COMPLIANCE PLANNING FOR DEXTROUS ASSEMBLY TASKS USING MULTI-FINGERED ROBOT HANDS

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ABSTRACT—This paper deals with compliance planning for effective peg-in-hole and peg-out-hole tasks using multi-fingered robot hands without inter-finger coupling. We first observe the fact that some of coupling stiffness elements of the spatial stiffness matrix cannot be planned arbitrary. Then, we analyze the conditions of the specified stiffness matrix in the operational space to successfully and more effectively achieve the given peg-in-out-hole tasks. It is clearly shown that the location of compliance center on the peg and the coupling stiffness element between the translational and the rotational direction play important roles for successful peg-in-out-hole tasks. Thus, the operational compliance characteristics should be carefully specified and the location of compliance center should be planned on-line for effective assembly and disassembly tasks. Simulation results are included to verify the feasibility of the analytic results.

Key Words: multi-fingered robot hands, peg-in-hole and peg-out-hole tasks, compliance planning

1. INTRODUCTION

Human hands have a variety of dexterity which is useful for fine manipulation, grasping tasks of objects having arbitrary shapes, textures and weights, complex assembly tasks, and other various tasks. Thus, a lot of research works on multi-fingered robot hands have been performed to accommodate a variety of tasks and objects in the field of industrial applications, service robots, and rehabilitation robots. Recently, several explicit force-based control techniques have been proposed for effective grasping and manipulation of object by multi-fingered hands, or multiple robot arms [1]-[3]. However, implementation of those methods may suffer from attaching force sensors in relating small finger mechanisms, as well as processing noisy force signals. Also, the integration of tactile and force information for individual finger control, and the combination of information from different fingers to guide the hand action are not still well-known [4]. Thus, instead of employing force signals, stiffness or compliance as successful alternatives has been known to be useful for characterizing the grasping and manipulation of robot hands [5, 6].

Many approaches have been reported in the field of grasp stiffness or compliance. The stiffness of objects grasped by virtual springs was analyzed in cases of planar and three-dimensional space [7]. Yokoi, et al. [8] proposed a direct compliance control method and applied the method to a parallel arm. Curtosky et al. [5] analyzed the effective grasp stiffness by considering the structural compliances in fingers and fingertips, servo gains at the joints of finger, and small changes in the grasp geometry that may affect the grasp forces acting upon the object. In [9], it is pointed out that a stiffness matrix containing some off-diagonal terms can be useful to prevent jamming of contact tasks. Li et al. [10] classified a grasp matrix into symmetric and anti-symmetric parts. Also, Kao, et al. [6] tried to apply stiffness models usually employed in robotics research to the analysis of human grasping behaviors. Kim, et al. [11] proposed an independent
finger/joint-based compliance control method for robot hands manipulating an object, and also the geometric condition for successful implementation of compliance control scheme have been addressed. They showed that an independent finger/joint-based compliance control via redundant actuation was more adequate to modulate the operational stiffness comparing with the case of the kinematically redundant structured fingers or manipulators. Some researchers have investigated the task-based stiffness characteristics [12, 13]. However, the study to achieve the desired stiffness characteristics is still understood as an everlasting research area.

Applications of compliance control to assembly tasks has been also active research field. Related to the peg-in-hole task using robot hands firstly, Whitney [15] classified the geometry of the inserted peg and analyzed the force relations in the peg-in-hole task by using a remote center compliance mechanism. Asada, et al. [16] analyzed the dynamic process of a peg insertion. Matsuoka, et al. [17] used a multi-sensors-based control system to perform a given peg-in-hole task and also proposed a method of executing tasks based on motion primitives for fine manipulation. In recent, a behavior-based peg-in-hole approach have been proposed in [18]. In other way, a few approaches to disassembly task have been proposed [20, 21]. Also, the general concept of compliance center has been analyzed [19]. Shimoga, et al. [13] presented that the desirable location of compliance center should be on the point of the grasped object which first touches or which already is in contact with the inserting workpiece. However, the determination of proper location of compliance center and stiffness characteristics to effectively achieve the given peg-in-hole and peg-out-hole tasks have not been deeply analyzed yet.

In this paper, we describe the operational compliance characteristics for effective peg-in/out-hole tasks and the selection problem of the proper location of compliance center. It will be also shown that the coupling stiffness element existing between translational and rotational direction plays an important role in the insertion task. The procedure of this paper is as follows. In section 2, we describe an independent finger-based compliance control method and also we discuss on the shape of stiffness matrix to be specified in the operational space by consideration of the grasp geometry by robot hand. In Section 3, a guideline specifying the operational compliance characteristics for effective handling of the given peg-in/out-hole tasks and also the selection problem of the location of compliance center is discussed. In section 4, we show several simulation results to verify the feasibility of the analytic results and also, we confirm that the location of compliance center on the peg and the role of a coupling stiffness element existing between the translational and the rotational direction are important for effective peg-in-hole and peg-out-hole tasks. Concluding remarks are drawn in section 5.

2. INDEPENDENT FINGER-BASED COMPLIANCE CONTROL

When a multi-fingered robot hand manipulates an object, the fine manipulation of the grasped object is not easy due to the interaction among fingers. In this section, we describe the independent finger-based compliance control method to cope with this problem.

Consider a rigid peg being inserted in a hole by a three-fingered robot hand in two-dimensional space as shown in Figure 1. The relation between the dynamic force vector in the operational space and the fingertip force vector is given by

\[ T_e = [G_e]^T T_f, \]  

where \( T_e \in \mathbb{R}^{3x1} \) denotes the generalized force vector in the operational space including the inertial load and external load, and the fingertip force vector \( T_f \in \mathbb{R}^{3x1} \) in the fingertip space is expressed as

\[ T_f = \begin{bmatrix} (r_1 \cdot T_f) \cdot (r_2 \cdot T_f) \cdot (r_3 \cdot T_f) \end{bmatrix}^T, \]

and the Jacobian matrix relating the operational space to the fingertip space \( [G_e] \in \mathbb{R}^{6x3} \) is given by

![Figure 1. Peg-in-hole task using a three-fingered robot hand.](image)
\[
\begin{bmatrix}
G'\end{bmatrix} = \begin{bmatrix}
G' & G' & G' \\
\end{bmatrix}
\]

with
\[
\begin{bmatrix}
G' \end{bmatrix} = \begin{bmatrix}
R, p \times R & 0 \\
\end{bmatrix}
\]

Here, \( R \) and \( R\times p \) denote the rotation matrix and the position vector directing from the operational space to the fingertip space, respectively.

When the motion trajectory of the object is pre-specified, the task of load distribution can be classified to determine the fingertip forces and moments in order to achieve the desired motion of the object and to maintain the grasp. The general solution of (1) is given by
\[
\begin{bmatrix}
T \end{bmatrix} = \begin{bmatrix}
G' \end{bmatrix} \begin{bmatrix}
T_4 \end{bmatrix} + \begin{bmatrix}
I - \begin{bmatrix}
G' \end{bmatrix} \begin{bmatrix}
G' \end{bmatrix} \end{bmatrix} \begin{bmatrix}
\xi_j \end{bmatrix}
\end{bmatrix}
\]

where \( \begin{bmatrix}
G' \end{bmatrix} \) is a pseudo-inverse of \( \begin{bmatrix}
G' \end{bmatrix} \) and \( \xi_j \) is an arbitrary \( 6 \times 1 \) vector. \( I \) denotes an \( 6 \times 6 \) identity matrix.

Using (2), we can perform explicit force control of robot hand by using force sensor signal, but the fine finger motion control is practically hard because force measurement at the fingertip is not easy and the real force signal is very noisy. The explicit force control method applied to robot hand may not be biomimetic. It is therefore worth studying the compliance control method of robot hand which is believed to be more human-like.

By taking the partial derivative of (1) with respect to the deflection vector \( u_i \) in the operational space, the \( 3 \times 3 \) stiffness matrix in the operational space including the effect of the change of contact configuration can be expressed as follows [11]
\[
\begin{bmatrix}
K' \end{bmatrix} = \begin{bmatrix}
G' \end{bmatrix} \begin{bmatrix}
K_j \end{bmatrix} \begin{bmatrix}
G' \end{bmatrix} + \begin{bmatrix}
I \end{bmatrix} \begin{bmatrix}
h_j \end{bmatrix}
\end{bmatrix}
\]

and we define
\[
\begin{bmatrix}
K_j \end{bmatrix} = \begin{bmatrix}
K_j \end{bmatrix} + \begin{bmatrix}
I \end{bmatrix} \begin{bmatrix}
h_j \end{bmatrix}
\end{bmatrix}
\]

Then, the relationship between the desired object stiffness matrix \( \begin{bmatrix}
K_j \end{bmatrix} \) in the operational space and the stiffness matrix \( \begin{bmatrix}
K_j \end{bmatrix} \) in the fingertip space is given as
\[
\begin{bmatrix}
K_j \end{bmatrix} = \begin{bmatrix}
K_j \end{bmatrix} \begin{bmatrix}
G' \end{bmatrix} \begin{bmatrix}
K_j \end{bmatrix} \begin{bmatrix}
G' \end{bmatrix}
\end{bmatrix}
\]

where
\[
\begin{bmatrix}
K_j \end{bmatrix} = \begin{bmatrix}
K_{xx} & K_{xy} & K_{xz} \\
K_{xy} & K_{yy} & K_{yz} \\
K_{xz} & K_{yz} & K_{zz}
\end{bmatrix}
\]

and the operator of \( (\cdot) \) and \( \begin{bmatrix}
h_j \end{bmatrix} \) represent the Generalized Scalar Dot Product [22] and the second-order kinematic influence coefficient matrix which is induced by the change of contact configuration [23], respectively.
In a robot hand system, the components of wrench transmitted through the contact between the fingertip and the contact point of object are limited by the contact constraint defined as the contact types. It is therefore very important to investigate how many fingers are required to modulate the desired compliance characteristic in the operational space. In [11], it is analyzed that a robot hand should have at least three fingers to modulate a 3 x 3 object stiffness characteristic in two-dimensional space.

Here, our objective is to eliminate inter-finger couplings as well as the coupling of the fingertip space for effective hybrid control in the fingertip space. So, the fingertip stiffness matrix satisfying this objective is set up as below

\[
[K_f] = \begin{bmatrix}
K_{fx} & 0 & 0 & 0 & 0 & 0 \\
0 & K_{fy} & 0 & 0 & 0 & 0 \\
0 & 0 & K_{fz} & 0 & 0 & 0 \\
0 & 0 & 0 & K_{fx} & 0 & 0 \\
0 & 0 & 0 & 0 & K_{fy} & 0 \\
0 & 0 & 0 & 0 & 0 & K_{fz}
\end{bmatrix}
\]  

(6)

Then, the equation (5) can be rearranged as a vector form

\[
K_{me} = [B_f^T] K_f
\]

(7)

\[
K_{me} = \begin{bmatrix}
K_{mxx} & K_{mxy} & K_{mxz} & K_{mwy} & K_{mwz} & K_{mww} \\
1.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 1.0 & 0.0 & 1.0 & 0.0 & 1.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 1.0 & 0.0 & 1.0 & 0.0 & 1.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0
\end{bmatrix}
\]

\[
[B_f^T] = \begin{bmatrix}
y_1 & y_2 & y_3 & x_1 & x_2 & x_3 \\
y_1^2 & y_2^2 & y_3^2 & x_1^2 & x_2^2 & x_3^2 \\
x_1^3 & x_2^3 & x_3^3 & y_1^3 & y_2^3 & y_3^3
\end{bmatrix}
\]

\[
K_f = \begin{bmatrix}
K_{fx} & K_{fy} & K_{fz} & K_{fx} & K_{fy} & K_{fz}
\end{bmatrix}
\]

and \(x_i, y_i\) denote the elements of position vectors directing from the \(i\) th finger contact position to the task position, and they are given all positive. \(K_{fx}\) and \(K_{fy}\) represent the \(x\) - and \(y\) -directional stiffness elements in the fingertip space of the \(i\) th finger, respectively.

Note that the elements of the second row of the mapping matrix \([B_f^T]\) in (7) are calculated as zero. This is because we excluded the coupling terms \('K_{fi}(i=1,2,3)\) in the fingertip space for independent compliance control. Thus, we have zero \(K_{me}\), which, in fact, is a linear combination of \('K_{fi}(i=1,2,3)\). Also, note that the third row of \([B_f^T]\) corresponds to modulation of \(K_{me}\). However, we can easily notice that zero \(K_{me}\) cannot be achieved by all positive stiffness components \(K_f\) defined in the fingertip space since the three influence coefficients (i.e., \(y_1, y_2,\) and \(y_3\)) are always positive in this grasped configuration. Therefore, the value of \(K_{me}\) is always positive. Accordingly, the operational stiffness matrix in Figure 1 can be specified as follows:

\[
[K_e] = \begin{bmatrix}
K_{me} & 0 & K_{me} \\
0 & K_{me} & 0 \\
K_{me} & 0 & K_{me}
\end{bmatrix}
\]

(8)

Next, consider a modified posture of grasp in Figure 2, in which the contact position of the third finger lies above the task position \(O\). In this case, the \((3, 5)\) element of \([B_f^T]\) is converted to \(-y_3\) and thus, zero
3. COMPLIANCE CHARACTERISTIC FOR DEXTROUS PEG-IN/OUT-HOLE TASKS

When a peg is assembled into a hole by human hand, the task can be accomplished by exact position control if the shape and the size of the peg and the hole are assumed to be known. However, if contact between the peg and the hole is occurred by control error or external disturbance in real system, additional control effort is necessary to successfully perform the peg-in/out-hole tasks. Also, we can experience that inserting the peg into a hole or extracting the inserted peg from the hole can be more easily achieved by intentionally giving small perturbation of orientation of the grasped peg.

In the conventional research [13], it is pointed out that the location of the compliance center should be specified in the distal point of the peg in a peg-in-hole task and also in a peg-out-task, it should be located in the proximal point of the peg. However, the determination of proper location of compliance center and stiffness characteristics to effectively achieve the given peg-in-hole task have not been fully understood yet. Moreover, for the case of peg-out-hole task, there are few research results considering the proper location of compliance center and the determination of stiffness characteristics.

In this section, we analyze the compliance characteristic for effective the peg-in/out-hole tasks by using multi-fingered robot hands and also discuss the role of the location of compliance center in the tasks. Also, it is assumed that the location of the hole, friction coefficient, and the contact status of the peg are known in advance. Specifically, Kaneko et al. [14] presented an approach for finding contact point for grasping an unknown object based on the self-posture changeability. In [17], it is shown that sensor fusion method using vision and force sensors can be used for peg assembly task.

3.1 Peg-In-Hole Task

A. Left-Side Contact

First, consider the case that a left-side of the peg is being contacted on the hole as shown in Figure 3. The springs attached to the point \( O \) denote the virtual springs. The point \( O \) is treated as a MCC point.

Then, the forces exerted on the virtual springs attached to the peg tip can be expressed as

\[
\begin{bmatrix}
    f_x \\
    f_y \\
    \tau_y
\end{bmatrix} =
\begin{bmatrix}
    K_{xx} & 0 & K_{xw} \\
    0 & K_{yy} & 0 \\
    K_{wy} & 0 & K_{ww}
\end{bmatrix}
\begin{bmatrix}
    \delta u_x \\
    \delta u_y \\
    \delta u_w
\end{bmatrix},
\]

where \( f_x \), \( f_y \), and \( \tau_y \) denote the \( x \)-, \( y \)-, and rotational forces, respectively. And the small deflections of the virtual springs in the \( x \)-, \( y \)-, and rotational directions, respectively, are given by

\[
\delta u_x = u_x' - u_x, \quad \delta u_y = u_y' - u_y, \quad \delta u_w = u_w' - u_w,
\]

and here \( u_x' \) and \( u_y' \) denote the \( j \)-directional desired and actual position, respectively.
Assuming that the insertion process takes place in a quasi-static state, the \( x \)-, \( y \)-directional forces, and torque induced by the reaction force (\( f_{\text{in}} > 0 \)) are given by

\[
\begin{align*}
    f_{\text{in}} &= -f_{\text{out}}, \\
    f_{\text{ex}} &= -\mu f_{\text{in}}, \\
    \tau_{\text{in}} &= f_{\text{in}}(l + \mu r),
\end{align*}
\]

(10) \hspace{1cm} (11) \hspace{1cm} (12)

here \( \mu \), \( l \), and \( r \) denote the friction coefficient at the contacting surface, the length between the compliance center and the point \( p \), and the radius of peg, respectively.

In this configuration, the desired path of the left-edge of the peg is located inside the surface of the hole, and thus, the small deflection \( \delta u_{\text{in}} \) of the virtual spring in the \( x \)-direction becomes negative. Consequently, a positive directional reaction force and its associated friction force are generated, and simultaneously, the orientation change of the peg is occurred by the torque caused by the reaction forces. From (9), the \( x \)-directional force and torque can be represented by

\[
\begin{align*}
    f_{\text{in}} &= K_{\text{ex}} \delta u_{\text{ex}} + K_{\text{ex}} \delta u_{\text{in}}, \\
    \tau_{\text{ex}} &= K_{\text{ex}} \delta u_{\text{ex}} + K_{\text{ex}} \delta u_{\text{in}}.
\end{align*}
\]

(13) \hspace{1cm} (14)

If we substitute (13) into (10), the reaction force can be expressed as

\[
    f_{\text{in}} = -K_{\text{ex}} \delta u_{\text{ex}} - K_{\text{ex}} \delta u_{\text{in}},
\]

(15)

and then, by substituting (15) into (12), the torque relation given by (12) can be rearranged by

\[
    \tau_{\text{in}} = -K_{\text{ex}}(l + \mu r) \delta u_{\text{ex}} - K_{\text{ex}}(l + \mu r) \delta u_{\text{in}}.
\]

(16)

Finally, we can obtain the following equality from the torque relations given by (14) and (16),

\[
    K_{\text{ex}} \delta u_{\text{ex}} + K_{\text{ex}} \delta u_{\text{in}} = -K_{\text{ex}}(l + \mu r) \delta u_{\text{ex}} - K_{\text{ex}}(l + \mu r) \delta u_{\text{in}}.
\]

(17)

By rearranging (17), the orientation change of the peg can be expressed as

\[
    \delta u_{\text{in}} = \frac{K_{\text{ex}}(l + \mu r)}{K_{\text{ex}} + K_{\text{ex}}(l + \mu r)} \delta u_{\text{in}}.
\]

(18)

Note that the sign of the \( K_{\text{ex}} \) is always positive in the grasp configuration shown in Figure 3, the sign of the value inside parenthesis is always given positive. However, the sign of \( \delta u_{\text{in}} \) is negative in this case. Therefore, the orientation change \( \delta u_{\text{in}} \) of (18) is at least greater than zero and hence the inserted peg will rotate to the counterclockwise direction about the compliance center. Therefore, this phenomena facilitates the insertion task. Even though \( l = 0 \) and \( \mu = 0 \), existence of positive \( K_{\text{ex}} \) makes the insertion job successful.

### B. Right-Side Contact

In this subsection, consider the case that a right-side of the peg is being contacted on the hole as shown in Figure 4. The RCC point \( O \) is located at the tip of the peg.
The forces exerted on the virtual springs of attached to the peg are derived as
\[ f_x = f_r \left( \cos(\phi) + \mu \sin(\phi) \right), \]
\[ f_y = -f_r \left( \sin(\phi) + \mu \cos(\phi) \right), \]
\[ \tau_w = -f_r \sigma, \]
where
\[ \sigma = \left( \frac{\tan(\phi) - \mu}{1 + \mu \tan(\phi)} \right) \geq 0, \quad \phi_{\min} \leq \phi \leq 2\pi, \quad \phi_{\max} = \frac{3\pi}{2} + \cos^{-1} \left( \frac{r}{r_k} \right). \]
\( \phi_{\min} \) and \( r_k \) denote the minimum orientation angle of the peg and the radius of the hole, respectively.

Similar to the left-side contact, the torque relation at the compliance center can be obtained from (9), (19), and (21), by
\[ K_{uw} \delta u_w + K_{aw} \delta u_a = -r K_{uw} \delta u_{uw} - r K_{aw} \delta u_a. \]
By rearranging (22), we have
\[ \delta u_w = -\left( \frac{K_{uw} + r K_{uw} \sigma}{K_{uw} + r K_{aw} \sigma} \right) \delta u_a. \]
From (23), note that the sign of the value inside parentheses is always positive. Since \( \delta u_a \) is always taken positive in this contact type, \( \delta u_w \) becomes negative. This results in clockwise rotation of the peg, which is undesirable for peg insertion.

**C. Modified Right-Side Contact**

Now, consider the peg-in-hole task shown in Figure 5, where the location of compliance center is modified. In Figure 5, the length parameters, \( \alpha \), and \( a \), and \( \alpha \) angle are computed as
\[ c = r |\tan(\phi)|, \]
\[ a = \sqrt{r^2 + b^2}, \]
\[ \alpha = \cos^{-1} \left( \frac{r}{a} \right), \]
where \( c \) denotes the distance between the top-edge of the peg and the point of the center-line that \( f_y \) is applied, \( b \) is the distance from the peg tip to the compliance center and it is a design parameter to be determined.

When the RCC point is located in such position of Figure 5, \( b \) is always greater than \( c \). Then, the \( x \)- and \( y \)-directional forces, and torque induced by the \( x \)-directional reaction force \( f_x > 0 \) are given by
\[ f_x = f_r \left( \cos(\phi) + \mu \sin(\phi) \right), \]
\[ f_y = -f_r \left( \sin(\phi) + \mu \cos(\phi) \right), \]
\[ \tau_w = a f_w \lambda, \]
where \( \lambda \) is defined as
\[
\lambda = \frac{\sin(\alpha) + \cos(\alpha) \tan(\phi_c)}{1 + \mu \tan(\phi_c)}
\]
and here, if the \( \alpha \) angle is properly determined by setting the distance \( b \), the sign of \( \lambda \) can be set up to be positive in most cases.

From (9), (27), and (29), the torque relation at the compliance center can be given by
\[
K_{acc} \delta u_{acc} + K_{cm} \delta u_{cm} = a K_{cm} \lambda \delta u_{cm} + a K_{cm} \lambda \delta u_{cm}.
\]
By rearranging (30), the orientation change of the peg can be expressed as
\[
\delta u_{cm} = -\left(\frac{K_{cm} - a K_{cm} \lambda}{K_{cm} - a K_{cm} \lambda}\right) \delta u_{cm}.
\]
In this case, the orientation change of the peg is at least greater than zero if either of the following conditions are satisfied:
\[
\delta u_{cm} = -\left(\frac{K_{cm} - a K_{cm} \lambda}{K_{cm} - a K_{cm} \lambda}\right) \delta u_{cm} > 0, \quad \text{for} \ K_{cm} > a K_{cm} \lambda \quad \text{and} \ K_{cm} < a K_{cm} \lambda.
\]
\[
\delta u_{cm} = -\left(\frac{K_{cm} - a K_{cm} \lambda}{K_{cm} - a K_{cm} \lambda}\right) \delta u_{cm} > 0, \quad \text{for} \ K_{cm} < a K_{cm} \lambda \quad \text{and} \ K_{cm} > a K_{cm} \lambda.
\]
From (32) and (33), we can notice that the stiffness elements in the operational space should be carefully selected for effective handling of the given peg-in-hole task.

3.2 Peg-Out-Hole Task

A. Left-Side Contact

Consider the task of disassembling peg from a hole, as shown in Figure 6, where the peg contacts the left-side of the hole.

When the location of compliance center lies in the origin denoted \( O \), in Figure 6, the generalized force relations induced at the peg-tip can be represented by
\[
f_{cm} = -f_{cm}.
\]
\[
f = \mu f_{cm},
\]
\[
r_{cm} = f_{cm} (l_c - \mu r),
\]
where \( l_c \) denotes the distance between the contact point and the location of compliance center, \( O \).

From (9), (34), and (36), the torque relation at the compliance center can be expressed as
\[
K_{cm} \delta u_{cm} + K_{cm} \delta u_{cm} = -K_{cm} (l_c - \mu r) \delta u_{cm} - K_{cm} (l_c - \mu r) \delta u_{cm}.
\]
By rearranging (37), we have the orientation change of the peg as follows:
\[ \delta u_{\alpha} = -\frac{K_{\alpha} + K_{\alpha}(l - \mu r)}{K_{\alpha} + K_{\alpha}(l - \mu r)} \delta u_{\alpha}, \quad (38) \]

By similar procedure, the orientation change of the peg as the location of compliance center, \( \alpha \), can be described as, respectively,

\[ \delta u_{\alpha} = -\frac{K_{\alpha} + K_{\alpha}(l_1 - \mu r)}{K_{\alpha} + K_{\alpha}(l_1 - \mu r)} \delta u_{\alpha}, \quad (39) \]

\[ \delta u_{\omega} = -\frac{K_{\omega} - K_{\omega}(l_1 + \mu r)}{K_{\omega} - K_{\omega}(l_1 + \mu r)} \delta u_{\omega}, \quad (40) \]

and

\[ \delta u_{\omega} = -\frac{K_{\omega}(l_1 + \mu r)}{K_{\omega}} \delta u_{\omega}, \quad (41) \]

where \( l_i (i = 2, 3, 4) \) denotes the distance between the contact point and the location of the \( i \)th compliance center, respectively. Particularly, if the compliance center moves from \( O_i \) to \( O_{\alpha} \), \( K_{\omega} \) in (40) can be specified zero as described in section 2. Thus, the orientation change \( \delta u_{\omega} \) of the peg can be expressed by (41) and also its sign is always negative since \( \delta u_{\omega} \) is given negative.

For a slender peg, \( l_i \) and \( l_1 \) are usually greater than \( r \). Then, the signs of the value inside parentheses in (38) and (39) are positive, while that of the value inside parentheses in (41) is always negative. However, that of (40) is conditionally negative. Since \( \delta u_{\alpha} \) is taken negative always, \( \delta u_{\omega} \) in (38) and (39) becomes positive, and hence the peg rotates to the counterclockwise direction about the compliance center in those cases. Also, \( \delta u_{\omega} \) becomes larger as the location of the compliance center is located near the peg tip. On the contrary, \( \delta u_{\omega} \) in (41) becomes negative, resulting in clockwise rotation of the peg which is undesirable for this task. This observation seems to be somewhat surprising, since compliance center for disassembly task of peg-in-hole has been usually believed to be chosen as \( O_i \) or \( O_\alpha \). Thus, it is necessary to properly plan the location of the compliance center, in on-line fashion, for effective disassembly task.

**B. Right-Side Contact**

Next, consider the case when the peg contacts the right-side of the hole. When the location of compliance center lies in the origin denoted \( O_i \), in Figure 7, the generalized force relations induced at the peg tip can be represented by

\[ f_{\mu} = f_\mu (\cos(\theta) - \mu \sin(\theta)), \quad (42) \]

\[ f_{\omega} = -f_\omega (\sin(\theta) - \mu \cos(\theta)), \quad (43) \]

\[ \tau_{\omega} = f_\omega (r \sin(\theta) + \mu r \cos(\theta)) \quad (44) \]

**Figure 7. Peg-out-hole: right-side contact.**

Combining (9), (42), and (44), the torque relation at the compliance center can be given by:

\[ K_{\omega} \delta u_{\alpha} + K_{\omega} \delta u_{\omega} = K_{\omega} r \left( \frac{\mu + \tan(\theta)}{1 - \mu \tan(\theta)} \right) \delta u_{\alpha} + K_{\omega} r \left( \frac{\mu + \tan(\theta)}{1 - \mu \tan(\theta)} \right) \delta u_{\omega}. \quad (45) \]

By rearranging (45), we have the orientation change of the peg as follows:
\[ \delta u_{m} = -\left( \frac{K_{m} - aK_{m}/\mu}{K_{m} - aK_{m}/\mu} \right) \delta u_{k}, \]  

(46)

where a variable \( \mu \) is defined as

\[ \mu = \frac{\mu + \tan(\phi)}{1 - \mu \tan(\phi)} \geq 0, \quad \text{for} \quad \mu \geq |\tan(\phi)|. \]

Also, we can obtain the orientation change of the peg as the location of compliance center moves to \( O_{1} \) and \( O_{2} \) as follows:

\[ \delta u_{m} = -\left( \frac{a_{i}K_{m}/\mu}{a_{i}K_{m}/\mu} \right) \delta u_{k}, \]  

(47)

where \( a_{i} (i = 2, 3) \) denotes the distance between the contact point and the \( i \)th compliance center and the positive variable \( \eta_{i} (i = 2, 3) \) for the \( i \)th compliance center is defined as

\[ \eta_{i} = \frac{\sin(\alpha_{i}) + \cos(\alpha_{i}) \tan(\phi_{i})}{1 - \mu \tan(\phi_{i})} + \mu \left[ \cos(\alpha_{i}) - \sin(\alpha_{i}) \tan(\phi_{i}) \right] \]  

(48)

Particularly, if the location of compliance center lies in the origin \( O_{i} \), \( K_{m} \) can be specified zero, and therefore we have always positive \( \delta u_{m} \) as follows:

\[ \delta u_{m} = \left( \frac{a_{i}K_{m}/\mu}{K_{m}} \right) \delta u_{k}, \]  

(49)

where \( \eta_{i} \) is determined by (48).

Since \( \delta u_{m} \) is given positive, we can notice from (46), (47), and (49) that if the distance between the peg tip and the compliance center is greater or equal to \( e \), \( \delta u_{m} \) can be made positive with proper selection of stiffness elements.

### 3.3 Discussion

In this subsection, we discuss the role of the location of compliance center for effectively achieving the given peg-in-hole and peg-out-hole tasks. If the peg happens to contact the edge of the hole, the mode of stiffness planning should be changed so that the peg should have its desired rotational motion to avoid jamming. Otherwise, the motion planning of the peg is such that the peg leave the hole before the other edge of the peg contact the wall.

Now, we can provide a guideline for selecting the location of compliance center for effective peg-in/out-hole task in Table I. The terminologies, PFH, POH, and LCC, in Table I denote the abbreviations of peg-in-hole, peg-out-hole, and the location of compliance center, respectively, and \( l_{i} \) is the distance between the peg tip and the compliance center. And \( O_{1}, A_{1} \), and \( X \) mean effective, conditionally effective, and not desired status for the given task, respectively. From Table I, we can conclude that the location of compliance center on the peg should be properly planned for more effective peg-in/out-hole tasks and also, the range of the compliance center denoted \( O_{1} \) may be suitable for conditionally stable task by planning the stiffness characteristics in the operational space.

### 4. Simulation Results

This section provides simulation results to confirm the orientation change of the peg when the peg contacts the hole. In simulations, we use a three-fingered robot hand equipped with five bar mechanism [11, 24]. The contact type between the fingertip and the object is assumed as a point contact with friction, and the slip at the contact points is ignored.
The generalized force in the operational space of the grasped peg in Figure 1 can be expressed as

\[ T_c(t) = [M_{p}] \dot{u}_c(t) + [B_c] \ddot{u}_c(t) + [K_c] \delta u_c(t), \]

where

\[ \delta u_c(t) = u_{d}(t) - u_{e}(t), \]

and \( u_{d}(t), u_{e}(t) \) denote the desired and actual positions of the manipulated object, respectively. \([M_c]\) and \([B_c]\) denote the inertial and damping matrices of the object, respectively, and \(\ddot{u}_c\) denotes the acceleration vector of the object.

When the grasped peg by robot hand contacts the left- or right-sides of the hole, the torque \(\tau_{\omega}\) is induced by the reaction force. As a result, we can notice that the induced-torque changes the orientation of the peg. Thus, we observe the orientation change for several peg-in/out-hole tasks.

In this simulation, we assume that the effect caused by the inertia and damping of the object is negligible in the quasi-static state. The desired stiffness matrix in the operational space is specified as

\[ [K_c] = \begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix} = \begin{bmatrix} 100 & 0 & 1.37 \\ 0 & 1500 & 0 \\ 1.37 & 0 & 0.5 \end{bmatrix}, \]

where \(K_{xx}\) is determined by (53) (See Appendix).

To be specific, in the assembly task, since the force control is desired in the \(x\)-direction and also the position control is desired in the \(y\)-direction, we set a lower stiffness value for the \(x\)-direction than the value for the \(y\)-direction. Also, the rotational stiffness is taken relatively small to allow compliant rotational motion.

The contact points of the three-fingered robot hand are given \((-x_c, y_c) = (-0.03, -0.06), (x_c, -y_c) = (0.03, -0.06),\) and \((x_c, y_c) = (0.0, -0.1),\) where the sign of all parameters are set positive and those unit is meter. The material of the peg and the hole is assumed the wood. The friction coefficient \(\mu\) and the initial orientation of the peg are set as 0.3 and 350°, respectively.

The first simulation corresponds to the case of Figure 3, where the parameter \(l\) is fixed as 0.03m. Figures 8 shows that the inserted peg rotates counterclockwise upon contacting the left-side of the peg on the hole. Thus, the peg goes back to the vertically straight configuration.

The second simulation considers the case of Figure 4. The third simulation treats the same task with modified compliance center as shown in Figure 5, where the distance parameter \(b\) is set as 0.05m. Note that the orientation of the peg in the case of second simulation (Figure 9(a)) decreases for the \(x\)-directional deflection, while that of the peg in the case of third simulation (Figure 9(b)) increases. Consequently, it can be said that the given peg-in-hole task is more easily achieved in the case (b) in comparison to the case (a). Figure 10 shows that the necessary conditions described in (33) are satisfied for the modified compliance
center. Finally, simulations for the peg-out-hole task are performed. Simulation results for the peg-out-hole task are shown in Figures 11 and 12, where the length parameters (unit: m) \( l, d, l_1, d_1, l_2, \) and \( d \) are set as 0.04, 0.01, 0.01, 0.035, 0.1, and 0.02, respectively. Figure 11 shows that when the location of compliance center locates in either \( O_i \) or \( O_j \), the peg rotates properly, while the resulting orientation of the peg for either \( O_j \) or \( O_i \) causes jamming. Figure 12 shows the orientation of the peg for right-side contact. We can see the peg goes back to the vertically straight configuration upon contact for all cases except \( O_i \). Also, it is observed the peg may not properly rotates when a compliance center on the peg is located in \( O_i, O_j, \) and \( O_r \). Thus, the compliance center \( O_i \) is best for effective peg-in/out-hole tasks.

Figure 9. Orientation of the peg for right-side contact: (a) is the case that the compliance center lies in the peg tip and (b) is the case that the location of compliance center is modified.

Figure 10. Trend of parameters: (a) \( K_{x,y} \), (b) \( \alpha K_{x,y} \), (c) \( K_{x,y} \), (d) \( \alpha K_{x,y} \).

Figure 11. Orientation of the peg for left-side contact in the peg-out-hole task: (a) \( O_i \), (b) \( O_j \), (c) \( O_r \), (d) \( O_i \).

Figure 12. Orientation of the peg for right-side contact in the peg-out-hole task: (a) \( O_i \), (b) \( O_j \), (c) \( O_r \), (d) \( O_i \).
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Conclusively, it is remarked that planning the location of the compliance center on the peg is necessary for effective peg-in/out-hole tasks and also the coupling stiffness element existing between the x-direction and the rotational direction plays important roles in peg-in/out-hole tasks.

5. CONCLUDING REMARKS

In this paper, we analyzed the conditions of the specified stiffness matrix in the operational space to successfully and more effectively achieve the defined peg-in/out-hole tasks along with the analysis of the location of compliance center. Through the analysis, it is concluded that the location of compliance center on the peg and the coupling stiffness element existing between the translational and the rotational direction play important roles for successful insertion and disassembly tasks. Therefore, the operational compliance characteristics should be carefully specified and the location of compliance center should be planned online for effective assembly and disassembly tasks. The fundamental analysis on compliance characteristics performed in this work can be extended to general robotic assembly tasks. On going study is the experimental verification of the proposed methodology for assembly tasks.

REFERENCES


APPENDIX

Here, we describe the procedure of determining the independent fingertip stiffness elements. The structure of $[B_f]$ in (7) may differ according to the geometry of grasping an object by robot hand. To be specific, the sign of some element of $[B_f]$ may be changed from positive to negative or zero. Therefore, the operational stiffness elements should be considerably specified as the grasp geometry. If all elements of some row of $[B_f]$ are zero, the corresponding element of operational stiffness matrix equals zero. Specially, if the sign of all elements of some row of $[B_f]$ corresponding to the coupling element of operational stiffness matrix is positive or negative, the coupling element of operational stiffness matrix always exist and also it is dependent on the independent elements of operational stiffness matrix [11].

Then, $K_{op}$ can be obtained by the following procedure. By rearranging (7), we have

$$K_{op} = [D_f]K_f,$$

where $[D_f]$ be the matrix excluding the second and third rows of $[B_f]$ and $K_{op}$ be the vector excluding $K_{op}$ and $K_{op}$ in $K_{op}$.

Next, we obtain the fingertip stiffness elements by solving the linear programming problem given in (52). After that, by substituting the fingertip stiffness elements in (7), the coupling stiffness element $K_{op}$ can be determined by

$$K_{op} = [B_f]K_f,$$

where $[B_f]$ denotes the third row of $[B_f]$.

Also, the proper procedure of determining the joint stiffness is needed to successfully implement the fingertip stiffness $[K_f]$ and an algorithm for obtaining the joint stiffness is described in [11].