On the Robustness and Performance of Disturbance Observers for Second-Order Systems

Youngjin Choi, Kwangjin Yang, Wan Kyun Chung,
Hong Rok Kim, Il Hong Suh

Abstract— **The disturbance observer (DOB) has been widely utilized for high precision and high speed motion control applications. In this paper, we suggest the robustness measure of DOB, as a criterion to design the robust DOB systems. Also, we suggest its design guidelines especially for second-order systems. Experimental results for an optical disk drive system show the validity of design guidelines.**

Index Terms— **Disturbance Observer(DOB), Robustness Measure, Optical Disk Drive(ODD) System**

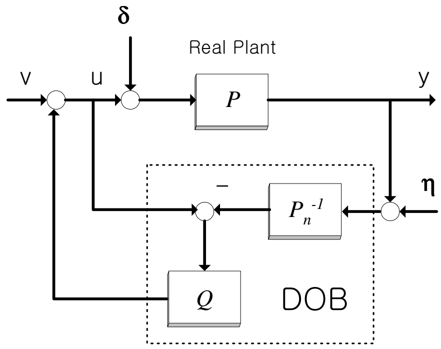


Fig. 1. The structure of disturbance observer (DOB)

I. Introduction

Most control systems have their own disturbance sources according to their structure and objective. For instance, the optical disk drive (ODD) system, *e.g.*, CD or DVD player, has various disturbances such as the disk surface vibration, disk eccentric vibration and resonances caused by actuator itself. These disturbance sources have been obstacles to the development of high speed ODD system[1], [2], [3]. For these cases, the disturbance observer (DOB) can be a good alternative in rejecting the effect of disturbance acting on ODD system. The concept of DOB is that the disturbance can be compensated efficiently by feedback of the observed disturbance[4], [5]. Actually, many experimental results in [3], [4], [5] showed that the performance against disturbances could be improved thanks to the DOB.

Many articles of [6], [7], [8], [9], [10], [11], [12] dealt with the design methods of robust DOB. Especially, the doubly coprime factorization was used to design the robust DOB in [10], [11], [12]. In this paper, we propose a robustness measure, which expresses quantitatively the degree of robustness acquired by DOB. In DOB systems, the low pass filter $Q(s)$ is essentially utilized for the causality and it determines several characteristics of DOB system. The time constant of Q filter is related to the disturbance rejection performance[4], [5], [11], [12]. Also, the relationship between the order of Q filter and the disturbance rejection performance was revealed partially in [10], [13], [14]. However, we do not still have systematic design guidelines for Q filter. Especially, we suggest six design guidelines for second-order systems: two guidelines are for the robustness, next two for the disturbance rejection performance and last two for the sensor noise effect.

For future notations, the Hardy space of stable and proper function is expressed by $\mathcal{RH}_\infty$, which denotes an analytic function in the right half region of complex plane. Also, the coprime factorization of SISO plant, *e.g.*, $P_n(s) = M^{-1}(s)N(s)$, is called a normalized coprime factorization in [15], [16], if $[\ M(s)\ N(s)\ ]$ has the norm preserving property, *e.g.*, $\|A(s)\,[\ M_l(s)\ \ N_l(s)\ ]\,\|_\infty = \|A(s)\|_\infty$. In section II, we will derive the robustness measure of DOB system using the norm preserving property of normalized coprime factorization. Additionally, we will suggest the design guidelines of DOB for second-order systems. In section III, experimental results will demonstrate the validity of these guidelines and section IV draws the conclusion.

Y. Choi is with Intelligent System Control Research Center, Korea Institute of Science and Technology (KIST), Seoul, KOREA, Fax: +82-2-958-5749 (E-mail: yjchoi@postech.edu)

K. Yang is with Department of Mechanical Engineering, Korea Air Force Academy, Chungju, KOREA, (E-mail: kjyang@afa.ac.kr)

W. K. Chung is with Robotics and Bio-Mechatronics Laboratory, Pohang University of Science and Technology (POSTECH), Pohang, KOREA, Fax: +82-54-279-5899 (E-mail: wkchung@postech.edu)

H. R. Kim and I. H. Suh are with the Intelligent Control and Robotics Laboratory, Hanyang University, KOREA, (E-mail : hrkim@incorl.hanyang.ac.kr and ihsuh@email.hanyang.ac.kr)

II. Robustness and Performance of DOB

The DOB has been generally used as a part of controller compensating for disturbances. Also, the DOB has the property of model shaping such as it forces the input-output behavior of real plant to follow that of nominal plant. To begin with, let us consider the real plant as following form:

$$P(s) = (M(s) - \Delta_M(s))^{-1}(N(s) + \Delta_N(s)), \tag{1}$$

where $M(s), N(s) \in \mathcal{RH}_\infty$ are normalized coprime factors and $\Delta_N, \Delta_M \in \mathcal{RH}_\infty$ are coprime factor uncertainties. As a matter of fact, it is difficult to identify the real plant exactly since the control system has many uncertain components such as friction nonlinearity, plant parameter perturbations and so on. Here, coprime factor uncertainties Δ_N, Δ_M were introduced into (1) as the expression for uncertainties of the physical system. The robustness measure against these uncertainties will be proposed in the following section.

A. Robustness of DOB

Let us assume that the real plant of (1) is controllable and observable, then the nominal plant can be obtained through the modeling for a real plant and it can be factorized as following form:

$$P_n(s) = M(s)^{-1}N(s). \tag{2}$$

The DOB has the form as shown in Figure 1, where Q is the low pass filter, v the main controller output, u the control input, δ the external disturbance, y the plant output and η the sensor noise. If we use the coprime factor representation for a real plant (1) and the nominal plant (2), then the DOB system of Figure 1 can be changed to that of Figure 2 using coprime factors and uncertainties. The DOB structure of Figure 2 has an advantage as proved in [8]: it can be used even for the unstable plant. In Figure 2, the transfer function from three external inputs (v, δ, η) and perturbation (ϕ) to output (y) is obtained as follows:

$$\begin{aligned} y &= M^{-1}Nv + (1-Q)M^{-1}N\delta + (1-Q)M^{-1}\phi - Q\eta \\ &= P_n v + (1-Q)P_n\delta + (1-Q)M^{-1}\phi - Q\eta. \end{aligned} \tag{3}$$

The characteristics of DOB is determined by the filter time constant, numerator order and denominator order (or relative degree) of Q filter. If $|Q| \approx 1$ in low frequencies, then the DOB

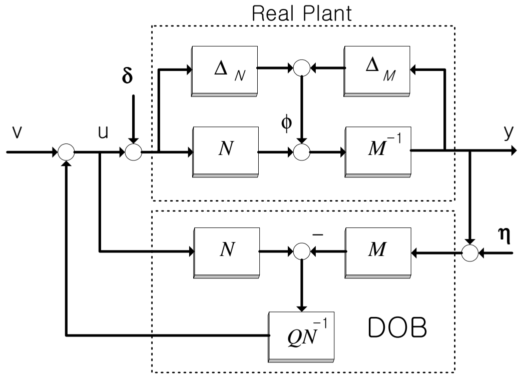


Fig. 2. The DOB system expressed by coprime factors and uncertainties

system (3) can be approximated as the following form:

$$y \approx P_n v - \eta.$$

This is an important characteristics of DOB in low frequencies, in other words, the disturbance(δ) can be rejected, the perturbation(ϕ) does not appear and the real plant $P(s)$ behaves like the nominal plant $P_n(s)$ affected only by the sensor noise(η). For instance, since most mechanical systems do not behave fast due to the inertia effect, if the bandwidth of Q filter can be designed wider than that of the mechanical system, then the DOB system shows a good disturbance rejection performance for most mechanical control systems. However, we can have a doubt whether the DOB may degrade the robustness against perturbation as the trade-off for a good disturbance rejection performance. From now on, we are to derive the robustness measure of DOB against the perturbation from Figure 2. First, let us obtain the perturbation quantity from Figure 2 as following form:

$$\phi = [\Delta_M \quad \Delta_N] \begin{bmatrix} y \\ u+\delta \end{bmatrix}, \qquad (4)$$

and assume that the bound on the uncertainties is as follows:

$$\| [\Delta_M \quad \Delta_N] \|_\infty < \frac{1}{\gamma},$$

where the small γ means large uncertainties and the large γ expresses small uncertainties. However, it is very difficult to calculate the size of uncertainties directly. Second, let us obtain another relation from Figure 2 as follows:

$$\begin{aligned} \begin{bmatrix} y \\ u+\delta \end{bmatrix} &= \begin{bmatrix} P_n \\ 1 \end{bmatrix} v + \begin{bmatrix} (1-Q)M^{-1} \\ -QN^{-1} \end{bmatrix} \phi \\ &+ \begin{bmatrix} (1-Q)P_n \\ (1-Q) \end{bmatrix} \delta - \begin{bmatrix} Q \\ QP_n^{-1} \end{bmatrix} \eta. \quad (5) \end{aligned}$$

Now, if the small gain theorem is applied to (4) and (5), then the robustness bound of DOB can be obtained as the following form:

$$\left\| \begin{bmatrix} (1-Q)M^{-1} \\ -QN^{-1} \end{bmatrix} \right\|_\infty < \gamma.$$

If the norm preserving property of normalized coprime factors is utilized for above equation, *e.g.*, $[\ M \quad N\ ]$ is multiplied by above equation, then we can get the following equation:

$$\sigma_{max} \triangleq \left\| \begin{bmatrix} (1-Q(s)) & (1-Q(s))P_n(s) \\ -Q(s)P_n(s)^{-1} & -Q(s) \end{bmatrix} \right\|_\infty < \gamma. \quad (6)$$

Since equation (6) implies the degree of robustness of DOB system against the perturbation, we define it as "robustness measure of a DOB system" denoted by σ_{max}. In other words, if a small σ_{max} can be achieved by designing an adequate $Q(s)$ filter for a given nominal plant $P_n(s)$, then the DOB system can be stabilized in spite of the large perturbation or uncertainties. Hence, the robustness measure of (6) can guide the design of Q filter that achieves a robust DOB. Actually, there are three important factors in designing a Q filter: the filter time constant, numerator order and denominator order (or relative degree) of Q filter. The filter time constant and orders of Q filter should be designed so that the small σ_{max} can be achieved for a good robustness. These will be explained through the example of second-order system in the following.

Now, we are to use the robustness measure (6) as the design method of a Q filter for second-order systems. First, we assume the Q filter of the following form:

$$Q_{mn}(s) = \frac{\sum_{i=0}^{n} a_{mi}(\tau s)^i}{(\tau s + 1)^m}, \qquad (7)$$

where τ is the filter time constant, $a_{mi} = \frac{m!}{(m-i)!i!}$ the binomial coefficient, m the denominator order and n the numerator order. Hence, $m \geq n+2$. Second, since most mechanical systems can be described by the second order transfer function, we assume that the nominal plant is given as follows:

$$P_n(s) = \frac{k}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \qquad (8)$$

where ζ is a damping ratio, ω_n the undamped natural frequency and k a constant. If the Q_{mn} filter of (7) and the nominal plant of (8) are put in (6), then the robustness measure can be calculated using the MATHEMATICA[1] as follows:

$$(\sigma_{max})_{mn} = \max_{\omega \geq 0} \sqrt{\frac{\left(b_{mn} A \omega_n^{2(m-n)} + k^2 c_{mn}\right)\left(A + \frac{k^2}{\omega_n^4}\right)}{k^2 A \left(\frac{1}{\omega_n^2} + \left(\frac{\tau\omega}{\omega_n}\right)^2\right)^m}}, \qquad (9)$$

where

$$A = \left(\frac{\omega}{\omega_n}\right)^4 + 2(2\zeta^2 - 1)\left(\frac{\omega}{\omega_n}\right)^2 + 1,$$

and $(\sigma_{max})_{mn}, b_{mn}, c_{mn}$ express the robustness measure and coefficients when Q_{mn} filter was used. As a matter of fact, the coefficients b_{mn}, c_{mn} for widely used filters (*e.g.*, $Q_{20}, Q_{30}, Q_{31}, Q_{41}, Q_{42}, Q_{52}$) comply with the general forms suggested in the last row of Table I. However, those of exceptional filters (*e.g.*, $Q_{40}, Q_{50}, Q_{51}, Q_{53}$) do not comply with the general forms and they are arranged separately in Table II.

To perceive the relationship between the robustness and design factors (orders and the filter time constant) of widely used filters in Table I, let us assume that the damping ratio is very small. Then, we can know that the maximum value of (9) is approximately achieved at $\omega \approx \omega_n$, though its maximum value is exactly obtained at the resonance frequency $\omega = \omega_n\sqrt{1-2\zeta^2}$ for $(0 \leq \zeta \leq 0.707)$. At $\omega = \omega_n$, we can easily see that $A = 4\zeta^2$, $b_{mn} = \frac{1}{\omega_n^{2m}} \sum_{i=0}^{n} e_{mi}(\tau\omega_n)^{2i}$ and $c_{mn} = \frac{1}{\omega_n^{2m}} \sum_{i=n+1}^{m} e_{mi}(\tau\omega_n)^{2i}$. Also, if the filter time constant satisfying $\tau < 1/\omega_n$ is determined, then the robustness measure (9) for second-order systems can be approximated as follows:

[1] Symbolic calculation software

TABLE I
COEFFICIENTS OF THE ROBUSTNESS MEASURE FOR Q_{mn} FILTERS

	b_{mn}	c_{mn}
Q_{20}	$\frac{1}{\omega_n^4}$	$\frac{4}{\omega_n^2}\left(\frac{\tau\omega}{\omega_n}\right)^2+\left(\frac{\tau\omega}{\omega_n}\right)^4$
Q_{30}	$\frac{1}{\omega_n^6}$	$\frac{9}{\omega_n^4}\left(\frac{\tau\omega}{\omega_n}\right)^2+\frac{3}{\omega_n^2}\left(\frac{\tau\omega}{\omega_n}\right)^4+\left(\frac{\tau\omega}{\omega_n}\right)^6$
Q_{31}	$\frac{1}{\omega_n^6}+\frac{9}{\omega_n^4}\left(\frac{\tau\omega}{\omega_n}\right)^2$	$\frac{9}{\omega_n^2}\left(\frac{\tau\omega}{\omega_n}\right)^4+\left(\frac{\tau\omega}{\omega_n}\right)^6$
Q_{41}	$\frac{1}{\omega_n^8}+\frac{16}{\omega_n^6}\left(\frac{\tau\omega}{\omega_n}\right)^2$	$\frac{36}{\omega_n^4}\left(\frac{\tau\omega}{\omega_n}\right)^4+\frac{4}{\omega_n^2}\left(\frac{\tau\omega}{\omega_n}\right)^6+\left(\frac{\tau\omega}{\omega_n}\right)^8$
Q_{42}	$\frac{1}{\omega_n^8}+\frac{4}{\omega_n^6}\left(\frac{\tau\omega}{\omega_n}\right)^2+\frac{36}{\omega_n^4}\left(\frac{\tau\omega}{\omega_n}\right)^4$	$\frac{16}{\omega_n^2}\left(\frac{\tau\omega}{\omega_n}\right)^6+\left(\frac{\tau\omega}{\omega_n}\right)^8$
Q_{52}	$\frac{1}{\omega_n^{10}}+\frac{5}{\omega_n^8}\left(\frac{\tau\omega}{\omega_n}\right)^2+\frac{100}{\omega_n^6}\left(\frac{\tau\omega}{\omega_n}\right)^4$	$\frac{100}{\omega_n^4}\left(\frac{\tau\omega}{\omega_n}\right)^6+\frac{5}{\omega_n^2}\left(\frac{\tau\omega}{\omega_n}\right)^8+\left(\frac{\tau\omega}{\omega_n}\right)^{10}$
Q_{mn}	$\sum_{i=0}^{n}\frac{e_{mi}}{\omega_n^{2(m-i)}}\left(\frac{\tau\omega}{\omega_n}\right)^{2i}$ if $i=n$, $e_{mi}=a_{mi}^2$ else, $e_{mi}=a_{mi}$	$\sum_{i=n+1}^{m}\frac{e_{mi}}{\omega_n^{2(m-i)}}\left(\frac{\tau\omega}{\omega_n}\right)^{2i}$ if $i=n+1$, $e_{mi}=a_{mi}^2$ else, $e_{mi}=a_{mi}$

TABLE II
COEFFICIENTS OF THE ROBUSTNESS MEASURE FOR EXCEPTIONAL FILTERS

	b_{mn}	c_{mn}
Q_{40}	$\frac{1}{\omega_n^8}$	$\frac{16}{\omega_n^6}\left(\frac{\tau\omega}{\omega_n}\right)^2+\frac{4}{\omega_n^4}\left(\frac{\tau\omega}{\omega_n}\right)^4+\frac{4}{\omega_n^2}\left(\frac{\tau\omega}{\omega_n}\right)^6+\left(\frac{\tau\omega}{\omega_n}\right)^8$
Q_{50}	$\frac{1}{\omega_n^{10}}$	$\frac{25}{\omega_n^8}\left(\frac{\tau\omega}{\omega_n}\right)^2+\frac{10}{\omega_n^4}\left(\frac{\tau\omega}{\omega_n}\right)^6+\frac{5}{\omega_n^2}\left(\frac{\tau\omega}{\omega_n}\right)^8+\left(\frac{\tau\omega}{\omega_n}\right)^{10}$
Q_{51}	$\frac{1}{\omega_n^{10}}+\frac{25}{\omega_n^8}\left(\frac{\tau\omega}{\omega_n}\right)^2$	$\frac{100}{\omega_n^6}\left(\frac{\tau\omega}{\omega_n}\right)^4+\frac{5}{\omega_n^2}\left(\frac{\tau\omega}{\omega_n}\right)^8+\left(\frac{\tau\omega}{\omega_n}\right)^{10}$
Q_{53}	$\frac{1}{\omega_n^{10}}+\frac{5}{\omega_n^8}\left(\frac{\tau\omega}{\omega_n}\right)^2+\frac{100}{\omega_n^4}\left(\frac{\tau\omega}{\omega_n}\right)^6$	$\frac{25}{\omega_n^2}\left(\frac{\tau\omega}{\omega_n}\right)^8+\left(\frac{\tau\omega}{\omega_n}\right)^{10}$

$$
\begin{aligned}
(\sigma_{max})_{mn} &\approx \sqrt{\frac{\left(\frac{1}{\omega_n^{2m}}\sum_{i=0}^{n} e_{mi}(\tau\omega_n)^{2i}(4\zeta^2)\omega_n^{2(m-n)}+k^2\frac{1}{\omega_n^{2m}}\sum_{i=n+1}^{m} e_{mi}(\tau\omega_n)^{2i}\right)\left(4\zeta^2+\frac{k^2}{\omega_n^4}\right)}{k^2(4\zeta^2)\frac{1}{\omega_n^{2m}}\left(1+(\tau\omega_n)^2\right)^m}} \\
&= \sqrt{\frac{\left((4\zeta^2\omega_n^{2(\boldsymbol{m}-\boldsymbol{n})})\sum_{i=0}^{n} e_{mi}(\boldsymbol{\tau}\omega_n)^{2i}+k^2\sum_{i=n+1}^{m} e_{mi}(\boldsymbol{\tau}\omega_n)^{2i}\right)\left(4\zeta^2+\frac{k^2}{\omega_n^4}\right)}{(4\zeta^2k^2)\left(1+(\boldsymbol{\tau}\omega_n)^2\right)^{\boldsymbol{m}}}}. \qquad (10)
\end{aligned}
$$

To have a robust DOB, the σ_{max} in (10) should be as small as possible. In other words, for the given plant parameters ζ, ω_n, k, we should determine the filter parameters τ, m and n so that σ_{max} can be small for a good robustness. Finally, the following design guidelines can be obtained from the robustness measure above.

(Guideline 1) *The relative degree $(m-n)$ of filter and robustness*
As we can see in equation (10), the smaller the relative degree, the better the robustness. To enhance the robustness of a DOB system, we should choose the Q filter with small relative degree, *e.g.*, the Q_{31} filter makes the DOB system more robust than Q_{30} filter.
(Guideline 2) *The denominator order (m) of filter and robustness*
As shown in (10), the larger the denominator order, the better the robustness. Hence, we should choose a large denominator order to have a robust DOB system. For example, the Q_{31} filter makes the DOB system more robust than Q_{20} filter.

Although the filter time constant (τ) has some relationship with robustness, we can not simply mention its characteristics for robustness. As shown in (10), the filter time constant shows a complex relation, especially for the numerator part of (10).

Remark 1: In [10], we could find a general comment "we can see that the robustness improved as n increases". Actually, this is equivalent to (Guideline 2) since they considered only three Q-filters (Q_{10}, Q_{21}, Q_{32}) for a first order plant. However, they did not mention (Guideline 1) for the robustness.

B. Disturbance Rejection Performance of DOB

Many experiments in [3], [4], [5], [10], [11], [14] have reported a good disturbance rejection performance of the DOB. Now, let us consider the transfer function from the disturbance δ to output y in (3) as follows:

$$G_{\delta y}(s) = (1 - Q(s))P_n(s), \qquad (11)$$

then its magnitude can be calculated using a Q_{mn} filter (7) as follows:

$$|G_{\delta y}(j\omega)_{mn}| = \sqrt{\frac{k^2 c_{mn}}{\left(\frac{1}{\omega_n^2}+\left(\frac{\tau\omega}{\omega_n}\right)^2\right)^m A\omega_n^4}}.$$

Also, let us obtain its decibel magnitude as follows:

$$
\begin{aligned}
& 20\log|G_{\delta y}(j\omega)_{mn}| \\
=\ & 20\log\left|\frac{k}{\omega_n^2}\right| - 20\log\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+\left(2\zeta\frac{\omega}{\omega_n}\right)^2}
\end{aligned}
$$

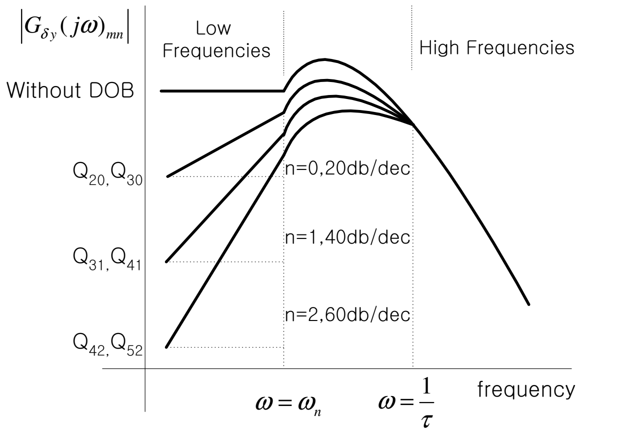


Fig. 3. Disturbance rejection performance according to the numerator order of Q_{mn} filters

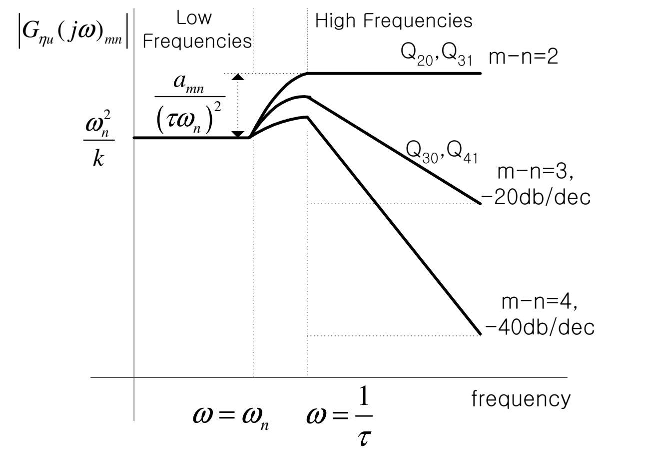


Fig. 4. Sensor noise effect according to the relative degree of Q_{mn} filters

$$+10\log\left\{\frac{e_{m(n+1)}}{\omega_n^{2(m-n-1)}}+\frac{e_{m(n+2)}}{\omega_n^{2(m-n-2)}}\left(\frac{\tau\omega}{\omega_n}\right)^2+\cdots+\left(\frac{\tau\omega}{\omega_n}\right)^{2(m-n-1)}\right\}+20(n+1)\log\left|\frac{\tau\omega}{\omega_n}\right|$$
$$-20m\log\sqrt{\frac{1}{\omega_n^2}+\left(\frac{\tau\omega}{\omega_n}\right)^2}. \quad (12)$$

First, if we approximate the equation (12) in low frequencies such as $\omega \ll \omega_n$, then (12) can be approximately given by:

$$\begin{aligned} & 20\log|G_{\delta y}(j\omega)_{mn}| \\ \approx\ & 20\log\left|\frac{k}{\omega_n^2}\right|+20\log\frac{\sqrt{e_{m(n+1)}}}{\omega_n^{m-n-1}} \\ & +20(n+1)\log\left|\frac{\tau\omega}{\omega_n}\right|-20m\log\left|\frac{1}{\omega_n}\right| \\ =\ & 20\log\left|\frac{k\boldsymbol{a}_{m(n+1)}}{\omega_n^2}\right|+20(\boldsymbol{n}+1)\log|\boldsymbol{\tau}\omega|, \end{aligned} \quad (13)$$

by using $e_{m(n+1)} = a^2_{m(n+1)}$. As we can see in equation (13), the disturbance rejection performance is dependent only on $a_{m(n+1)}$, n and τ in low frequencies. Second, if we approximate the equation (12) in high frequencies such as $\omega \gg 1/\tau$, then $|G_{\delta y}(j\omega)_{mn}|$ recovers the magnitude of nominal plant $|P_n(j\omega)|$, regardless of Q_{mn} filters. Therefore, the plot for function (12) can be approximately obtained like Figure 3. Actually, Figure 3 depicts the disturbance rejection performance according to Q_{mn} filters. Also, since most disturbances exist in low frequencies, the following two design guidelines can be obtained from Figure 3 and equation (13):

(Guideline 3) *The numerator order (n) of filter and disturbance rejection*
As shown in Figure 3, the larger the numerator order, the better the disturbance rejection performance, *e.g.*, the performance of Q_{31} filter is better than that of Q_{30} filter. Additionally, for Q_{mn} filters with same numerator order, the disturbance rejection performance varies a bit according to the values of $a_{m(n+1)}$ as shown in equation (13), *e.g.*, the Q_{20} filter shows somewhat better performance for disturbance rejection than the Q_{30} filter, because $a_{21} = 2$ for Q_{20} and $a_{31} = 3$ for Q_{30}.

(Guideline 4) *The filter time constant (τ) and disturbance rejection*
The filter time constant determines the effective frequency range for disturbance rejection. For instance, small τ means wider frequency range for disturbance rejection. As shown in Figure 3, since the second order plant of (8) has the advantage of $-40db/decade$ roll-off in high frequencies over the natural frequency, we should determine τ satisfying at least $\tau < \frac{1}{\omega_n}$.

Remark 2: It was shown in [10], [13], [14] that the disturbance rejection performance becomes better as the numerator order increases. They explained the reason by analysis for steady-state error. However, they did not consider the effect of coefficient $a_{m(n+1)}$ for disturbance rejection.

C. Sensor Noise Effect of DOB

Since the DOB system has the sensing devices, sensor noises necessarily appear in high frequencies. As a matter of fact, the sensor noise of DOB system affects the control input (u) more seriously than the output (y) because most mechanical systems have the characteristics of low pass filter. Now, let us consider the transfer function from sensor noise (η) to control input (u) in (5) given by:

$$G_{\eta u}(s) = Q(s)P_n(s)^{-1}, \quad (14)$$

then its magnitude is obtained as follows:

$$|G_{\eta u}(j\omega)_{mn}| = \sqrt{\frac{b_{mn}A\omega_n^4}{k^2\left(\frac{1}{\omega_n^2}+\left(\frac{\tau\omega}{\omega_n}\right)^2\right)^m}},$$

and its decibel magnitude is obtained as follows:

$$\begin{aligned} & 20\log|G_{\eta u}(j\omega)_{mn}| \\ =\ & 20\log\frac{\omega_n^2}{k}+20\log\sqrt{1+e_{m1}(\tau\omega)^2+\cdots+e_{mn}(\tau\omega)^{2n}} \\ & +20\log\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+\left(2\zeta\frac{\omega}{\omega_n}\right)^2} \\ & -20m\log\sqrt{1+(\tau\omega)^2}. \end{aligned} \quad (15)$$

First, if we approximate the equation (15) in low frequencies such as $\omega \ll \omega_n$, then $|G_{\eta u}(j\omega)_{mn}|$ becomes a constant $\frac{\omega_n^2}{k}$, irrespective of Q_{mn} filters. Second, we can approximate the equation (15) in high frequencies such as $\omega \gg 1/\tau$ as follows:

$$\begin{aligned} & 20\log|G_{\eta u}(j\omega)_{mn}| \\ \approx\ & 20\log\frac{\omega_n^2}{k}+20\log\sqrt{e_{mn}}(\tau\omega)^n+20\log\left(\frac{\omega}{\omega_n}\right)^2 \end{aligned}$$

$$\begin{aligned} & -20m\log|\tau\omega| \\ = \; & 20\log\frac{\boldsymbol{a}_{mn}}{k\boldsymbol{\tau}^2} - 20(\boldsymbol{m}-\boldsymbol{n}-2)\log|\boldsymbol{\tau}\omega|, \end{aligned} \quad (16)$$

by using $e_{mn} = a_{mn}^2$. The plot for function (15) can be approximately obtained like Figure 4, which explains the effect of sensor noise according to the relative degree. Also, since the sensor noise appears in high frequencies, the following two design guidelines can be obtained from Figure 4 and equation (16):

(Guideline 5) *The relative degree of filter $(m-n)$ and sensor noise*
As shown in Figure 4, the larger the relative degree, the better the performance against the sensor noise, *e.g.*, the Q_{30} filter is less sensitive to the sensor noise than the Q_{31} filter. Additionally, if the relative degree of QP_n^{-1} is zero, then the control input is affected by the noise amplified with the considerable amplification ratio $\frac{a_{mn}}{(\tau\omega_n)^2}$ as shown in Figure 4. Actually, the amplification ratio is proportional to the coefficient a_{mn}, *e.g.*, the Q_{31} filter is somewhat more sensitive to sensor noise than the Q_{20} filter, since $a_{31} = 3$ for Q_{31} and $a_{20} = 1$ for Q_{20}.

(Guideline 6) *The filter time constant (τ) and sensor noise*
The filter time constant affects the amplification ratio $\frac{a_{mn}}{(\tau\omega_n)^2}$ for the sensor noise, *e.g.*, small τ makes the DOB system more sensitive to the sensor noise.

When we are to determine the numerator and denominator orders (n and m) of Q filter, Guidelines 1, 2, 3 and 5 should be considered, especially, Guidelines 1 and 5 contradict each other. Hence, more important guideline should be chosen among six guidelines according to the objective and situation of control system. In the case of filter time constant (τ), since Guideline 4 in low frequencies and Guideline 6 in high frequencies complement each other, they help to design the filter time constant. Till now, we proposed six guidelines for design parameters of Q filter, *e.g.*, the filter time constant, numerator/denominator order and relative degree. In the next section, we will show the validity of guidelines experimentally.

III. Experiments : ODD System

There exist many disturbances in the optical disk drive(ODD) system, the representative and dominant disturbances are classified as the disk surface vibration, eccentricity vibration, radial vibration and resonance. The first influences the focusing servo, the second affects the tracking servo, the third and fourth are caused by the structure of ODD control system. Therefore, the DOB can be utilized to reject these disturbances efficiently. As shown in Figure 5, the real ODD system consists of the following components: the first is the focusing/tracking VCM (Voice Coil Motor) drivers, the second is VCM's and the third is RF amplifier which generates the focusing/tracking errors from optical spots reflected on the optical disk. The VCM driver is second-order transfer function, the VCM is second-order one and the RF Amp is first-order one. Hence, the real ODD system is fifth-order plant. However, since the bandwidth of VCM is smaller than those of RF Amp and driver, we can approximate fifth-order real plant by a second-order nominal plant. For instance, the nominal plant for Samsung 12X DVD tracking servo system is described as follows:

$$P_n(s) = \frac{5\times 193600\times 6880}{s^2 + 2(0.158)(440)s + (440)^2}, \ [\text{Volt/Volt}] \quad (17)$$

where 5 is the DC gain of driver, 193600 is that of VCM, 6880 is that of RF, $\zeta = 0.158$ (the damping ratio of VCM) and $\omega_n = 440$ (undamped natural frequency of VCM).

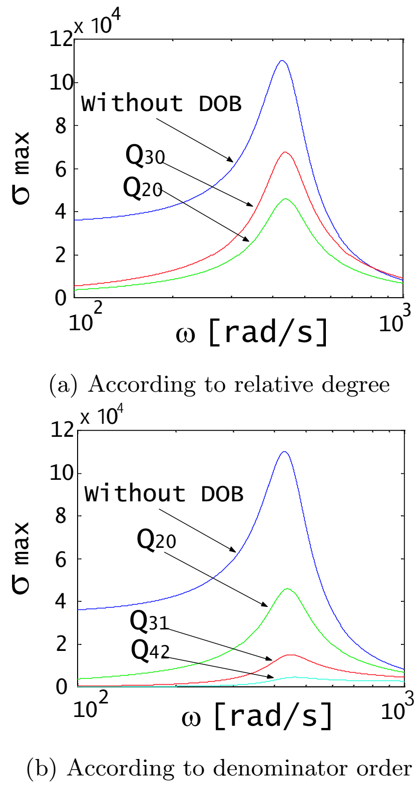


Fig. 6. Robustness comparison according to Q_{mn} filters

Figure 6 depicts maximum singular values of (6) according to frequencies. The robustness measure corresponds to the maximum among maximum singular values for each Q_{mn} filter. As we suggested in Guidelines 1 and 2, the robustness becomes better as the relative degree decreases and as the denominator order increases with the same relative degree.

The DOB was realized as a 320C44 DSP Chip (manufactured by TI Co.). We added it to the original servo as shown in Figure 5. In experiments, we set the filter time constant $\tau = 0.002$. As a matter of fact, the filter time constant was determined by considering both Guideline 4 for the disturbance rejection and Guideline 6 for the sensor noise. Also, a $150\mu m$ eccentric DVD disk (manufactured by ALMEDIO Co.) was used to generate the external disturbance (eccentric vibration). The eccentric vibration has the form of periodic function because it is generated whenever the eccentric disk is rotated in an ODD system. First, the experimental result without DOB is shown in Figure 7.(a). Second, the experimental results using DOB are obtained as Figure 7.(b),(c),(d) for Q_{20}, Q_{31}, Q_{42} filters, respectively. In Figure 7, we should note that there exists the bias of about 1.82 [volt] and the spikes appear whenever the optical spot returns to the given original address on the rotating disk for the task of maintaining given address. This is a problem peculiar to an ODD systems, because CD/DVD disk has a spiral data structure.

As we can see in Figure 7, the disturbance rejection performance becomes better as the numerator order increases like $Q_{20} \to Q_{31} \to Q_{42}$. This agrees with Guideline 3. Though Q_{42} DOB shows the best disturbance rejection performance in Figure 7, it becomes the most sensitive to sensor noise because the amplification ratios for sensor noise become 1.29 for Q_{20}, 3.87 for Q_{31} and 7.74 for Q_{42} according to Guideline 5.

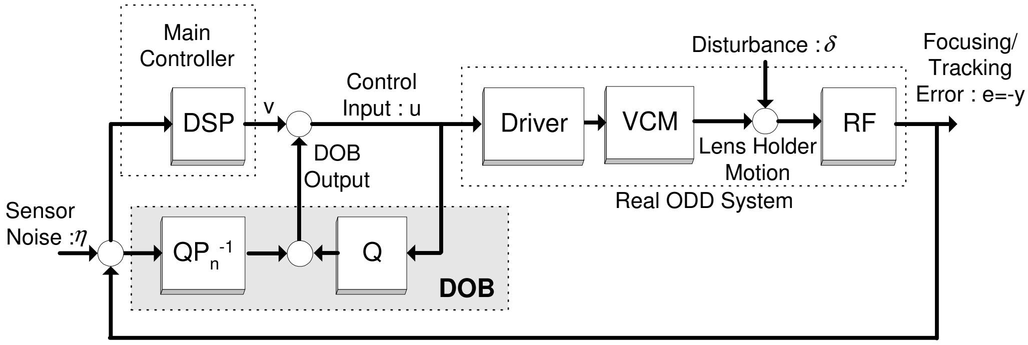
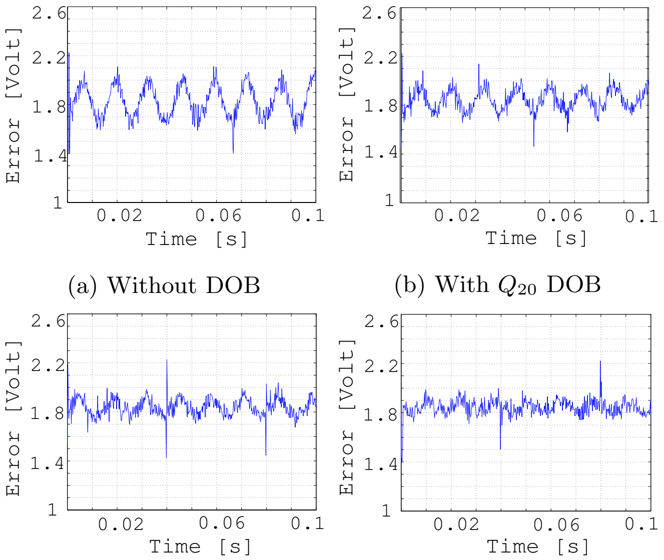


Fig. 5. The DOB and main control system for an ODD system



(a) Without DOB (b) With Q_{20} DOB

(c) With Q_{31} DOB (d) With Q_{42} DOB

Fig. 7. Tracking servo performance according to numerator order when $150\mu m$ eccentric DVD disk was used

IV. Concluding Remarks

A robustness measure was proposed quantitatively for DOB systems. Also, the disturbance rejection performance and the sensor noise effect of DOB were analytically analyzed for the second order plant. For the good performance and robustness of DOB system, we suggested six design guidelines for a Q filter and showed their validity through experiments.

References

[1] K. Fujiyama, M. Tomizuka, and R. Katayama, "Digital tracking controller design for CD player using disturbance observer," *Int. Workshop on Advanced Motion Control*, pp. 598–603, 1998.

[2] K. Ohishi and K. Kudo, "Robust high speed tracking servo system for optical disk system," *Int. Workshop on Advanced Motion Control*, pp. 92–97, 2000.

[3] K. Yang, Y. Choi, W. K. Chung, I. H. Suh, and S. R. Oh, "Robust tracking control of optical disk drive systems using error based disturbance observer and its performance measure," *Proc. of the American Control Conference*, pp. 1395–1400, 2002.

[4] M. T. White, M. Tomizuka, and C. Smith, "Improved track following in magnetic disk drives using a disturbance observer," *IEEE/ASME Trans. on Mechatronics*, vol. 5, no. 1, pp. 3–11, 2000.

[5] A. Tesfaye, H. S. Lee, and M. Tomizuka, "Sensitivity optimization approach to design of a disturbance observer in digital motion control systems," *IEEE/ASME Trans. on Mechatronics*, vol. 5, no. 1, pp. 32–38, 2000.

[6] E. Schrijver and J. van Dijk, "On the design of robust disturbance observers for mechatronic systems," *in Mechatronic Systems, R. Isermann, published for the International Federation of Automatic Control by Pergamon, Oxford, UK*, pp. 819–824, 2001.

[7] B. A. Güvencc and L. Güvencc, "Robustness of disturbance observers in the presence of structural real parametric uncertainty," *Proc. of the American Control Conference*, pp. 4222–4227, 2001.

[8] Y. Choi, W. K. Chung, and Y. Youm, "Disturbance observer in $\mathcal{H}_\infty$ frameworks," *Int. Conf. on Industrial Electronics, Control, and Instrumentation*, pp. 1394–1400, 1996.

[9] T. Mita, M. Hirata, K. Murata, and H. Zhang, "$\mathcal{H}_\infty$ control versus disturbance-observer-based control," *IEEE Trans. on Industrial Electronics*, vol. 45, no. 3, pp. 488–495, 1998.

[10] T. Umeno and Y. Hori, "Robust speed control of DC servomotors using modern two degrees-of-freedom controller design," *IEEE Trans. on Industrial Electronics*, vol. 38, no. 5, pp. 363–368, 1991.

[11] T. Umeno, T. Kaneko, and Y. Hori, "Robust servosystem design with two degrees of freedom and its application to nevel motion control of robot manupulators," *IEEE Trans. on Industrial Electronics*, vol. 40, no. 5, pp. 473–485, 1993.

[12] K. Ohishi, T. Miyazaki, and Y. Nakamura, "High performance ultra low speed range based on doubly coprime factorization and instantaneous speed observer," *IEEE/ASME Trans. on Mechatronics*, vol. 1, no. 1, pp. 89–98, 1996.

[13] K. Yamada, S. Komada, M. Ishida, and T. Hori, "Characteristics of servo system using high order disturbance observer," *IEEE Conf. on Decision and Control*, pp. 3252–3257, 1996.

[14] S. Komada, N. Machii, and T. Hori, "Control of redundant manipulators considering order of disturbance observer," *IEEE Trans. on Industrial Electronics*, vol. 47, pp. 413–420, 2000.

[15] K. Glover and D. McFarlane, "Robust stabilization of normalized coprime factor plant description with $\mathcal{H}_\infty$-bounded uncertainty," *IEEE Trans. on Automatic Control*, vol. 34, pp. 821–830, 1989.

[16] K. Zhou, J. C. Doyle, and K. Glover, *Robust and Optimal Control*, Prentice Hall, 1995.