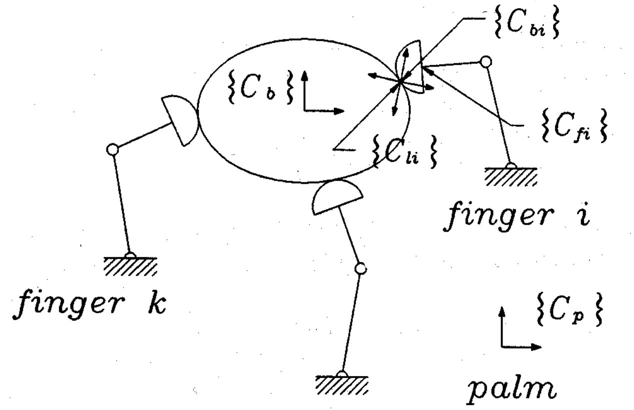

Fig. 2. A k-fingered hand grasping an object.

the mass center of the object; the finger frame, $\{C_{fi}\}$, is fixed to the last link of finger i; at the i-th contact point between the finger i and the object, the local frame of the object, $\{C_{bi}\}$, is fixed with respect to $\{C_b\}$ and the local frame of the finger i, $\{C_{li}\}$, is fixed with respect to $\{C_{fi}\}$, where their z-axes coincide with the outward pointing normal to the object surface and the fingertip surface, respectively and their x- and y-axes lie in the common tangent plane as well as they share a common origin at the contact point.

Let $r_{\beta,\alpha} \in R^3$ and $A_{\beta,\alpha} \in SO(3)$ be denoted as the position vector and the rotation matrix of a coordinate frame $\{C_\beta\}$ with respect to a coordinate frame $\{C_\alpha\}$, respectively. If $(r_{\beta,\alpha}(t), A_{\beta,\alpha}(t))$ in any curve in $SE(3) \equiv R^3 \times SO(3)$ representing the trajectory of $\{C_\beta\}$ with respect to $\{C_\alpha\}$, the translational and rotational velocities of $\{C_\beta\}$ with respect to $\{C_\alpha\}$ can be described by

$$v_{\beta,\alpha} = A^t_{\beta,\alpha}\dot{r}_{\beta,\alpha} \quad \text{and} \quad \omega_{\beta,\alpha} = S^{-1}(A^t_{\beta,\alpha}\dot{A}_{\beta,\alpha}), \tag{1}$$

where S is an operator defined by

$$\omega = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}, \quad S(\omega) = \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix} \tag{2}$$

and superscript t implies the transpose.

For any three coordinate frames $\{C_\alpha\}$, $\{C_\beta\}$, and $\{C_\gamma\}$, the following relation holds between their relative velocities:

$$v_{\gamma,\alpha} = A^t_{\gamma,\beta}(v_{\beta,\alpha} + \omega_{\beta,\alpha} \times r_{\gamma,\beta}) + v_{\gamma,\beta} \tag{3-1}$$

$$\omega_{\gamma,\alpha} = A^t_{\gamma,\beta}\omega_{\beta,\alpha} + \omega_{\gamma,\beta} \tag{3-2}$$

In particular, when $\{C_\gamma\}$ is fixed with respect to $\{C_\beta\}$, the velocity $\{C_\gamma\}$ is related to that of $\{C_\beta\}$ by a constant transformation, given by,

$$\begin{pmatrix} v_{\gamma,\alpha} \\ \omega_{\gamma,\alpha} \end{pmatrix} = \begin{pmatrix} A^t_{\gamma,\beta} & -A^t_{\gamma,\beta}S(r_{\gamma,\beta}) \\ 0 & A^t_{\gamma,\beta} \end{pmatrix} \begin{pmatrix} v_{\beta,\alpha} \\ \omega_{\beta,\alpha} \end{pmatrix} \tag{4}$$

Define (v^i_x, v^i_y, v^i_z) and $(\omega^i_x, \omega^i_y, \omega^i_z)$ as the translational and rotational velocities of $\{C_{bi}\}$ with respect to $\{C_{li}\}$, respectively. In case of contact motions, v^i_x and v^i_y represent sliding, ω^i_x and ω^i_y rolling, and ω^i_z spin motions, respectively. These are velocities of the object with respect to finger i expressed in local frames. Denote ψ_i as the contact angle and $(r_{bi,b}, A_{bi,b})$ and $(r_{li,fi}, A_{li,fi})$ as contact parameters for the object and the fingertip, respectively. Specifically, ψ_i is defined as the angle between the x-axes of $\{C_{bi}\}$ and $\{C_{li}\}$. We choose the sign of ψ_i such that a rotation of $\{C_{bi}\}$ through $-\psi_i$ around its z-axis aligns the x-axes. Also define A_{ψ_i} as the orientation matrix of $\{C_{bi}\}$ with respect to $\{C_{li}\}$ given by

$$A_{\psi_i} = \begin{pmatrix} \cos\psi_i & -\sin\psi_i & 0 \\ -\sin\psi_i & -\cos\psi_i & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{5}$$

Using (3), then, the translational and rotational velocities of $\{C_{bi}\}$, $v_{bi,p}$ and $\omega_{bi,p}$, respectively, can be expressed as

$$\begin{pmatrix} v_{bi,p} \\ \omega_{bi,p} \end{pmatrix} = \begin{pmatrix} A_{\psi_i} & 0 \\ 0 & A_{\psi_i} \end{pmatrix} \begin{pmatrix} v_{li,p} \\ \omega_{li,p} \end{pmatrix} + \begin{pmatrix} v^i_x \\ v^i_y \\ v^i_z \\ \omega^i_x \\ \omega^i_y \\ \omega^i_z \end{pmatrix} \tag{6}$$

On the other hand, the velocity of $\{C_{bi}\}$ is related to the velocity of $\{C_b\}$ by

$$\begin{pmatrix} v_{bi,p} \\ \omega_{bi,p} \end{pmatrix} = \begin{pmatrix} A^t_{bi,b} & -A^t_{bi,b}S(r_{bi,b}) \\ 0 & A^t_{bi,b} \end{pmatrix} \begin{pmatrix} v_{b,p} \\ \omega_{b,p} \end{pmatrix}. \tag{7}$$

Similarly, for finger i, we can obtain that

$$\begin{pmatrix} v_{li,p} \\ \omega_{li,p} \end{pmatrix} = \begin{pmatrix} A^t_{li,fi} & -A^t_{li,fi}S(r_{li,fi}) \\ 0 & A^t_{li,fi} \end{pmatrix} \begin{pmatrix} v_{fi,p} \\ \omega_{fi,p} \end{pmatrix}. \tag{8}$$

Moreover, the velocity of the finger frame, $\{C_{fi}\}$, is related to the velocity of the finger joints, $\dot{q}_i$, by using the finger Jacobian, $J_i(q_i)$,

$$\begin{pmatrix} v_{fi,p} \\ \omega_{fi,p} \end{pmatrix} = J_i(q_i)\dot{q}_i. \tag{9}$$

Denote J_{iu} and J_{il} as the upper and lower matrices of the finger Jacobian, respectively. For given desired object motions, $v_{b,p}$ and $\omega_{b,p}$, then, (v^i_x, v^i_y, v^i_z) and $(\omega^i_x, \omega^i_y, \omega^i_z)$ can be expressed in terms of the finger Jacobian, the velocity of the finger joints, and the contact parameters by substituting equations (7), (8), and (9) into equation (6):

$$\begin{pmatrix} v^i_x \\ v^i_y \\ v^i_z \end{pmatrix} = [A^t_{bi,b}v_{b,p} - A^t_{bi,b}S(r_{bi,b})\omega_{b,p}] - [A_{\psi_i}A^t_{li,fi}J_{iu}\dot{q}_i - A_{\psi_i}A^t_{li,fi}S(r_{li,fi})J_{il}\dot{q}_i] \tag{10-1}$$

$$\begin{pmatrix} \omega^i_x \\ \omega^i_y \\ \omega^i_z \end{pmatrix} = [A^t_{bi,b}\omega_{b,p}] - [A_{\psi_i}A^t_{li,fi}J_{il}\dot{q}_i] \tag{10-2}$$

4. MONTANA'S KINEMATIC EQUATIONS OF CONTACT

This section describes the motion ot a point of contact over the surfaces of two contacting bodies in response to a relative motion of these bodies. When the object roll or slide over the fingertip, the contact parameters $(r_{bi,b}, A_{bi,b})$ and $(r_{li,fi}, A_{li,fi})$ evolve according to the Montana's kinematic equations of contact.[3] If the object and the fingertip surfaces are parametrized by their surface variables, we can describe $(r_{bi,b}, A_{bi,b})$ and $(r_{li,fi}, A_{li,fi})$ by these variables. Let the symbols K, T, and M be represented as the curvature form, torsion form, and metric, respectively with respect to its coordinate system at time t at the point of contact.[11] Let R_ψ be represented as the orientation matrix of the x- and y-axes of $\{C_{li}\}$ with respect to the x- and y-axes of $\{C_{fi}\}$ and the subscripts o and f be denoted as the object and fingertip, respectively. Also let $\tilde{K}_f$ be defined as $R_\psi K_f R_\psi$ and let $K_o + \tilde{K}_f$ be defined as the relative curvature form. At a point of contact, if the relative curvature form is invertible, then the point of contact and the angle of contact evolve according to

$$\dot{u}_0 = M_o^{-1}(K_o + \tilde{K}_f)^{-1}([-\omega_y \quad \omega_x]^t - \tilde{K}_f[v_x \quad v_y]^t), \quad (11)$$

$$\dot{u}_f = M_f^{-1}R_\psi(K_o + \tilde{K}_f)^{-1}([-\omega_y \quad \omega_x]^t + K_o[v_x \quad v_y]^t), \quad (12)$$

$$\dot{\psi} = \omega_z + T_oM_o\dot{u}_o + T_fM_f\dot{u}_f, \quad (13)$$

$$0 = v_z. \quad (14)$$

Thus, the contact equations give time derivatives of the surface variables and the contact angle by receiving the relative velocities of two contacting object. Equations (11) thru (13) are called the first, second, and third contact equations, respectively. Equation (14) is the kinematic constraint of contact imposing the constraint on the relative motion necessary to maintain contact.

Montana imposed the constraints on the above equations that $v_x = v_y = \omega_z = 0$ such that the fingers must roll without slipping.[3] It may be impractical to manipulate an object using only rolling contacts in the sense that the grasping stability may be maintained even though one of the fingers are exposed to sliding contacts. Thus, we will handle the complete contact equations not dropping any terms to consider the general relative velocities which can be obtained by using equations (10-1) and (10-2).

In dealing with the contact equations, we may encounter a numerical difficulty due to geometrical singularity of both bodies in contact. When the surface variables ξ and/or β are very near to $\pi/2$ radians with respect to a prescribed body coordinate frame, the denominators of contact equations go to zero, thus, $\dot{u}_0$, $\dot{u}_f$, or $\dot{\psi}$ to infinity. To eliminate this difficulty, two ad hoc tactics can be employed: 1) a perturbation approach and 2) a transformation approach.

1) Perturb ξ or β by a small amount to keep $\dot{u}_0$, $\dot{u}_f$, or $\dot{\psi}$ within the bounds of finite values.

2) Transform the body coordinate frame occasionally so that the contact points can be described by the surface variables which will not invoke any numerical difficulty.

While the perturbation approach results in the positional errors between the real positions and the numeric data, the transformation approach may suffer from the accumulation of truncation errors due to the multiplications of rotational matrices. In this paper, the perturbation approach is simply tried for convenience' sake. But, an improved tactic involving the transformation approach should be developed to overcome the aforementioned difficulties.

5. SOLUTION APPROACHES FOR PROBLEM I; NONLINEAR PROGRAMMING APPROACH

5.1 Force/moment equilibrium equation

In Figure 3, all vectors are represented with respect to the object coordinate system $\{C_b\}$ and a point contact with friction model is assumed at the contact point. Denote F_i and $\tilde{T} = [F_e^t, M_e^t]^t \in R^{6\times1}$ as the force vectors applied to the object at i-th contact point by each finger and the resultant force and moment vectors, respectively. Also denote p_i and n as the position vector from the origin of the object coordinate system to i-th contact point and the number of contact points, respectively. Then, the force/moment equilibrium equation can be written as follows.

$$F_e = \sum_{i=1}^{n} F_i \quad (15\text{-}1)$$

and

$$M_e = \sum_{i=1}^{n} p_i \times F_i \quad (15\text{-}2)$$

equation (15) can be written in the matrix form as

$$\tilde{T} = GF, \quad (16)$$

where $G \in R^{6\times3n}$ is defined by

$$G \equiv \begin{pmatrix} I_3 & I_3 & \cdots & I_3 \\ P_1 & P_2 & \cdots & P_n \end{pmatrix}. \quad (17)$$

Here I_3's are 3×3 unit matrices and P_i are the 3×3 skew symmetric matrices with zero diagonal elements equivalent to the vector product of position vectors $p_i = (p_{ix}, p_{iy}, p_{iz})^t$ shown as

$$P_i \equiv \begin{pmatrix} 0 & -p_{iz} & p_{iy} \\ p_{iz} & 0 & -p_{ix} \\ -p_{iy} & p_{ix} & 0 \end{pmatrix}. \quad (18)$$

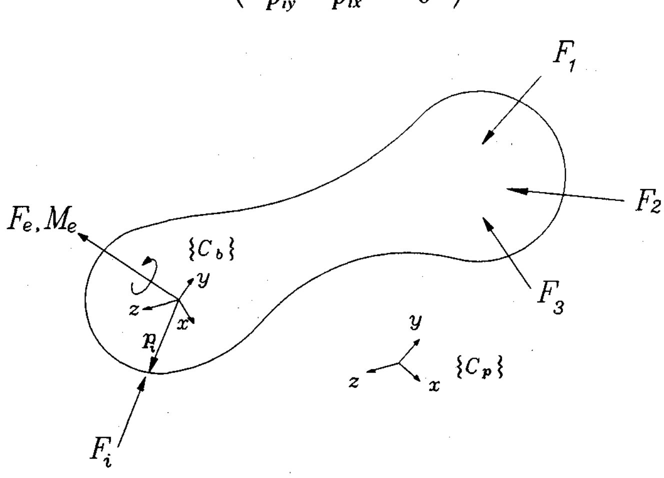

Fig. 3. Modelling of a manipulation of an object by a multifingered hand.

It is noted that P_i are time dependent as the contact points evolve and the dynamic equilibrium can be also maintained if the inertia force is included in $\tilde{T}$ in equation (16).

5.2 Forces transmitted at a point of contact

The resultant force transmitted from one surface to another through a point of contact is resolved into a normal force F_n acting along the common normal, which generally must be compressive, and a tangential force F_t in the tangent plane sustained by friction. The magnitude of F_t must be less than or equal to the force of static friction, i.e.

$$F_t \leq \mu_s F_n, \tag{19}$$

where μ_s is the coefficient of static friction.

After sliding occurs, a condition of kinetic friction accompanies the ensuing motion. Kinetic friction force is usually somewhat less than the maximum static friction force. We find also that the kinetic force is proportional to the normal force. Hence, the magnitude of F_t may be put in the form when sliding occurs

$$F_t = \mu_k F_n, \tag{20}$$

where μ_k is the coefficient of kinetic friction.

In this paper, a strategy was devised to allow sliding as well rolling contacts during whole manipulation time by adaptively changing the coefficient of friction according to the contact type. Initially, the friction constraint is set as equation (19). If rolling occurs at current time step, which implies F_t is less than $\mu_s F_n$, equation (19) will be continuously used for next time step. As the magnitude of F_t increases up to the magnitude of $\mu_s F_n$, however, then sliding impends, μ_s in equation (19) is changed to μ_k as

$$F_t \leq \mu_k F_n \tag{21}$$

for next time step. While using equation (21) as kinetic friction constraint, if F_t becomes less than $\mu_k F_n$, then the contact type is returned to rolling and equation (21) is replaced by equation (19). If F_t equals $\mu_k F_n$, kinetic friction constraint of equation (21) is continuously used for next time step.

5.3 Contact maintenance condition

If the contact between the surfaces of the fingertip and the object is continuous, their velocity component along the common normal must be equal such that the surfaces are neither separating nor overlapping. Thus, v_z^i which can be represented in terms of the finger Jacobian, the joint velocity, and contact parameters always equals zero as follows:

$$[A^t_{bi,b} v_{b,p} - A^t_{bi,b} S(r_{bi,b}) \omega_{b,p}]_z - [A_{\psi_i} A^t_{li,fi} J_{iu} \dot{q}_i - A_{\psi_i} A^t_{li,fi} S(r_{li,fi}) J_{il} \dot{q}_i]_z = 0, \tag{22}$$

where subscript z implies z-component.

5.4 Consistency of slide/roll mode between contact force and relative motion

When an object is manipulated by a multifingered hand, either rolling or sliding at the contact point may be resulted from the contact force. To generate the corresponding relative motion for the contact force applied at the contact point, the following mode parameter is defined:

$$\delta = \mu F_n - F_t \tag{23}$$

While the contact force results in rolling motion if δ is greater than zero, the contact force generates sliding motion if δ is equals to zero. Denote v_t as the translational relative velocity. Then, the consistency of the contact force and relative motion at the contact point is accomplished by satisfying the following compatibility equation:

$$\delta \cdot v_t = 0 \tag{24}$$

Thus, while v_t should be zero to imply rolling motion if δ is greater than zero, v_t has any nonzero finite magnitude to obtain sliding motion if δ is zero. When both quantities are zero at the same time, the sliding motion stops.

5.5 The direction of tangential force and sliding velocity

The tangential force of friction is constrained to be no greater than the product of the normal force with the coefficient of friction. In a purely sliding contact the friction force reaches its limiting value in a direction opposed to the sliding velocity. In this paper, the sliding velocity is defined as the translational relative velocity of the object with respect to the fingertip at the contact point. Thus, the direction of the friction force and sliding velocity should be coincident. Denote $||*||$ as the Euclidean norm. Then, the directional constraint of friction force and sliding velocity is described as follows:

$$\frac{F_t}{||F_t||} = \frac{v_t}{||v_t||} \tag{25}$$

5.6 Nonlinear optimization problem formulation

Now for given contact parameters, the motion planning problems can be formulated into a nonlinear optimization problem to find the minimum joint velocity and the minimum contact force at each time step satisfying the constraints derived in subsection 5.1 thru 5.5 and some physical constraints.

Minimize

$$||\dot{q}_i|| + ||F_i|| \tag{26}$$

Subject to

$$GF = \tilde{T} \tag{27}$$

$$F_t \leq \mu F_n \tag{28}$$

$$v_z^i = 0 \tag{29}$$

$$q_{i\,\min} \leq q_i \leq q_{i\,\max} \tag{30}$$

$$\dot{q}_{i\,\min} \leq \dot{q}_i \leq \dot{q}_{i\,\max} \tag{31}$$

$$\ddot{q}_{i\,\min} \leq \ddot{q}_i \leq \ddot{q}_{i\,\max} \tag{32}$$

$$F_{i\,\min} \leq F_i \leq F_{i\,\max} \tag{33}$$

$$\delta \cdot v_t = 0 \tag{34}$$

$$\frac{F_t}{||F_t||} = \frac{v_t}{||v_t||} \tag{35}$$

The summation of the Euclidean norm of joint velocity and the Euclidean norm of contact force are chosen as a cost function in equation (26). Equation (27) is the force/moment equilibrium equation. It is remarked that the risk of losing contact stability due to the sliding contact does not happen since the object should satisfy the force/moment equilibrium equation to ensure the force closure condition. Equations (28) and (35) are the Coulomb friction constraints. In equation (28), μ may be μ_s or μ_k according to the contact type. When pure rolling occurs, equation (35) becomes inactive and the friction force is determined by the equations of equilibrium. Equation (29) is the contact maintenance condition. Equations (30), (31) and (32) are the joint angle, velocity and acceleration constraints, respectively. Equation (32) is the constraint on the magnitude of the contact force. Equation (34) is the mode compatibility condition of slide or roll between contact force and relative velocity at the contact point. Thus, for given contact parameters, the minimum joint velocity and the minimum contact force of the fingers are obtained by solving the nonlinear optimization problem.

The procedure to find the joint velocity and the contact force at each time step can be summarized as follows:

[Step 0] Read the joint configurations and the contact parameters at the current time step with the informations of the object.

[Step 1] Calculate the object velocity at the contact point by equation (7).

[Step 2] Determine the minimum joint velocity and the minimum contact force of the fingers by solving the nonlinear optimization problem given in equations (26) thru (35).

[Step 3] Calculate the relative velocity at the contact point by equations (10-1) and (10-2).

[Step 4] Determine the time derivatives of the surface variables and contact angle at the current time step by the contact equations given in equations (11) thru (13).

[Step 5] Update the contact parameters and the joint configurations of the fingers.

[Step 6] Go to Step 1.

6. SIMULATION RESULTS

A re-orienting task of a sphere is considered to show the validities of our proposed algorithm for a robotic hand with three fingers each of which has four joints (Figure 4), where the z-axes define the joints' axes of rotation. The specifications of the hand and the object are given in Table I. The coefficients of static and kinetic friction are also given in Table I by assuming both contacting surfaces are unlubricated steel.[12] In Table II, the initial joint configurations of the fingers are given. The surface variables and the contact angles for initial contact points of the fingertip and the object are given in Table III. The orientation of reference and body coordinate frames are initially chosen to be coincident. A rotational motion

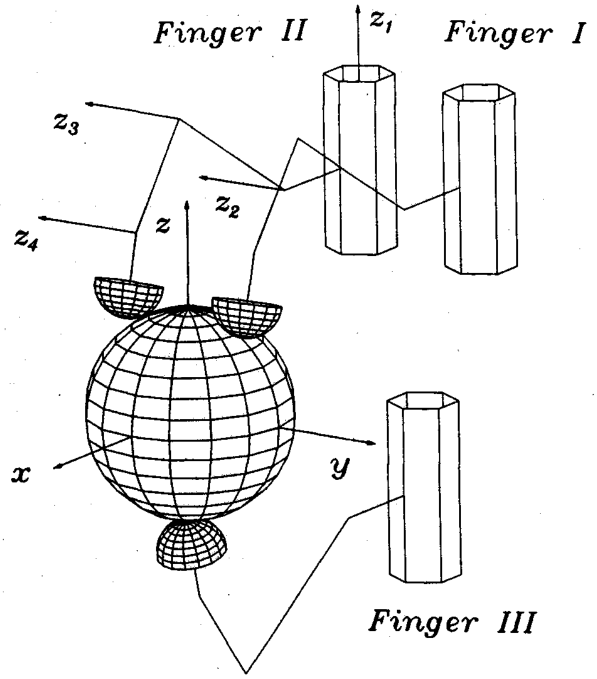

Fig. 4. A re-orienting of a sphere by a three-fingered hand.

about x-axis of reference frame with a velocity profile as shown in Figure 5 is given to the object.

The solution of the problem is obtained by utilizing a general purpose nonlinear optimization solver Integrated Design Optimization Library (IDOL) Ver. 1.5 developed at Applied Mechanics & Optimal Design Laboratory in Hanyang University and has been implemented on an IBM RISC/600 320H. In IDOL based on the Augmented Lagrange Multiplier Method,[13] several schemes are devised for computational enhancements of the ALM method in the sense of selecting good initial guesses for design variables and Lagrange multipliers with scalings of constraints, restartings of descent vectors, and dynamic stopping criterions. Specifically, descent vectors are determined by using the Broydon–Fletcher–Goldfarb–Shanno (BFGS) method.[13] For line search, the incremental search method is firstly used to find bounds on the solution, then the bounds are refined by the golden section method, and finally a cubic polynomial approximation technique is applied to interpolate for the solution using the last four function values.

Figures 6 thru 8 show the contact forces of each finger with respect to $\{C_b\}$ to generate the desired object motion. It is firstly observed that while the z-components of the contact forces of Finger I and Finger II have negative values, those of Finger III are always positive to sustain the weight of the object and the x-components of contact forces for all three fingers are approximately zero, since the object is manipulated only in the yz-plane. It is also observed that the contact forces are gradually decreased after $t = 0.80$ sec. due to the inertia moment.

Figures 9 thru 11 show the joint velocities of each finger. Finger I and Finger II move with small joint velocities. Define R_{sr} as the slide/roll ratio at the contact

Table I Specifications of Multifingered Hand and Object

No. of Fingers		3	
No. of Joints/Finger		4	
Link Length of Each finger [m]			
link 1: 0.028	link 2: 0.062	link 3: 0.036	link 4: 0.014
Geometry of Fingertip		Hemisphere	
Radius of Fingertip [m]		0.01	
Coefficient of Static friction		0.6	
Coefficient of Kinetic friction		0.4	
Geometry of Object		Sphere	
Radius of Sphere [m]		0.03	
Mass of Sphere [kg]		0.2	

Table II Initial Joint Configurations of Fingers [rad]

	Joint 1	Joint 2	Joint 3	Joint 4
Finger I	0	0.559	−1.431	−0.611
Finger II	0	0.559	−1.431	−0.611
Finger III	0	−0.559	1.431	0.611

Table III Initial Surface Variables and Contact Angles of Fingertips and Object

	Fingertip		Sphere		Contact Angle ψ
	α[rad]	β [rad]	η [rad]	ξ [rad]	
Contact Point 1	1.992	0.991	1.571	−1.047	0
Contact Point 2	−1.992	0.991	−1.571	−1.047	0
Contact Point 3	0	1.309	0	1.571	1.571

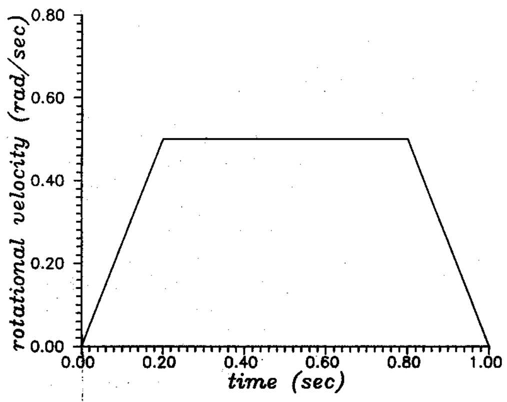

Fig. 5. A desired velocity profile about x-axis of $\{C_p\}$.

point given by

$$R_{sr} = \frac{\|v_t\|}{(\|\tilde{v}_{bi,p}\| + \|\tilde{v}_{li,p}\|)/2}, \tag{36}$$

where $\tilde{v}_{bi,p}$ and $\tilde{v}_{bi,p}$ imply the translational velcoity components in the common tangent plane. If R_{sr} is zero, the contact motion shows pure rolling. But if R_{sr} is positive, the contact motion includes sliding. Figure 12 shows slide/roll ratios for each finger. It is observed from Figure 12 that sliding contacts are not simultaneously generated at all three contact points, which may be necessary to maintain the grasp stability. Specifically, Finger III maintains nearly rolling contacts over the whole manipulation time, while Finger I and II frequently use sliding contacts. Since almost all contacts of Finger III rely on rolling contacts, the joint velocities of Finger III are rather similar with the desired profile of object velocity. At those instances when local peak values of R_{sr} are shown, the joint velocities of fingers also show relatively large values, which seems to be considered as the stick-slip phenomenon. The stick-slip phenomenon prevents smooth movements and gives rise to erratic jerky behavior. Under stick-slip conditions, the fingertip will remain stationary until the contact force can overcome the static friction resistance. The fingertip will then accelerate momentarily but will rapidly slow down and become stationary again. The changes from stick to slip and back again to stick are associated with variations in the friction forces, which prevail under the low speed conditions.[14]

The contact point evolutions on the object surface with respect to $\{C_b\}$ are illustrated in Figs. 13 thru 15. The x-coordinates of contact points for all three fingers are located around zero, because the object motion is executed in yz-plane. The y-coordinates of contact points for Finger I and Finger II move to the positive y-direction, which agrees to the intuition of human. The z-coordinates of contact points of Finger I move downward, while those of Finger II upward, which also

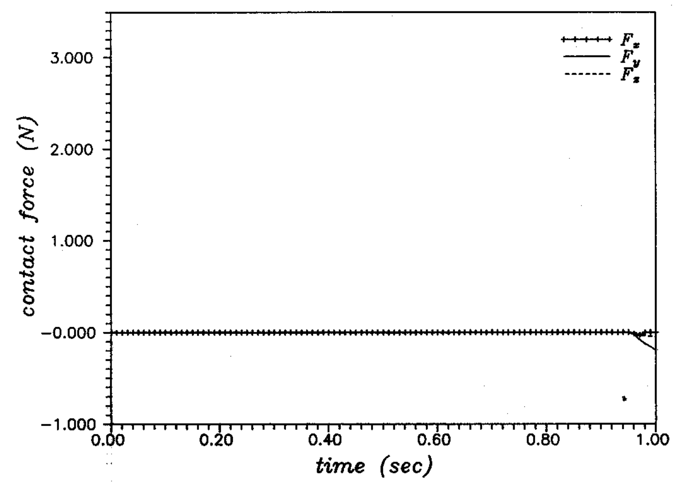

Fig. 6. The contact force of Finger I.

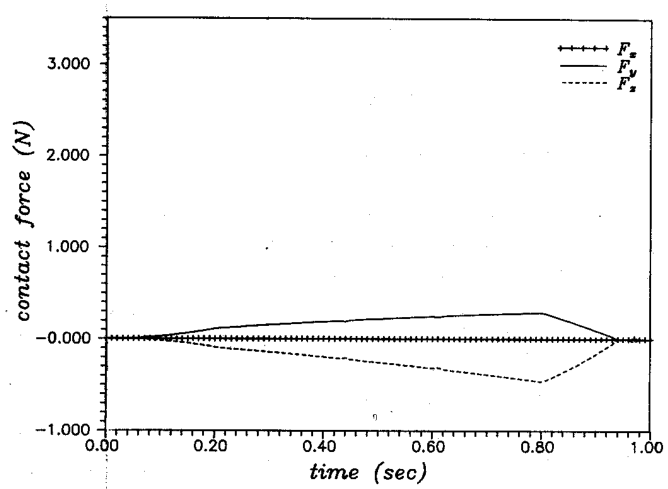

Fig. 7. The contact force of Finger II.

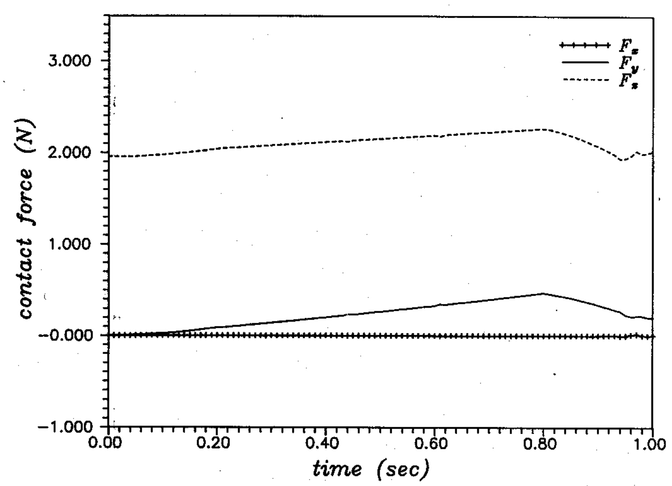

Fig. 8. The contact force of Finger III.

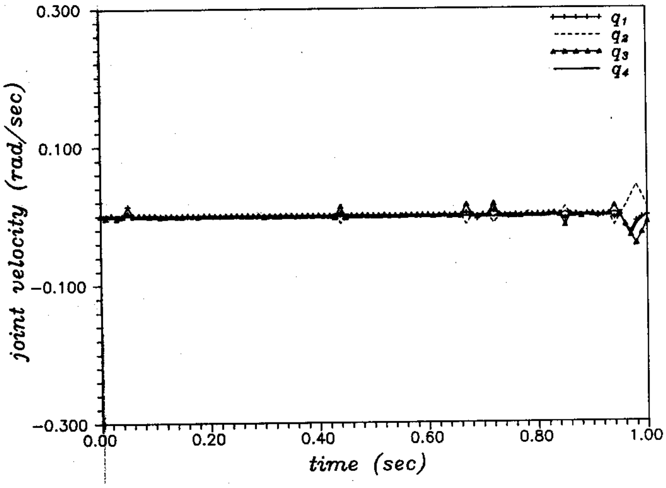

Fig. 9. The joint velocity of Finger I.

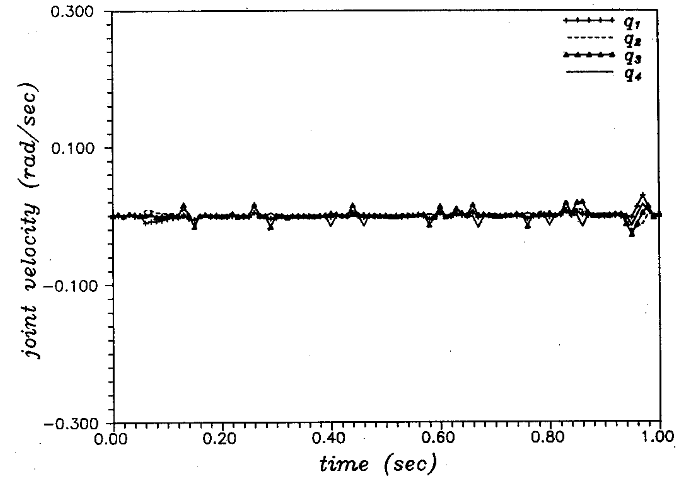

Fig. 10. The joint velocity of Finger II.

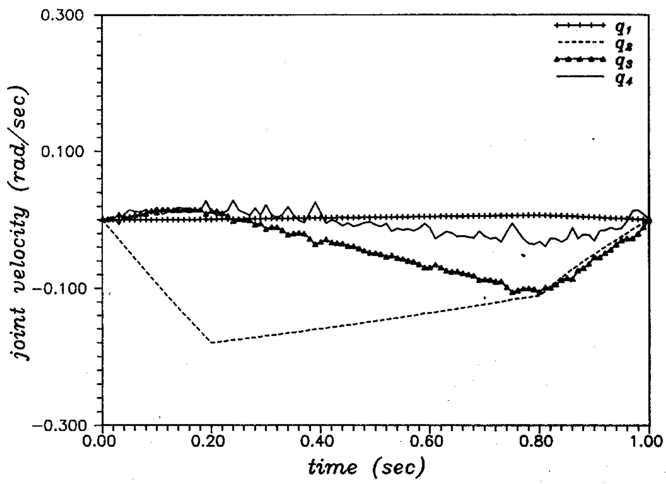

Fig. 11. The joint velocity of Finger III.

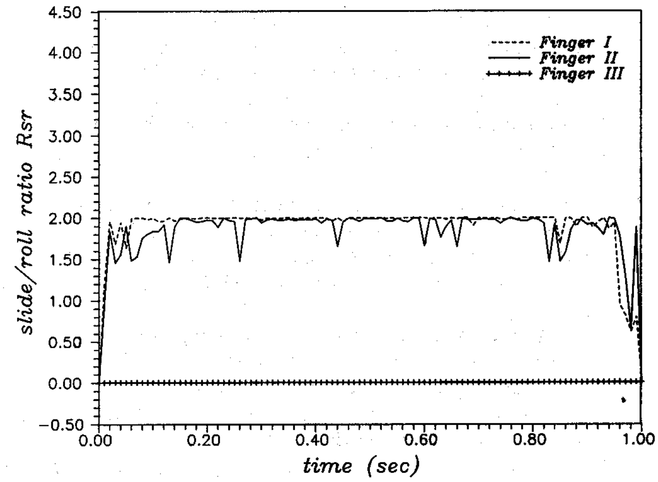

Fig. 12. Slide/roll ratios for each finger.

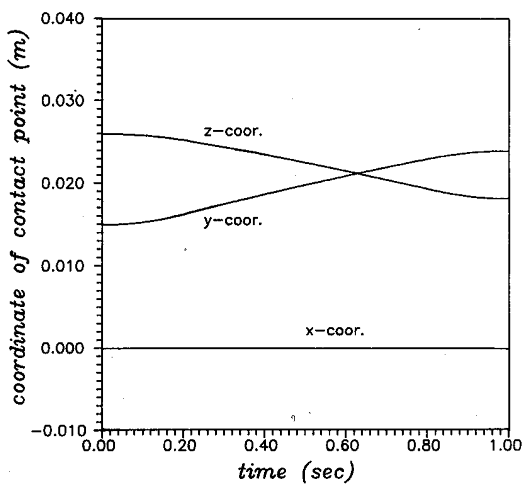

Fig. 13. Evolution of contact point of Finger I.

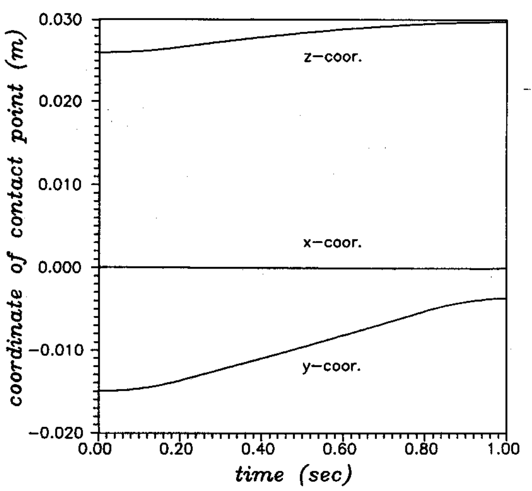

Fig. 14. Evolution of contact point of Finger II.

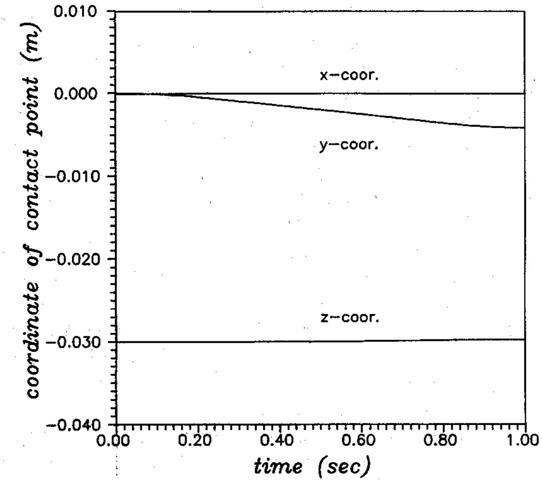

Fig. 15. Evolution of contact point of Finger III.

agrees to the intuition of human. The evolution rates are much similar with the desired profile of object velocity. In Figure 15, the z-coordinate of contact point of Finger III shows rather sluggish rising than the human intuition, which may be due to the numerical difficulty mentioned in Section 4. The contact points may evolve more appropriately for a given object motion than pure rolling contacts by using the extra *d.o.f.*'s generated from sliding contacts. This may be useful in re-positioning contact points on the object when good initial contact points are not available. The manipulation with pure rolling contacts will be straightforward by choosing the performance index as $||v_r||$.

7. CONCLUSION

A motion planning algorithm for multifingered robotic hands manipulating an object of arbitrary shape was presented. In this study, the general contact motions including sliding motions were considered to find the trajectory of fingertips' contact points over the object surface. The contact forces and the joint velocities to generate a desired object motion were found by utilizing a nonlinear optimization technique. A simulations was presented by employing a three-fingered robotic hand re-orienting a sphere.

In this paper, rigid-body models and frictional point contacts were assumed between the fingertips and the grasped object. Also the idealized Coulomb friction model was employed and the effect of finger dynamics was ignored for a quasistatic analysis. As our future research works, more sophisticated models will be considered by relaxing assumptions on friction and rigid-body models and the effect of finger dynamics will also be investigated.

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