Disturbance observer based path tracking control of robot manipulator considering torque saturation

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Abstract

This paper proposes a path-tracking algorithm to compensate for path deviation due to torque limits. The algorithm uses a disturbance observer with an additional saturation element at each joint of \( n \) degrees of freedom (DOF) manipulator to obtain a simple equivalent robot dynamic (SERD) model. This model is represented as an \( n \) independent double integrator system and is designed to ensure stability under input saturation. For an arbitrary trajectory generated for a given path in Cartesian space whenever any of the actuators is saturated, the desired acceleration of the nominal trajectory in Cartesian space is modified on-line by using SERD. An integral action with respect to the difference between the nominal and modified trajectories is utilized in the nonsaturated region of actuators to reduce the path error. To verify the effectiveness of the proposed algorithms, real experiments were performed for a two DOF SCARA-type direct-drive arm. 

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1. Introduction

For many industrial applications, rapid motion at a constant speed along straight lines or other paths described in Cartesian space, can be considered as one of the most important performance requirements of a robot manipulator. To perform such path-tracking tasks, a trajectory should be generated in Cartesian space based on kinematic relationships. Since the possible acceleration in Cartesian space is dependent on the configuration of a robot manipulator, actuator torques may be saturated for an arbitrarily generated trajectory. Under conventional feedback control, the manipulator may deviate from the desired path as a result of the torque limits.

Several off-line time-optimal trajectory planning algorithms have been proposed to follow a specified path considering the torque limits of actuators [1,2]. When the time-optimal trajectory of a manipulator along a geometrically prescribed path is planned taking into account the manipulator dynamics and the torque limits of its actuators, at least one of the joints should be at the torque limit. Trajectory tracking control is usually executed based on the position feedback while following a target point on the desired path. Minimum time trajectory tracking by such a control scheme results in torque saturation. Consequently, the control has no margin to suppress the tracking error and the manipulator may deviate from the path [3]. In addition, time-optimal trajectory planning requires exact manipulator dynamic model. Unfortunately, perfect dynamics cannot be obtained. Moreover, such a modeling error may also cause the manipulator to produce a fairly large path error.

To resolve these problems, on-line path-following algorithms have been suggested [3,7]. Arai and his coworkers [3] defined path coordinate based on the desired path and independently controlled the components normal to the path and the components along the path. This method shows a good tracking performance as long as the joint torques do not exceed a limit values. However, since this path-tracking control algorithm is based on the dynamic model of a robot manipulator, unmodeled dynamics and uncertainties may cause undesirable behavior.

Recently, disturbance observer based control algorithms have been reported that compensate for modeling uncertainties as well as for external disturbances [5,6]. The disturbance observer regards the difference between the actual output and the output of nominal model as an equivalent disturbance applied to the nominal model. It estimates the equivalent disturbance and the estimate is used as a cancellation signal. Therefore, when a disturbance observer is applied to each joint of a manipulator, the dynamics of each joint of robot manipulator can be considered as a simple inertia system that can be obtained without the complex computation of dynamic equations. To the best of our knowledge, such a disturbance observer has not previously been applied to the path-tracking control of a manipulator with torque limits.

Ohnishi and his coworkers proposed a robust control method that considered the saturation of controller output as well as torque [4]. In this scheme, an $H^\infty$ controller was applied to each joint to suppress any equivalent disturbance, and
thus the calculation of robot manipulator dynamics was not required. However, using the difference between the control input and torque limit value scaled down the acceleration command in joint space linearly, so it is not guaranteed to effectively compensate path deviation due to torque limits for any configuration of the manipulators.

In this paper, a disturbance observer based path-tracking algorithm is proposed for robot manipulators with torque limits. Specifically, a disturbance observer with an additional saturation element is applied to each joint of $n$ DOF manipulator to obtain a simple equivalent robot dynamics (SERD), represented as an $n$ independent double integrator system, to ensure stability under input saturation. Then, an arbitrary trajectory is generated for a given path in Cartesian space. The trajectory can be obtained by an off-line time-optimal method [2] or by using a simple trapezoidal velocity profile as in practice. Whenever torque saturation is met, the desired acceleration of the nominal trajectory in Cartesian space is modified in an on-line fashion by using the SERD. In addition, an integral action with respect to the difference between the nominal and modified trajectories is used in the nonsaturated region of the actuators to reduce the path error. The disturbance observer cannot function correctly if the magnitude of the disturbance signal is greater than the physical torque limit. Therefore, we determine the maximally admissible torque value of each joint of the actuator by using SERD to guarantee that the disturbance observer correctly eliminates the equivalent disturbance signal. Since the maximally admissible torque value necessary for calculating the maximum acceleration in Cartesian space is obtained by using SERD, the proposed algorithm is expected to be less sensitive to modeling uncertainties than methods employing complex dynamics [7,8].

To verify the effectiveness of the proposed algorithm, real experiments were performed for a two DOF SCARA type direct-drive arm shown in Fig. 1.

2. Robot dynamics with a disturbance observer

Consider the dynamics of an $n$ link robot manipulator given by a set of highly nonlinear, coupled differential equations as

$$
M(q)\ddot{q} + c(q, \dot{q}) + g(q) + f(\dot{q}) = \tau,
$$

where $M(q)$ is the $n \times n$ inertia matrix and $c(q, \dot{q}), g(q), f(\dot{q})$ are $n \times 1$ vectors of Coriolis and centrifugal forces, gravity loading, and friction force, respectively. Here, $\tau = [\tau_1 \cdots \tau_n]^T$ is the $n \times 1$ torque vector applied to the joints of the robot manipulator, while $q, \dot{q}$ and $\ddot{q}$ are $n \times 1$ vectors representing the angular position, velocity and acceleration, respectively. Now, the robot dynamics in Eq. (1) can be rewritten as a fixed inertia term plus an equivalent disturbance torque given by

$$
\hat{M} \ddot{q} + \tau_d(q, \dot{q}, \ddot{q}) = \tau,
$$

where $\hat{M} = \text{diag}(\hat{M}_{11} \cdots \hat{M}_{nn})$ is an $n \times n$ diagonal matrix. Here, $\hat{M}_{ii}$ is the
constant-valued nominal $i$th axis inertia term, which can be measured approximately using the frequency response. Specifically, a frequency response for the $i$th axis can be obtained by locking all the other actuators except the $i$th actuator. Then, by assuming that the dynamics of the $i$th axis can be treated as $\tau_i = M_i \ddot{q}_i$, $M_i$ can be measured experimentally by using the frequency response plot for torque input and velocity output. Fig. 2 shows a frequency response plot for the selected configuration, where the solid line indicates the experimental frequency response and the dashed line shows the result of curve fitting. From this figure, $M_{22}$ can be determined as 0.1 kg m$^2$. In Eq. (2), $\tau_d = [\tau_{d1} \cdots \tau_{dn}]^T$ is the $n \times 1$ vector implying an equivalent disturbance including all the remaining dynamic terms and unmodeled dynamics, such as nonlinearity, coupling effects and payload uncertainty. The disturbance of the $i$th axis can be represented as

$$\tau_{id} = \sum_{j=1, j \neq i}^{n} M_{ij}(q)\ddot{q}_j + \sum_{j=1}^{n} \sum_{k=1}^{n} c_{ijk} \dot{q}_j \dot{q}_k + g_i + f_i + \left( M_{ii}(q) - \dot{M}_{ii} \right) \ddot{q}_i. \quad (3)$$

If the equivalent disturbance in Eq. (3) can be obtained, the dynamics of each axis can be decoupled by eliminating the equivalent disturbance. Thus, a simple control strategy is sufficient to track a desired trajectory $q_d(t)$. The equivalent disturbance can be estimated by a disturbance observer [5,9] and can be suppressed by adding the estimated disturbance signal to the control input. Fig. 3 shows the structure of the disturbance observer for the $i$th single axis, based on

Fig. 1. Two DOF SCARA type direct-drive arm.
inverse model of a nominal plant. In this figure, \( P_{in}(s) \) is the nominal plant of the real system \( P_i(s) \) where \( P_{in}(s) \) is given as \( 1/\dot{M}_{is} \), and \( Q_i(s) \) is a low pass filter that is employed to realize \( P_{in}^{-1}(s) \).

From the block diagram in Fig. 3, the following input–output relation is obtained:

\[
y_i = G_{uiy}(s)u_i + G_{r_iy}(s)r_{id},
\]

(4)
where

$$G_{u,y}(s) = \frac{P_n(s)P_m(s)}{P_m(s) + (P_i(s) - P_m(s))Q_i(s)}.$$ (5)

and

$$G_{e,u}(s) = \frac{P_n(s)P_m(s)(1 - Q_i(s))}{P_m(s) + (P_i(s) - P_m(s))Q_i(s)}.$$ (6)

From these equations, we can observe that in the design of a disturbance observer, $Q_i(s)$ plays the most significant role in determining the robustness and disturbance suppression performance of the system. If $Q_i(s) \approx 1$, the transfer functions is reduced to

$$G_{u,y}(s) \approx P_m(s), \quad \text{and} \quad G_{e,u}(s) \approx 0.$$ (7)

This implies that for a disturbance signal whose maximum frequency is lower than the cut-off frequency of $Q_i(s)$, the disturbance signal is effectively rejected and the real plant behaves as a nominal plant. Therefore, if such a disturbance observer is employed for every joint of a manipulator, then the robot dynamics can be considered as SERD system given by

$$\ddot{M}_n \ddot{q} = \tau.$$ (8)

Note that because a disturbance observer for a system with saturation may cause the system to be unstable, a stabilization technique has to be considered. The next section analyzes the stability problem of the system with a disturbance observer under input saturation from the viewpoint of internal stability.

### 3. Stability under control input saturation

If we apply disturbance observer to the system, we can get the transfer function from $u$ to $y$ as $P_n$ approximately. Therefore, we can easily design any controller based on the nominal model $P_n$ for the system performance as shown in Fig. 4. The intrinsic characteristic of disturbances is that it is not predictable. When we apply disturbance observer to the system and if the disturbance signal is large enough to make the control input saturate, the performance of the whole system is not guaranteed. Now, we assume that $C$ is a general linear controller for discussion.

When there is no input saturation in Fig. 4, the input–output relationship can be derived as

$$y = G_{ey}r + G_{dy}d,$$ (9)

where
\[
G_{ry} = \frac{CPP_n}{\chi_c}, \quad (10)
\]
and
\[
G_{dy} = \frac{P_n(1 - Q)}{\chi_c}. \quad (11)
\]
Here, \(\chi_c = P_n(1 + CP) + (P - P_n)Q\), and \(\chi_c = 0\) is the characteristic equation of the closed loop system. Let the perturbed plant be given as
\[
P = P_n(1 + \Delta_p), \quad (12)
\]
where \(\Delta_p\) is assumed to be allowable [9]. Allowable means that no unstable poles of \(P_n(s)\) are canceled in forming \(P(s)\). Then, the robust stability of the above multiplicative uncertainties can be stated in the following theorem.

**Theorem 1 (Robust stability).** Let the plant be modeled as \(P = P_n(1 + \Delta_p)\) with allowable multiplicative uncertainties. Assume that the nominal model \(P_n\) be minimum phase system, and a linear controller \(C\) can stabilize \(P_n\). Then a sufficient condition for robust stability of the closed loop system for the uncertainty \(\Delta_p\) is given as
\[
|\Delta_p|_{s=j\omega} < \left| \frac{1 + CP_n}{Q + CP_n} \right|_{s=j\omega}, \quad \forall \omega. \quad (13)
\]

**Proof.** From Eqs. (10)–(12), \(\chi_c\) can be expressed as
\[
\chi_c = P_n(1 + CP_n) \left(1 + \frac{Q + CP_n}{1 + CP_n} \cdot \Delta_p\right). \quad (14)
\]

![Fig. 4. Block diagram of the control system.](image)
where \( P_n \) and \((1 + CP_n)\) are stable by assumptions. After all, for the robust stability of the closed loop system, the third term on the right hand side in Eq. (14) must be stable. By the small gain theorem [10], we can obtain that

\[
\left| \frac{Q + CP_n}{1 + CP_n} \cdot \Delta_p \right| < \left| \frac{Q + CP_n}{1 + CP_n} \right| |\Delta_p| < 1. \tag{15}
\]

Then, Eq. (13) can be easily obtained from Eq. (15). \( \square \)

It is also noted from Eq. (13) that \(|Q(j\omega)|\) must be small at high frequencies for robust stability, because uncertainty bound \(|\Delta_p|\) is usually large in the high frequency range. Thus, we cannot arbitrarily increase the cut-off frequency of \( Q \).

**Theorem 2 (Internal stability).** If \( P_n(s), P(s), C(s), \) and \( Q(s) \) are minimum phase system and if the multiplicative uncertainty satisfies the robust stability condition in Eq. (13), then the whole system in Fig. 4 is internally stable.

**Proof.** In Fig. 4, the relation of the input vector \([r, d]^T\) and system state \([u, u^*, y]^T\) can be described by

\[
\begin{bmatrix}
  u \\
  u^* \\
  y
\end{bmatrix} = \mathcal{P}(s) \cdot \mathcal{M} \begin{bmatrix} r \\ d \end{bmatrix}, \tag{16}
\]

where

\[
\mathcal{P}(s) = \frac{P_n}{P_n(1 + CP) + (P - P_n)Q},
\]

and

\[
\mathcal{M} = \begin{bmatrix}
  (1 - Q)C + CQPP_n^{-1} & -(1 - Q)C \\
  C & -(C + QPP_n^{-1}) \\
  CP & (1 - Q)
\end{bmatrix}.
\]

To be internally stable, six transfer functions in \( \mathcal{M} \) and \( \mathcal{P}(s) \) should all be stable and also not have any unstable pole-zero cancellation [10,11]. In Eq. (16), \( \mathcal{P}(s) \) is stable by Theorem 1, and since \( P_n(s), P(s), C(s), \) and \( Q(s) \) are assumed to be minimum phase systems, six transfer functions in \( \mathcal{M} \) are all stable, and thus, there are no unstable pole-zero cancellations. \( \square \)

Consider the case that there is an input saturation as shown in Fig. 5. When there is an input saturation, internal stability in Theorem 2 does not hold. If we assume the same conditions in Theorem 2, the relation of \([r, d]^T\), \( \hat{u} \), and system state \([u, u^*, y]^T\) can be described by
where \( \hat{u} \) will be equal to \( u^* \) if \( u^* \) is unsaturated, and, if \( u^* \) is saturated, \( \hat{u} \) can be regarded as an input of \( P \) which does not depend on feedback actions.

Note from Eq. (17) that under all internal stability conditions in Theorem 2, if \( Q(s) = N_q(s)^{-1}D_q(s) \), the transfer function from the input \( [r, d]^T \) and \( \hat{u} \) to the output \( u^* \) has \( (N_q(s) - D_q(s)) \) in the denominator. This implies that some transfer functions may have unstable poles. After all, internal stability is lost, and boundedness of the output \( u^* \) for the bounded input \( r, d \) and \( \hat{u} \) is not guaranteed.

To guarantee the stability in the case of input saturation, a disturbance observer

Fig. 6. Disturbance observer with an Additional Saturation Element (ASE).
with Additional Saturation Element (ASE) is employed. If we apply control input saturation element in front of the $Q$ filter, the block diagram is shown in the Fig. 6. In this case, the relation of $[r, d]^T$, $\dot{u}$, and system state $[u, u^*, y]^T$ can be described by

$$
\begin{bmatrix}
u \\
u^* \\
y
\end{bmatrix} =
\begin{bmatrix}
C & -C \\
C & -(C + QP_n^{-1}) \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
r \\
d
\end{bmatrix} +
\begin{bmatrix}
-CP \\
(Q - CP - PQP_n^{-1})P
\end{bmatrix} \dot{u}.
$$

It can be easily observed from Eq. (18) that if the internal stability condition described in Theorem 2 is satisfied, the system in Fig. 6 is internally stable. Thus, we can guarantee the bounded output $u, u^*$, and $y$ for bounded input $r, d$ and $\dot{u}$.

### 4. Path tracking control in Cartesian space considering torque saturation

Trajectory planning is an off-line procedure resulting in a nominal trajectory to be used as a reference trajectory in Cartesian space. By denoting the end-effector position with respect to the base coordinate as $p$, it is related to the joint position $q$ by the forward kinematics given as

$$p = k(q),$$

$$\dot{p} = J(q)\dot{q}$$

and

$$\ddot{p} = \dot{J}(q, \dot{q})\ddot{q} + J(q)\dddot{q},$$

where $J(q)$ is the Jacobian matrix and $\dot{J}(q, \dot{q})$ is the derivative of $J(q)$. If the disturbance observer cancels the disturbance in Eq. (2), simple PD control action is sufficient to drive $q$ to track a desired trajectory $q_d(t)$ [6]. Fig. 7 shows the

![Disturbance observer based independent joint control scheme in Cartesian space.](image)

Fig. 7. Disturbance observer based independent joint control scheme in Cartesian space.
disturbance observer based independent joint control scheme required to perform tasks planned in Cartesian space.

Every joint has torque limit, however, and this should be handled properly. If the magnitude of the disturbance signal is greater than the physical torque limit of the actuators, the disturbance observer cannot work correctly. Thus, the maximally admissible torque value of the joint actuator is determined here by using SERD to guarantee that the disturbance observer correctly eliminates the equivalent disturbance signal. For SERD to be valid, the maximum torque command $\tau_{\text{max}} = [\tau_{\text{max}1} \ldots \tau_{\text{max}n}]^T$, which is the maximum output of the PD controller, should be given as

$$
\tau_{\text{max}} = \tau_{\text{lim}} - \hat{\delta},
$$

(22)

where $\tau_{\text{lim}}$ and $\hat{\delta}$ are the respective vectors representing the physical torque limit of the actuator and the output of the disturbance observer.

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**Fig. 8.** Block diagram of the acceleration modification scheme based on a disturbance observer.

**Fig. 9.** Experimental setup.
From Eqs. (8) and (21), the relation between the acceleration in Cartesian space and joint torque can be obtained as

$$\ddot{p} = J(q, \dot{q})\dot{q} + J(q)\ddot{J}_n^{-1}T.$$

(23)

Note that since the maximally admissible acceleration in Cartesian space of dimension $n$ under torque limit, $p_{\text{max}} \triangleq [\dot{p}_{\text{max1}} \ldots \dot{p}_{\text{maxn}}]^T$, is dependent on the configuration of the manipulator, an arbitrarily generated trajectory for fast motion may cause the torque command to exceed $\tau_{\text{max}}$. Consequently, under conventional feedback control, the manipulator may deviate from the desired path. A constructive way to avoid this problem is to slow the velocity of the manipulator temporarily from a preplanned reference trajectory. For this, a modification method of reference acceleration, $\ddot{p}_{\text{r}} \triangleq [\ddot{p}_{\text{r1}} \ldots \ddot{p}_{\text{rn}}]^T$, is proposed by deriving the maximally admissible acceleration, $\ddot{p}_{\text{max}}$, only for the case when any actuator is saturated.

Now, the maximally admissible acceleration in the direction of $\ddot{p}_{\text{r}}$ is derived to obtain the time-optimal solution in the saturation region. For this, let us assume that the $i$th actuator is saturated for $\ddot{p}_{\text{r}}$. Then, $\ddot{p}_{\text{max}}$, which is the maximally admissible acceleration in the direction of $\ddot{p}_{\text{r}}$ under the torque limit of the $i$th actuator, $\tau_{\text{r}i}^{\text{max}}$, can be represented as

$$\ddot{p}_{\text{max}} = \alpha \ddot{p}_{\text{r}},$$

(24)

where $\alpha$ is a scalar. Substituting Eq. (24) into Eq. (23) yields
\[ x \ddot{\psi}_r = h(q, \dot{q}) + D(q)\tau, \]  

(25)

where

\[ h(q, \dot{q}) = J(q, \dot{q})\dot{\dot{q}}, \]  

(26)

and

\[ D(q) = J(q)\hat{M}^{-1}_n \triangleq [d_1 \cdots d_n]. \]  

(27)

In Eq. (27), \( d_i \) denotes the \( i \)th column vector of \( D(q) \). For \( q \) and \( \dot{q} \) in a configuration, \( \tau_{i}^{\text{max}} \) is replaced with \( \tau_i \) in Eq. (23). Then, we obtain

\[ \tilde{D}(q)\ddot{\tau} = \tilde{h}(q, \dot{q}), \]  

(28)

where

\[ \tilde{h}(q, \dot{q}) = h(q, \dot{q}) + d_i\tau_{i}^{\text{max}}, \]  

(29)
\[
\tilde{D}(q) = \left[ -d_1 \cdots -d_{i-1} \dot{p}_r - d_{i+1} \cdots -d_n \right],
\]
and
\[
\tilde{\tau} = [\tau_1 \cdots \tau_{i-1} \neq \tau_{i+1} \cdots \tau_n].
\]
Thus \( \tilde{\tau} \) can be obtained from Eq. (28) as
\[
\tilde{\tau} = \tilde{D}(q)^{-1}\tilde{h}(q, \dot{q}).
\]
In Eq. (32), \( z \) is obtained, and thus \( \tilde{p}_{\text{max}} \) can be also obtained in Eq. (24).

The computation of \( \tilde{p}_{\text{max}} \) is very simple for any configuration owing to SERD. Let \( \tilde{p}_m \) be the actual acceleration command to be obtained from \( \tilde{p}_r \) and \( \tilde{p}_{\text{max}} \). \( \tilde{p}_m \) should be \( \tilde{p}_{\text{max}} \) if the actuator is saturated. In the case when all actuators are nonsaturated, \( \tilde{p}_m \) should be given in such a way that the difference between \( \tilde{p}_r \) and \( \tilde{p}_m \) will not cause the manipulator to be incorrectly decelerated and thus eventually located at the wrong position. For this, \( \tilde{p}_c \) is designed to compensate

![Fig. 12. Experimental results for the PD controller with disturbance observer.](image)

(a) joint angle of axis 1; (b) actual path in Cartesian space; (c) velocity in Cartesian space; (d) acceleration in Cartesian space.
the integral of the difference between $\ddot{p}_r$ and $\ddot{p}_{\text{max}}$. This can be summarized as

$$\dddot{p}_m = \dddot{p}_r - \dddot{p}_c,$$

where $\dddot{p}_c$ is given by

$$\dddot{p}_c =\begin{cases} 
\dddot{p}_r - \dddot{p}_{\text{max}} & \text{if } \tau_i \geq \tau_{i,\text{max}} \text{ for any } i \\
K \int_0^t (\dddot{p}_r - \dddot{p}_{\text{max}}) \, dt & \text{otherwise,}
\end{cases}$$

(34)

where $K$ is an $n \times n$ gain matrix. The proposed acceleration modification method is depicted in Fig. 8.

5. Experimental results

To compare the proposed scheme with a simple PD control and PD plus a

![Fig. 13. Experimental results for the proposed acceleration modification method. (a) Joint angle of axis 1; (b) actual path in Cartesian space; (c) velocity in Cartesian space; (d) acceleration in Cartesian space.](image)
disturbance observer, experiments for motion at a constant speed along a straight line and in a circle were performed with the two DOF SCARA-type direct-drive arm shown in Fig. 1. The control algorithm was written in C language and tested on our prototype robot controller, which uses a 32-bit microprocessor (FORCE30) embedded VxWorks and a DSP board for servo control for real-time operation as sketched in Fig. 9. The respective sampling times for proposed acceleration modification and PD control were set at 1 and 0.1 ms.

A trapezoidal trajectory in Cartesian space generated for a desired path is shown in Fig. 10, where the velocity along the path is set at 2 m/s. For the given trajectory, starting of motion results in saturation of the first axis, and \( p_x^{\text{max}} \) and \( p_y^{\text{max}} \) are calculated as 5 and \(-22\) m/s\(^2\), respectively, from \( p_x \) and \( p_y \) which are given as \( 18\) m/s\(^2\) and \(-80\) m/s\(^2\), respectively.

The experimental results for the conventional PD controller are shown in Fig. 11, where the saturation of the first axis actuator produces a tracking error along the 1st axis and a large path error in Cartesian space near the starting location. This phenomenon stems from the fact that at the initiation of motion, an acceleration command larger than the admissible acceleration is generated, as shown in Fig. 11(d). Fig. 12 shows the performance of the PD controller with a disturbance observer. The purpose of this experiment was to see whether the disturbance observer could make a robot dynamics SERD in spite of actuator saturation. Tracking error still occurred along the 1st axis, as for the case of PD controller, due to the effect of saturation, and consequently the actual path deviates from the given linear path. This implies that a disturbance observer

![Fig. 14. Experimentally measured path deviation from the desired linear path.](image-url)
cannot guarantee SERD when any actuator is saturated. Fig. 13 shows the performance for the proposed scheme. Fig. 13 shows that the actual trajectory of 1st axis satisfactorily tracks the modified trajectory, and the actual path follows the given path with less path deviation, owing to the on-line modification of acceleration in Cartesian space. In addition, Fig. 13(c) shows that the velocity error is relatively small when compared to the performances in Fig. 12(c). The path errors for three experimental cases are depicted in Fig. 14, where the proposed method shows good path tracking performance in spite of simple SERD-based on-line calculation of the maximally admissible acceleration.

Similarly, a path tracking experiment was performed for circular motion in Cartesian space. A trapezoidal trajectory was generated for the desired path, which causes the saturation of each axes. The experimental results for conventional PD controller are shown in Fig. 15, where the saturation of the first and second axis actuators at 0.1 s produced the tracking error. However, the proposed acceleration modification method properly modified the acceleration in the saturation region as shown in Fig. 16, and thus the velocity is slowed and the

![Fig. 15. Experimental results for the PD controller. (a) Joint angle of axis 1; (b) actual path in Cartesian space; (c) velocity in Cartesian space; (d) acceleration in Cartesian space.](image-url)
desired tracking performance was achieved. Note, however, the task completion times were almost the same in both Figs. 15 and 16.

### 6. Conclusion

A path tracking algorithm was proposed to compensate for path deviation due to saturation of actuators, in which a disturbance observer based independent joint control scheme was employed to obtain simple equivalent robot dynamics (SERD) and a method of modifying acceleration in Cartesian space based on SERD was derived. The proposed algorithm was digitally implemented and tested using a two DOF SCARA-type direct-drive robot manipulator. The experimental results show that the proposed scheme successfully tracks the given path even with the saturation of actuators.
References