

# Robust Time Optimal Controller Design for Hard Disk Drives

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**Abstract**— Robust time optimal control algorithm based on internal loop compensator is proposed. Internal loop compensator effectively isolates hard disk system from the uncertain disturbances including modeling error, external disturbance, shock, and control torque saturation. We use the  $\mathcal{H}_\infty$  mixed sensitivity method for the optimization of internal loop compensator and proximate time optimal controller as a feedback controller to provide a time optimal property. The performance of the proposed controller is verified via simulation works.

**Index Terms**— Hard disk drive, internal loop compensator,  $\mathcal{H}_\infty$  mixed sensitivity method, time optimal control.

## I. INTRODUCTION

**H**ARD disk drive (HDD) servo control has two main purposes. The first is to move the head to the target track for reading or writing data as quickly as possible and the other is to precisely keep the read-write head on the desired track. The overall control algorithm requires minimum time control during track seeking and minimum variance control during track following. This paper focuses on the disturbance estimation and attenuation in order to guarantee these control specifications. Various methods for this purpose have been proposed. Disturbance observer based controllers [1]-[3] which make the behavior of the real system as a nominal system are good examples, but these methods are not so generic, nor have a way to provide optimal concept in the controller.

The objectives of this paper are to design a robust compensator for disturbance attenuation and to provide optimization concept in this compensator design procedure. The controller consists of two independent control loop, namely, internal and external loop. The internal loop is used as a compensator rather than a controller, referred to as *internal loop compensator* and the external loop is used as a controller, referred to as *external loop controller* in this paper. The internal loop compensator is for the rejection of uncertain disturbances and proximate time optimal servomechanism (PTOS) [4] is integrated as an external loop controller after the internal loop design.

In the next section, we propose a robust internal loop compensator and optimize it using  $\mathcal{H}_\infty$  mixed sensitivity method. In section III, we deal with the design of robust time optimal controller based on internal loop compensator. The simulation results of the proposed controller

are presented and finally we draw conclusions.

## II. OPTIMAL INTERNAL LOOP COMPENSATOR DESIGN

We consider the internal loop compensator for a single-input, single-output system which can be described as follows:

$$\text{Plant } P : \begin{aligned} \dot{x} &= Ax + bu \\ y &= Cx \end{aligned} \quad (1)$$

$$\text{Model } P_m : \begin{aligned} \dot{x}_m &= A_m x_m + b_m u_m \\ y_m &= C_m x_m \end{aligned} \quad (2)$$

$$\text{Control Input } u : \quad u = u_m + Ky_m - Ky \quad (3)$$

where  $x \in \mathbb{R}^n$  is the plant state,  $u$  is the plant input,  $y$  is the plant output,  $x_m \in \mathbb{R}^n$  is the model state,  $u_m$  is the reference input,  $y_m$  is the model output,  $A, b, C, A_m, b_m, C_m$  are matrices of appropriate dimension, and  $K$  is the compensator gain. If we assume that the plant and the model are stabilizable and the model has piecewise continuous and uniformly bounded reference input  $u_m$  and output  $y_m$ , then the closed loop system of (1)-(3) approaches the model  $P_m$  as  $K \rightarrow \infty$  [5].

### A. Optimal Compensator Design in the $\mathcal{H}_\infty$ Framework

We design a robust internal loop compensator (RIC) considering the frequency characteristics of the transfer function from  $u_m$  to  $y$ . In Fig. 1, if there exists external disturbance and measurement noise, the control input can be written as follows:

$$u(s) = \{1 + K(s)P_m(s)\} u_m(s) - K(s) \{y(s) + \xi(s)\} + d(s), \quad (4)$$

where  $\xi(s)$  is a measurement noise and  $d(s)$  is a disturbance which comes through the input channel.

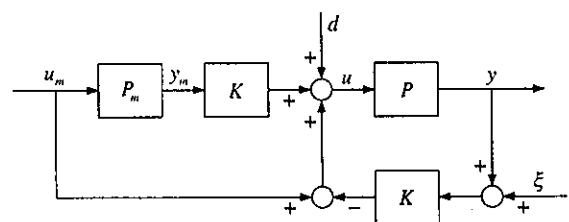
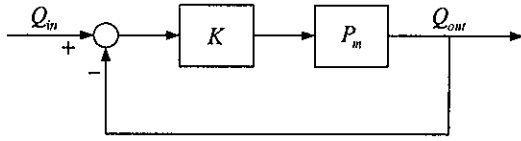


Fig. 1. Robust Internal Loop Compensator Structure

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Fig. 2. Transfer Function  $Q$ 

Now, let's define the transfer function  $Q$  which controls  $P_m$  using feedback controller  $K$  as shown in Fig. 2.  $Q$  is described as

$$Q = \frac{P_m K}{1 + P_m K} \quad (5)$$

If we recalculate this equation in terms of  $K$ , and plug this  $K$  into Fig. 1, we obtain Fig. 3 which is a well known structure of disturbance observer [6]. Therefore, the optimal compensator design problem becomes the optimization problem of  $K$  for the reference model  $P_m$ .

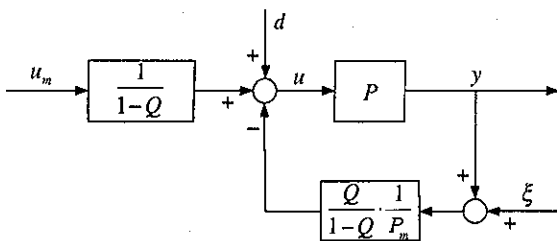
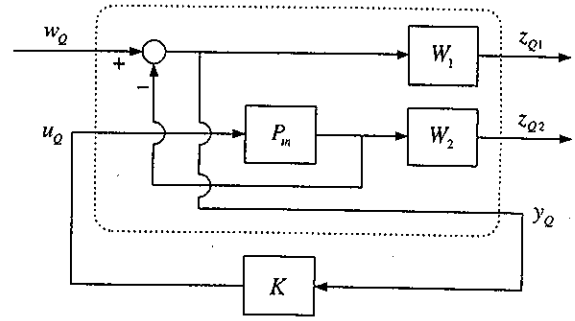
The design of a low-pass filter  $Q$  in the conventional disturbance observer structure is very heuristic because in the disturbance observer, it is very difficult to apply an optimal concept in the design of  $Q$ . However, in the RIC structure, it is very simple. As mentioned previously, the RIC is determined by the reference model  $P_m$  and the feedback controller  $K$ , and simultaneously can be described by the low-pass filter  $Q$ . From Fig. 2, the sensitivity  $S_Q$  and complementary sensitivity  $T_Q$  of  $Q$  are obtained by

$$S_Q = \frac{1}{1 + P_m K}, \quad T_Q = \frac{P_m K}{1 + P_m K} \quad (6)$$

We use the  $H_\infty$  mixed sensitivity method to determine the optimal compensator gain  $K$  as shown in Fig. 4. This is another advantage of the RIC structure since the structure accepts the optimal concept in designing  $Q$  or  $K$ . In the mixed sensitivity problem formulation, nominal disturbance attenuation specifications and stability margin specifications are combined into a single infinity norm specification. The mixed  $H_\infty$  sensitivity problem is formulated as follows:

$$\min_K \left\| \begin{bmatrix} W_1(1 + P_m K)^{-1} \\ W_2 P_m K(1 + P_m K)^{-1} \end{bmatrix} \right\|_\infty < 1. \quad (7)$$

Since (7) uses reference model parameters, this equation can be solved easily and the optimal frequency characteristics can be assigned to the  $Q$ .

Fig. 3. Equivalent Controller Structure of RIC using  $Q$ Fig. 4. Optimization of RIC by  $H_\infty$  Mixed Sensitivity Method

### B. RIC Design under Actuator Saturation.

In the case of the model based control algorithm, the mismatch between model and plant leads to instability under the saturation constraints. If the controller is implemented in the RIC framework, this problem can be solved as shown in Fig. 5. In this figure, the saturation block in front of the reference model would cause the output of the model to behave similarly like the output of the real plant when the input reaches saturation values. Therefore, RIC can handle the saturation problem and guarantee the predicted robust performance of the control system also. On the other hand, the disturbance observer structure has limitations in this case. A remediable algorithm such as self-adjusting saturation method was proposed [7], but this algorithm also cannot be a complete solution since the low-pass filter  $Q$  has a fixed shape.

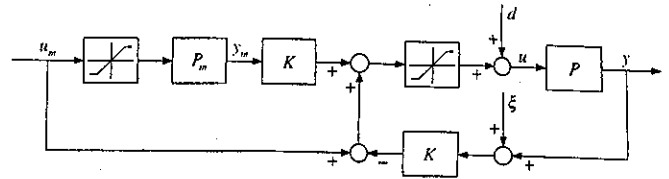


Fig. 5. RIC Implementation in the Presence of Actuator Saturation

## III. ROBUST TIME OPTIMAL CONTROLLER DESIGN

The PTOS consists of two controllers, one is the velocity controller based on a proximate time optimal curve, and the other is a linear controller that operates within a given linear control region. The size of the linear control region has to be chosen to satisfy the continuity of the control input. The PTOS control law is described as follows:

$$u = \bar{u} \cdot \text{sat} \left( \frac{k_2 \{f(y_e) - \dot{y}\}}{\bar{u}} \right), \quad (8)$$

$$f(y_e) = \begin{cases} \frac{k_1}{k_2} \cdot y_e & |y_e| \leq y_l \\ \text{sgn}(y_e) \left( \sqrt{\frac{2\bar{u}k_v\alpha}{k_v}} |y_e| - \frac{\bar{u}}{k_2} \right) & |y_e| > y_l \end{cases}, \quad (9)$$

where  $y_e$  is the tracking error and  $\bar{u}$  is the allowable maximum control input. Constraints of  $y_l (= \bar{u}/k_1)$  and  $\alpha (= 2k_1k_y/k_vk_2^2)$  are used to facilitate a jerk-free transition during the motion.

Since the PTOS is derived from the double integrator system, when the plant cannot be modeled by the double integrator, it cannot be applied directly to the real system. Moreover the plant usually has modeling uncertainty, parameter variation, system resonance, etc., then it is very difficult to apply PTOS to the HDD system, especially for uncertain environments.

RIC estimates and compensates modeling uncertainty as well as parameter variation, internal and external disturbance. Practically, RIC generates the output signal to eliminate the above mentioned disturbance effects, and this signal is fed back to the actuator along with the control input calculated by the PTOS as shown in Fig. 6. We designed the RIC to make the whole system behave as a reference model of a double integrator system after compensating uncertain nonlinear forces. PTOS as a controller stabilizes the whole system and makes the system have a time optimal property. Although the control input due to the PTOS alone is reached to the maximum, we can handle the control input saturation using a saturation avoidance algorithm considered in the previous section.

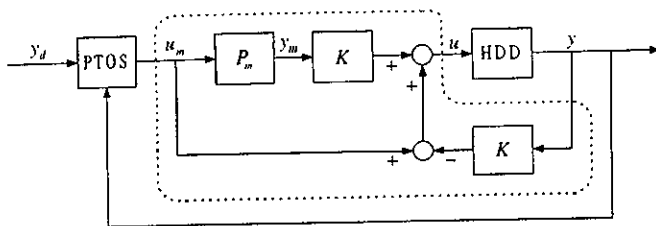


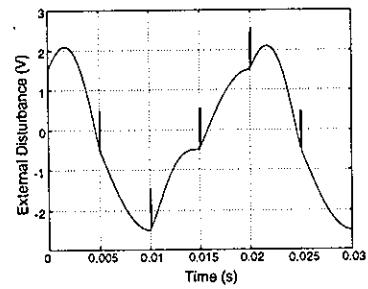
Fig. 6. PTOS Based on RIC

#### IV. NUMERICAL SIMULATION

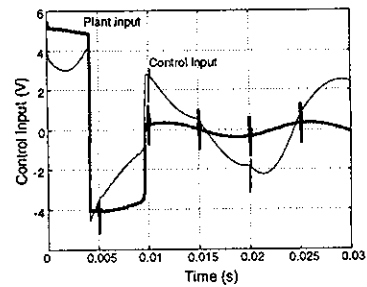
A common HDD model with two flexible modes was used. Fig. 7 shows the simulation results for the input saturation and the external disturbance. The target track was 2000 tracks and the saturation voltage was set to 5.0 volt. The external disturbances are shown in Fig. 7(a) with sinusoids plus series of impulses. As can be seen here, the tracking error shows a good performance even for the large disturbances of the system and control input saturation.

#### V. CONCLUSIONS

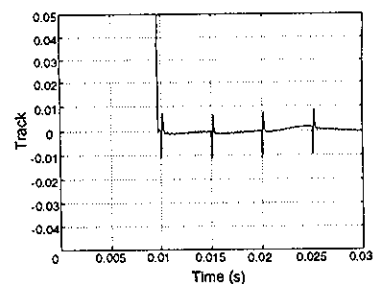
In this paper, we propose the controller which can handle the repetitive and non-repetitive disturbances and guarantee the time optimal property. The robust internal loop compensator makes the plant behave like the reference model by canceling out the disturbance terms and its time response is fast enough to meet the specification due to the proximate time optimal controller.  $H_\infty$  mixed sensitivity problem is used to optimize the compensator and the saturation avoidance algorithm provides optimal control input under the saturation limitation. Through the numerical simulation, the performance of the proposed



(a) Disturbances



(b) Control Input and Plant Input



(c) Position Error

Fig. 7. Simulation Results of Robust Time Optimal Controller

controller was evaluated and the results show that suggested controller achieves the specified performances.

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