A Guideline for Specifying Compliance in Multi-Fingered Operations

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In this paper, we present a fundamental compliance analysis for multi-fingered hands and also provide a guideline for specifying compliance characteristics in the three-dimensional operational space of multi-fingered hands. Through the analysis of the stiffness relation between the operational space and the fingertip space of multi-fingered hands, it is shown that some of the coupling stiffness elements cannot be planned arbi-

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trarily. Also, an independent finger-based compliance control method to achieve the given compliance characteristics is described. Conclusively, when we specify the operational stiffness matrix, the contents of the operational stiffness matrix cannot be fully achieved by influence of the grasp geometry of multi-fingered hands. The grasp positions of fingers as well as the RCC point, which are shown important factors, should be chosen to achieve the specified operational stiffness matrix for the given task. The cases of a point contact with friction and soft contact are examined as illustrative examples. It is observed that a five-fingered robotic hand is sufficient for implementation of proper three-dimensional compliance characteristics.

1. INTRODUCTION

Related to grasping and manipulation of an object by using robot hands or multiple robot arms, many researchers have proposed explicit force-based control methods.[1–8] However, implementation of those methods may not be easy practically because it is not only hard to employ force sensors in finger mechanisms, but also problematic to successfully process noisy force signals. Also, the integration of tactile and force information for individual finger control, and the combination of information from different fingers to guide the hand action, are still not well known.[9] Therefore, it has been pointed out that instead of employing force signals, stiffness or compliance can be used as successful alternatives for characterizing the grasping and manipulation of robot hands.[10,11] Importantly, when an object grasped by a multi-fingered hand is being manipulated in constrained space, the appropriate stiffness planning of the hand is very important for successful compliant tasks.[12–14]

Many approaches have been reported in the field of grasp stiffness or compliance.[15–21] Shimoga[15] summarized the conventional grasp synthesis algorithms for robot hands. The stiffness of objects grasped by virtual springs was analyzed in cases of planar and three-dimensional space.[16] Yokoi et al.[17] proposed a direct compliance control method and applied the method to a parallel arm. Cutkosky et al.[10] analyzed the effective grasp stiffness by considering the structural compliances in fingers and fingertips, servo gains at the joints of finger, and small changes in the grasp geometry that may affect the grasp forces acting upon the object. In ref. 18, it is pointed out that a stiffness matrix containing some off-diagonal terms can be useful to prevent the jamming of contact tasks. Li et al.[19] classified a grasp matrix into symmetric and antisymmetric parts. An experimental investigation was performed for successful object stiffness control using a multi-fingered robot hand.[20] Also, Kao et al.[11] tried to apply stiffness models usually employed in robotics research to the analysis of human grasping behaviors. Recently, Kim et al.[21] proposed an independent finger/joint-based compliance control method for robot hands manipulating an object, and also the geometric condition for successful implementation of a compliance control scheme has been addressed. They showed that an independent finger/joint-based compliance control via redundant actuation was more adequate to modulate the operational stiffness in comparison to the case of the kinematically redundant structured fingers or manipulators. Some approaches to achieve the desired stiffness characteristics have been presented based on grasp geometry[22,23] and task-based stiffness characteristics.[24–26] However, it is still an open research field.

Thus, the main objective of this paper is to analyze the achievable contents of operational compliance characteristics for compliant manipulation of an object grasped by multi-fingered robot hands and provide a guideline of specifying compliance characteristics in the operational space of multi-fingered hands. This paper is organized as follows. In Section 2, we describe a two-step stiffness relation among the operational space, the joint space, and the fingertip space of multi-fingered hands. Then, we analyze the geometric conditions for successful implementation of the compliance control in the object operational space. In Section 3, we provide a necessary condition for independent finger-based compliance control of robot hands in terms of the number of fingers and contact types. Also, an independent finger-based compliance control method for multi-fingered robot hands is presented. In Section 4, through analyzing the stiffness relation between the operational space and the fingertip space of multi-fingered hands, we provide a guideline of specifying compliance characteristics in the operational space of multi-fingered hands. Also, it is shown that a five-fingered hand is necessary to implement a 6×6 operational stiffness matrix by using the independent finger-based compliance control method. Finally, concluding remarks are drawn in Section 5.

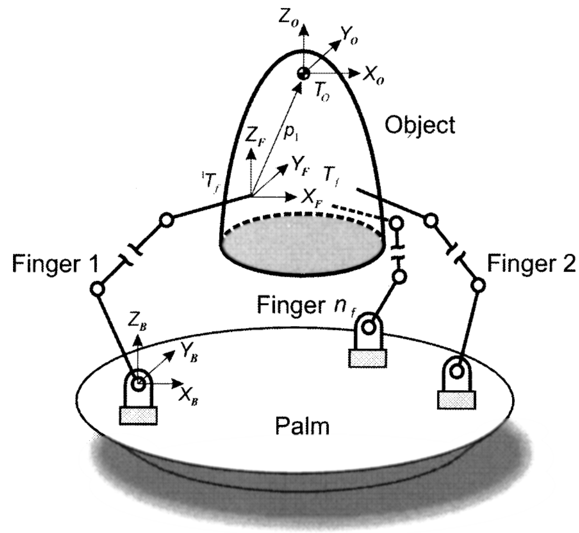

Figure 1. A multi-fingered robot hand.

2. STIFFNESS RELATION OF MULTI-FINGERED HAND

The stiffness or compliance can be employed for characterizing the grasping and manipulation of robot hands in the case that it is specially dominated in approximated linear analysis where low velocities and small relative motions lead to small inertial forces. In this section, we present the stiffness relation between the operational space and the fingertip space.

Consider a rigid object being manipulated by a n_f-fingered robot hand as shown in Figure 1, where each finger has ${}^in_j$-joints, and the relation between the generalized force vector in the operational space and the fingertip force vector is given by

$$T_o=[\mathbf{G}_o^f]^T T_f, \tag{1}$$

where $T_o \in \mathcal{R}^{n\times 1}$ denotes the generalized force vector in the operational space (o) including the inertial load and external load. The fingertip force vector $T_f \in \mathcal{R}^{m\times 1}$ in the fingertip space (f) is expressed as

$$T_f=[({}^1T_f)^T \ \ ({}^2T_f)^T \ \ \cdots \ \ ({}^{n_f}T_f)^T]^T$$

and the Jacobian matrix relating the operational space to the fingertip space $[\mathbf{G}_o^f] \in \mathcal{R}^{m\times n}$ is given by

$$[\mathbf{G}_o^f]=[[{}^1\mathbf{G}_o^f]^T \ \ [{}^2\mathbf{G}_o^f]^T \ \ \cdots \ \ [{}^{n_f}\mathbf{G}_o^f]^T]^T,$$

$$[{}^i\mathbf{G}_o^f]=\begin{bmatrix} {}^f_o\mathbf{R}_i & {}^f_op_i \times {}^f_o\mathbf{R}_i \\ 0 & {}^f_o\mathbf{R}_i \end{bmatrix}.$$

Here, ${}^f_o\mathbf{R}_i$ and ${}^f_op_i$ denote the rotation matrix and the position vector from the operational space to the fingertip space, respectively. Also, m ($m=\Sigma_{i=1}^{n_f}{}^in_{fp}$, where ${}^in_{fp}$ denotes the dimension of the ith fingertip) denotes the total dimension of wrenches applied to the grasped object by n_f fingers.

When the trajectory of the grasped object is prespecified, the task of load distribution will be the determination of the fingertip forces and moments in order to achieve a desired motion of the object and to maintain the grasp. The general solution of (1) is given by

$$T_f=([\mathbf{G}_o^f]^T)^+T_o+(\mathbf{I}-([\mathbf{G}_o^f]^T)^+[\mathbf{G}_o^f]^T)\xi_f, \tag{2}$$

where the superscript T implies the transpose of a matrix, $([\mathbf{G}_o^f]^T)^+$ is a pseudo-inverse of $[\mathbf{G}_o^f]^T$, and ξ_f is an arbitrary $m\times 1$ vector. $\mathbf{I}$ denotes an $m\times m$ identity matrix.

According to (2), we need three fingers to control 6×1 operational force and 3×1 internal force vectors under the assumption that the contact type is in point contact with friction. However, the number of required fingers will be different for the case of compliance control.

Using (2), we can perform explicit force control of robot hands by using a force sensor signal, but the fine finger motion control is practically hard because force measurement at the fingertip is not easy and the real force signal is very noisy. As an alternative, a stiffness or compliance control method can be usefully applied to the robot hand instead of using an explicit force control method.

By taking the partial derivative of (1) with respect to the 6×1 displacement vector u_o in the operational space and including the effect of the change of contact configuration, the stiffness matrix in the operational space can be expressed as follows:

$$\begin{aligned}[\mathbf{K}_o'] &= -\frac{\partial T_o}{\partial u_o} = -[\mathbf{G}_o^f]^T\frac{\partial T_f}{\partial u_f}\frac{\partial u_f}{\partial u_o} - \frac{\partial[\mathbf{G}_o^f]^T}{\partial u_o}T_f \\ &= [\mathbf{G}_o^f]^T[\mathbf{K}_f][\mathbf{G}_o^f] - \frac{\partial[\mathbf{G}_o^f]^T}{\partial u_o}T_f \\ &= [\mathbf{G}_o^f]^T[\mathbf{K}_f][\mathbf{G}_o^f] - ([T_f]^T \circ [\mathbf{H}_{oo}^f]),\end{aligned} \tag{3}$$

where $[\mathbf{K}_f]$ represents the $m\times m$ stiffness matrix in the fingertip space. The operator of ($\circ$) and $[\mathbf{H}_{oo}^f]$ represent the generalized scalar dot product[27] and the second-order kinematic influence coefficient matrix which represents the change of $[\mathbf{G}_o^f]$ with respect to contact configuration,[28] respectively.

Also, (3) is rearranged as

$$[\mathbf{K}_o]=[\mathbf{K}_o']+([T_f]^T\circ[\mathbf{H}_{oo}^f])=[\mathbf{G}_o^f]^T[\mathbf{K}_f][\mathbf{G}_o^f], \quad (4)$$

where $[\mathbf{K}_f]$ representing the $m\times m$ stiffness matrix in the fingertip space is expressed as

$$[\mathbf{K}_f]=\begin{bmatrix} {}^1\mathbf{K}_f & {}^{12}\mathbf{K}_f & \cdots & {}^{1n_f}\mathbf{K}_f \\ {}^{21}\mathbf{K}_f & {}^2\mathbf{K}_f & \cdots & {}^{2n_f}\mathbf{K}_f \\ \vdots & \vdots & \ddots & \vdots \\ {}^{n_f1}\mathbf{K}_f & {}^{n_f2}\mathbf{K}_f & \cdots & {}^{n_f}\mathbf{K}_f \end{bmatrix},$$

in which $[{}^i\mathbf{K}_f]$ $(i=1,\ldots,n_f)$ is given by

$$[{}^i\mathbf{K}_f]=\begin{bmatrix} {}^i\mathbf{K}_{fxx} & {}^i\mathbf{K}_{fxy} & {}^i\mathbf{K}_{fxz} \\ {}^i\mathbf{K}_{fyx} & {}^i\mathbf{K}_{fyy} & {}^i\mathbf{K}_{fyz} \\ {}^i\mathbf{K}_{fzx} & {}^i\mathbf{K}_{fzy} & {}^i\mathbf{K}_{fzz} \end{bmatrix},$$

specifically for the case of a point contact with friction, and ${}^{ij}\mathbf{K}_f$ in $[\mathbf{K}_f]$ denotes the interfinger coupling stiffness matrix between the ith finger and the jth finger. It is noted from (4) that the second term is not utilized in Salisbury's algorithm.[29] This becomes crucial when the magnitudes of the grasping forces T_f are considerable.[28,30,31] $[\mathbf{K}_o]$ is also symmetric since the second term of (4) becomes symmetric when linear fingertip forces are applied.

Next, we describe the stiffness relation between the fingertip space and the joint space. It is associated with the structure of a finger of the hand. With a closed-loop finger, the stiffness matrix ${}^i\mathbf{K}_f'({}^in_{fp}\times{}^in_{fp})$ in the ith fingertip space including the effect of the change of joint configuration can be represented as follows:[30]

$$[{}^i\mathbf{K}_f']=[{}^i\mathbf{G}_f^q]^T[{}^i\mathbf{K}_q][{}^i\mathbf{G}_f^q]-([{}^iT_q]^T\circ[{}^i\mathbf{H}_{ff}^q]), \quad (5)$$

and we define

$$[{}^i\mathbf{K}_f]=[{}^i\mathbf{K}_f']+([{}^iT_q]^T\circ[{}^i\mathbf{H}_{ff}^q])=[{}^i\mathbf{G}_f^q]^T[{}^i\mathbf{K}_q][{}^i\mathbf{G}_f^q], \quad (6)$$

where $[{}^i\mathbf{K}_q]$ represents the ${}^in_j\times{}^in_j$ stiffness matrix in the joint space (q) of each finger, $[{}^i\mathbf{G}_f^q]$ denotes a Jacobian (i.e., backward mapping) relating the joint space to the fingertip space, and $[{}^i\mathbf{H}_{ff}^q]$ represents the second-order kinematic influence coefficient matrix induced by the change of joint configuration described in ref. 27, and more detail for the second-order kinematic influence coefficient matrix is described in refs. 28 and 37.

The number of finger-joints and the number of actuators in the joint space should be taken into account to realize the calculated fingertip stiffness characteristic given by (6). First, we assume that we employ a finger having four degrees of freedom since most of robotic fingers have been designed to have four degrees of freedom by taking into account the structure of the human finger.

Also, recently,[30,32] a rule similar to Tables I and II has been derived to resolve the compliance from the fingertip space to the joint space. The number of actuators in the finger space should be more than the number of independent compliance elements in the fingertip compliance matrix. Thus, a parallel or hybrid structure will be beneficial for a robotic finger to

Table I. Necessary condition for stiffness control in two-dimensional cases.

Finger	(*a*)	(*b*)	(*c*)	(*d*)	Remarks
2	6	6	10	−2	$[\mathbf{K}_o]_{(3\times3)}$
	3	6	10	1	$[\mathbf{K}_o]_{(2\times2)}$
3	6	15	21	0	
4	6	28	36	2	$[\mathbf{K}_o]_{(3\times3)}$
5	6	45	55	4	

Table II. Necessary condition for stiffness control in three-dimensional cases.

Finger	Contact type	(*a*)	(*b*)	(*c*)	(*d*)	Remarks
2	Point	21	15	21	−15	
	Soft	21	28	36	−13	
3	Point	21	36	45	−12	
	Soft	21	66	78	−9	
4	Point	21	66	78	−9	
	Soft	21	120	136	−5	$[\mathbf{K}_o]_{(6\times6)}$
5	Point	21	105	120	−6	
	Soft	21	190	210	−1	
6	Point	21	153	171	−3	
	Soft	21	276	300	3	
7	Point	21	210	231	0	
	Soft	21	378	406	7	

satisfy this requirement since those structures have many potential input locations where actuators can be attached. In ref. 32, a planar five-bar finger mechanism involving two redundant actuators was employed to realize the proposed compliance control scheme.

3. A COMPLIANCE CONTROL METHOD FOR MULTI-FINGERED HANDS

In this section, we provide a necessary condition for independent finger-based compliance control of robot hands in terms of the number of fingers and contact types. We also describe an independent finger-based compliance control without interfinger coupling so as to achieve the desired stiffness in the operational space.

3.1. Necessary Finger Condition for Compliance Control

In a robot hand system, the components of the wrench transmitted through the contact between the fingertip and the contact point of the object are limited by the contact constraint defined according to the contact types.[33] It is therefore very important to determine the number of fingers and the number of joints for implementation of the desired compliance characteristic given in the operational space.

In a common robot hand system, there exists interfinger coupling. If the interfinger coupling can be eliminated, each finger can be independently controlled which makes the hand control relatively easy. For this, it is useful to rearrange (4) as follows:

$$\begin{bmatrix} K_{oo} \\ \vec{0} \end{bmatrix} = \begin{bmatrix} \mathbf{M}_1 \\ \mathbf{M}_2 \end{bmatrix} K_{ff}, \tag{7}$$

where K_{oo} and K_{ff} denote $n(n+1)/2\times 1$ and $m(m+1)/2\times 1$ vectors, consisting of independent elements of $[\mathbf{K}_o]$ and $[\mathbf{K}_f]$, respectively. $\mathbf{M}_1$ denotes the $n(n+1)/2\times m(m+1)/2$ matrix relating the independent elements of the object stiffness matrix to those of the fingertip stiffness matrix, and $\mathbf{M}_2$ denotes the $m(m-1)/2\times m(m+1)/2$ selection matrix which relates all the off-block diagonal elements of $[\mathbf{K}_f]$ to those of the fingertip stiffness matrix. The vector $\vec{0}$ is the $m(m-1)/2\times 1$ zero vector corresponding to the off-block diagonal elements of $[\mathbf{K}_f]$. Thus, the number of fingers for successful implementation of the stiffness control in the operational space can be analyzed from (7).

Table I shows the number of fingers required to implement the corresponding stiffness characteristic in the two-dimensional space, where the contact type is assumed to be point contact with friction. Also, the number of fingers required to implement the corresponding stiffness characteristic in the three-dimensional space is shown in Table II, where two contact types, such as point contact with friction (Point) and soft finger contact (Soft), are discussed.

In Tables I and II, (a), (b), and (c) denote the number of independent elements of $[\mathbf{K}_o]$, the number of finger coupling elements of $[\mathbf{K}_f]$, and the number of independent elements of $[\mathbf{K}_f]$, respectively, while $(d)[=(c)-(a)-(b)]$ represents the remaining degree of freedom for $[\mathbf{K}_f]$. In other words, if (d) is greater than 0, it implies that the hand with the specified fingers can fully implement the defined compliance characteristics in the operational space. That is, the term of (a) means the degree of freedom for implementing the compliant characteristics desired in the operational space. The term of (c) represents the degree of freedom in the fingertip space for the implementation and also it represents practically the number of drivers needed in the joint space. Specifically, the case of a soft finger needs more drivers that of point contact type. For example, consider a planar two-fingered hand. For a point contact with friction, the specified 3×3 object stiffness matrix consists of six independent elements, and the dimension of the fingertip stiffness matrix is 4×4, which has ten independent stiffness elements, and six coupling elements exist among them. Thus, we can notice that the dimensions of $\mathbf{M}_1$ and $\mathbf{M}_2$ are both 6×10. This implies that ten input parameters are not enough to solve for the 12 equations given in (7). It is therefore confirmed that a two-fingered hand cannot implement 3×3 object stiffness characteristic in the two-dimensional space. Hence, a robot hand should have at least three fingers to implement 3×3 object stiffness characteristic in the two-dimensional space as shown in Table I. For the three-dimensional case, we can see that seven fingers are needed to implement 6×6 object stiffness characteristic in the case of point contact with friction. On the other hand, Table II shows that six fingers are required to implement 6×6 object stiffness characteristic in the case of the soft finger. As a general rule, full stiffness implementation is possible when (d) in Table II is equal to or greater than 0.

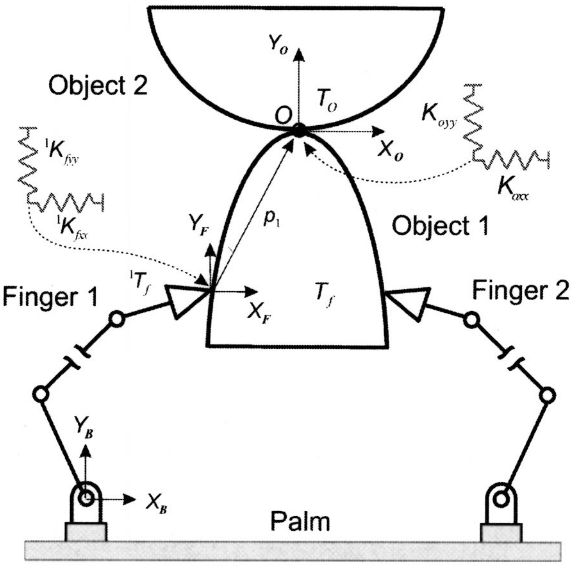

Figure 2. Compliance control by two fingers.

3.2. Independent Finger-Based Compliance Control

For convenience of describing the independent finger-based compliance control, we particularly consider the case of a two-fingered hand in two-dimensional space, where its contact type is assumed a point contact with friction and slip at the contact point is ignored. The operational stiffness matrix is assumed to be symmetric. And then general formulation is abridged.

When a two-fingered hand manipulates an object in the three-dimensional space as shown in Figure 2, let the desired 2×2 object stiffness matrix in the operational space(o) be given as

$$[\mathbf{K}_o]=\begin{bmatrix}\mathbf{K}_{oxx} & \mathbf{K}_{oxy}\\ \mathbf{K}_{oyx} & \mathbf{K}_{oyy}\end{bmatrix}. \tag{8}$$

According to the necessary finger condition (two fingers) for stiffness control, this two-fingered system satisfies the necessary condition.[21] The stiffness matrix in the fingertip space is generally represented by

$$[\mathbf{K}_f]=\begin{bmatrix}{}^1\mathbf{K}_{fxx} & {}^1\mathbf{K}_{fxy} & {}^{12}\mathbf{K}_{fxx} & {}^{12}\mathbf{K}_{fxy}\\ {}^1\mathbf{K}_{fyx} & {}^1\mathbf{K}_{fyy} & {}^{12}\mathbf{K}_{fyx} & {}^{12}\mathbf{K}_{fyy}\\ {}^{21}\mathbf{K}_{fxx} & {}^{21}\mathbf{K}_{fxy} & {}^2\mathbf{K}_{fxx} & {}^2\mathbf{K}_{fxy}\\ {}^{21}\mathbf{K}_{fyx} & {}^{21}\mathbf{K}_{fyy} & {}^2\mathbf{K}_{fyx} & {}^2\mathbf{K}_{fyy}\end{bmatrix}, \tag{9}$$

where the off-diagonal blocks denote the interfinger coupling matrices, and ${}^i\mathbf{K}_{fxy}$ $(i=1,2)$ denotes the coupling stiffness elements between the x- and y-directions in the ith fingertip space.

A grip Jacobian matrix relating the small displacement of the operational position to that of the finger positions is given by

$$\begin{bmatrix}{}^1\mathbf{G}_o^f\\ {}^2\mathbf{G}_o^f\end{bmatrix}=\begin{bmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\\ b_{31} & b_{32}\\ b_{41} & b_{42}\end{bmatrix}. \tag{10}$$

Here, our objective is to eliminate interfinger couplings as well as the coupling of the fingertip space for effective hybrid control in the fingertip space. So, the stiffness matrix satisfying this objective is set up as below,

$$[\mathbf{K}_f^d]=\begin{bmatrix}{}^1\mathbf{K}_{fxx}^d & 0 & 0 & 0\\ 0 & {}^1\mathbf{K}_{fyy}^d & 0 & 0\\ 0 & 0 & {}^2\mathbf{K}_{fxx}^d & 0\\ 0 & 0 & 0 & {}^2\mathbf{K}_{fyy}^d\end{bmatrix}, \tag{11}$$

where ${}^i\mathbf{K}_{fxx}^d$ and ${}^i\mathbf{K}_{fyy}^d$ denote the desired x- and y-directional stiffness elements in the fingertip space of the ith finger, respectively.

Thus, the stiffness relation of (4) in the two-dimensional space can be expressed by

$$\begin{bmatrix}\mathbf{K}_{oxx} & \mathbf{K}_{oxy}\\ \mathbf{K}_{oyx} & \mathbf{K}_{oyy}\end{bmatrix} = \begin{bmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\\ b_{31} & b_{32}\\ b_{41} & b_{42}\end{bmatrix}^T \begin{bmatrix}{}^1\mathbf{K}_{fxx}^d & 0 & 0 & 0\\ 0 & {}^1\mathbf{K}_{fyy}^d & 0 & 0\\ 0 & 0 & {}^2\mathbf{K}_{fxx}^d & 0\\ 0 & 0 & 0 & {}^2\mathbf{K}_{fyy}^d\end{bmatrix} \times \begin{bmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\\ b_{31} & b_{32}\\ b_{41} & b_{42}\end{bmatrix}. \tag{12}$$

Then, (12) can be rearranged as a vector form,

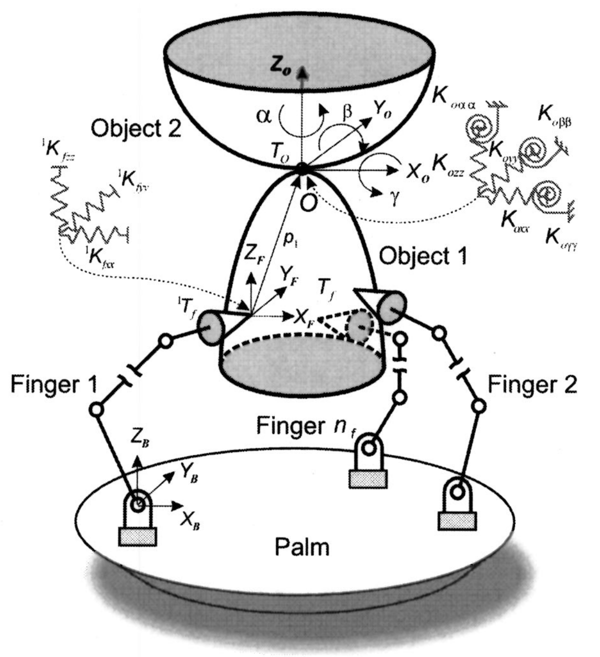

Figure 3. Compliance control by a multi-fingered hand.

$$K_{oo}=[\mathbf{B}_f^o]K_{ff}, \tag{13}$$

where

$$K_{oo}=[\mathbf{K}_{oxx}\ \ \mathbf{K}_{oxy}\ \ \mathbf{K}_{oyy}]^T,$$

$$K_{ff}=[{}^1\mathbf{K}_{fxx}^d\ \ {}^1\mathbf{K}_{fyy}^d\ \ {}^2\mathbf{K}_{fxx}^d\ \ {}^2\mathbf{K}_{fyy}^d]^T,$$

and

$$[\mathbf{B}_f^o]=\begin{bmatrix}(b_{11})^2 & (b_{21})^2 & (b_{31})^2 & (b_{41})^2\\ b_{11}b_{12} & b_{21}b_{22} & b_{31}b_{32} & b_{41}b_{42}\\ (b_{12})^2 & (b_{22})^2 & (b_{32})^2 & (b_{42})^2\end{bmatrix}.$$

For most cases as shown in Figure 3, the general form of $[\mathbf{B}_f^o]$ relating the fingertip space to the operational space is described in the Appendix. The structure of $[\mathbf{B}_f^o]$ may change according to the grasping geometry. To be specific, the sign of some elements of $[\mathbf{B}_f^o]$ may be changed from positive to negative or zero. Accordingly, a change in the values of operational stiffness elements should be considerably observed. Generally, the problem of computing the fingertip stiffness for a given operational stiffness can be transferred to a linear programming problem of the form: find $K_{ff}(\geqslant 0)$ from the linear matrix equation given by (A2) in Appendix.

Consequently, the procedure of computing the fingertip stiffness for the given object stiffness can be summarized as follows:

1. Specify $[\mathbf{K}_o]$ and construct K_{oo}.
2. Check the necessary conditions from Table I and Table II.
3. Determine the grip Jacobian $[\mathbf{G}_o^f]$.
4. Rearrange (4) into (13).
5. Solve the linear programming problem given in (A2) to determine K_{ff}.
6. Find $[\mathbf{K}_f]$ by reconstructing K_{ff}.

Another merit of the modified stiffness relation given by (13) is that it reduces the computation time as compared to the original formulation given by (4). In the case of the first example using two fingers, the computation time is reduced to one third of the original formulation, and that of the three fingers is one fourth of the original formulation.

Additionally, the stiffness characteristics in the fingertip space can be independently resolved into the joint space of each finger.[30] Thus, each joint of all fingers can be satisfactorily controlled.

4. COMPLIANCE ANALYSIS FOR MULTI-FINGERED HANDS

In this section, through the observation of the stiffness relation between the operational space and the fingertip space of multi-fingered hands, we analyze a guideline to appropriately specify compliance characteristics in the two- and three-dimensional spaces. Fundamentally, it is considered for the case of point contact with friction.

4.1. Two-Dimensional Multi-Fingered Operation

When an object is assembled by multi-fingered hands as shown in Figure 4, the performance of the given task is closely related to the grasp geometry and the location of remote compliance center (RCC).[34] Also, the desired compliance characteristics in the operational space can be achievable by solving the linear programming problem described in Section 3.2.

Consider the RCC point lying in the point $\mathbf{O}_1$ of

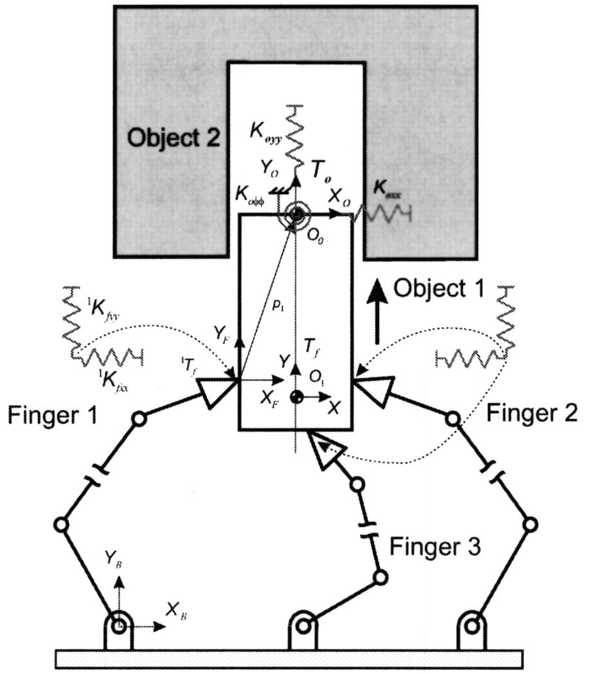

Figure 4. A peg-in-hole task using a multi-fingered hand.

Figure 4. By using (4), (13), and (A2), the independent stiffness relation between the operational space and the fingertip space is given by

$$[K_{oo}]_{(6\times1)}=[\mathbf{B}_f^o]_{(6\times6)}[K_{ff}]_{(6\times1)}$$
$$=\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -y_1 & 0 & -y_2 & 0 & y_3 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & -x_1 & 0 & x_2 & 0 & x_3 \\ y_1^2 & x_1^2 & y_2^2 & x_2^2 & y_3^2 & x_3^2 \end{bmatrix} K_{ff}, \tag{14}$$

where

$$K_{oo}=[\mathbf{K}_{oxx}\ \ \mathbf{K}_{oxy}\ \ \mathbf{K}_{ox\phi}\ \ \mathbf{K}_{oyy}\ \ \mathbf{K}_{oy\phi}\ \ \mathbf{K}_{o\phi\phi}]^T,$$

$$K_{ff}=[{}^1\mathbf{K}_{fxx}\ \ {}^1\mathbf{K}_{fyy}\ \ {}^2\mathbf{K}_{fxx}\ \ {}^2\mathbf{K}_{fyy}\ \ {}^3\mathbf{K}_{fxx}\ \ {}^3\mathbf{K}_{fyy}]^T.$$

Also, x_i and y_i denote the elements of the position vectors directing from the ith finger contact position to the task position, and they are given to be all positive. ${}^i\mathbf{K}_{fxx}$ and ${}^i\mathbf{K}_{fyy}$ represent the x- and y-directional stiffness elements in the fingertip space of the ith finger, respectively.

Note that the elements of the second row of the mapping matrix $[\mathbf{B}_f^o]$ in (14) are calculated as zero. This is because we excluded the coupling terms ${}^i\mathbf{K}_{fxy}$ $(i=1,2,3)$ in the fingertip space so as to facilitate the fingertip's control. Thus, we have zero $\mathbf{K}_{oxy}$, which, in fact, is a linear combination of ${}^i\mathbf{K}_{fxy}$ $(i=1,2,3)$.

Thus, the resultant stiffness matrix in the operational space can be specified as follows:

$$\begin{bmatrix} \mathbf{K}_{oxx} & \mathbf{K}_{oxy} & \mathbf{K}_{ox\phi} \\ \mathbf{K}_{oyx} & \mathbf{K}_{oyy} & \mathbf{K}_{oy\phi} \\ \mathbf{K}_{o\phi x} & \mathbf{K}_{o\phi y} & \mathbf{K}_{o\phi\phi} \end{bmatrix} = \begin{bmatrix} S & 0 & 0,\pm\psi_1 \\ 0 & L & 0,\pm\psi_2 \\ 0,\pm\psi_1 & 0,\pm\psi_2 & S \end{bmatrix}, \tag{15}$$

where ψ_1 and ψ_2 are all positive parameters. And S and L mean that small and large value of stiffness, respectively. The diagonal elements can be determined by considering the task characteristics for each direction.[24–26] ψ_1 and ψ_2 can be arbitrarily determined and also those parameters can be suitably adjusted by considering additional control performance (e.g., the jamming effect of assembly tasks).

Next, consider the RCC point lying in the point $\mathbf{O}_0$ of Figure 4. In this case, the signs of y_1 and y_2 in the third row of $[\mathbf{B}_f^o]$ are changed by moving the RCC point from $\mathbf{O}_1$ to $\mathbf{O}_0$. Thus, the stiffness matrix in the operational space can be specified as follows:[35]

$$\begin{bmatrix} \mathbf{K}_{oxx} & \mathbf{K}_{oxy} & \mathbf{K}_{ox\phi} \\ \mathbf{K}_{oyx} & \mathbf{K}_{oyy} & \mathbf{K}_{oy\phi} \\ \mathbf{K}_{o\phi x} & \mathbf{K}_{o\phi y} & \mathbf{K}_{o\phi\phi} \end{bmatrix} = \begin{bmatrix} S & 0 & +\psi_1 \\ 0 & L & 0,\pm\psi_2 \\ \pm\psi_1 & 0,\pm\psi_2 & S \end{bmatrix}. \tag{16}$$

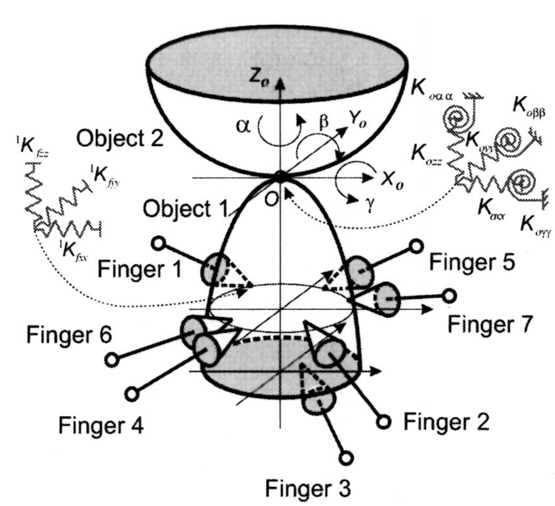

Figure 5. Compliant manipulating task by a seven-fingered hand in three-dimensional space.

As a result, in order to effectively perform various assembly tasks using multi-fingered hands, we can notice that the compliance characteristics in the operational space should be properly specified by considering the location of the compliance center.

4.2. Three-Dimensional Multi-Fingered Operation

For most cases, consider a common compliant manipulating task by multi-fingered hands in three-dimensional space as shown in Figure 3. To implement the desired 6×6 object stiffness matrix in the operational space, a seven-fingered hand is fundamentally necessary according to the necessary condition for stiffness control in Table II. Thus, the hand shown in Figure 5 satisfies the necessary condition.

Consider the RCC point lying in the point **O** of Figure 5. By using (4), (13), and (A2), the independent stiffness relation between the operational space and the fingertip space is given by

$$[K_{oo}]_{(21\times 1)} = [\mathbf{B}_f^o]_{(21\times 21)}[K_{ff}]_{(21\times 1)},$$

$$\begin{bmatrix} \mathbf{K}_{oxx} \\ \mathbf{K}_{oxy} \\ \mathbf{K}_{oxz} \\ \mathbf{K}_{ox\gamma} \\ \mathbf{K}_{ox\beta} \\ \mathbf{K}_{ox\alpha} \\ \mathbf{K}_{oyy} \\ \mathbf{K}_{oyz} \\ \mathbf{K}_{oy\gamma} \\ \mathbf{K}_{oy\beta} \\ \mathbf{K}_{oy\alpha} \\ \mathbf{K}_{ozz} \\ \mathbf{K}_{oz\gamma} \\ \mathbf{K}_{oz\beta} \\ \mathbf{K}_{oz\alpha} \\ \mathbf{K}_{o\gamma\gamma} \\ \mathbf{K}_{o\gamma\beta} \\ \mathbf{K}_{o\gamma\alpha} \\ \mathbf{K}_{o\beta\beta} \\ \mathbf{K}_{o\beta\alpha} \\ \mathbf{K}_{o\alpha\alpha} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -z_1 & 0 & 0 & -z_2 & 0 & 0 & -z_3 & 0 & 0 & -z_4 & 0 & 0 & -z_5 & 0 & 0 & -z_6 & 0 & 0 & -z_7 & 0 & 0 \\ -y_1 & 0 & 0 & y_2 & 0 & 0 & -y_3 & 0 & 0 & y_4 & 0 & 0 & -y_5 & 0 & 0 & y_6 & 0 & 0 & -y_7 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & z_1 & 0 & 0 & z_2 & 0 & 0 & z_3 & 0 & 0 & z_4 & 0 & 0 & z_5 & 0 & 0 & z_6 & 0 & 0 & z_7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -x_1 & 0 & 0 & x_2 & 0 & 0 & x_3 & 0 & 0 & -x_4 & 0 & 0 & x_5 & 0 & 0 & x_6 & 0 & 0 & x_7 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & y_1 & 0 & 0 & -y_2 & 0 & 0 & y_3 & 0 & 0 & -y_4 & 0 & 0 & y_5 & 0 & 0 & -y_6 & 0 & 0 & y_7 \\ 0 & 0 & x_1 & 0 & 0 & -x_2 & 0 & 0 & -x_3 & 0 & 0 & x_4 & 0 & 0 & -x_5 & 0 & 0 & x_6 & 0 & 0 & -x_7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & z_1^2 & y_1^2 & 0 & z_2^2 & y_2^2 & 0 & z_3^2 & y_3^2 & 0 & z_4^2 & y_4^2 & 0 & z_5^2 & y_5^2 & 0 & z_6^2 & y_6^2 & 0 & z_7^2 & y_7^2 \\ 0 & 0 & x_1y_1 & 0 & 0 & x_2y_2 & 0 & 0 & -x_3y_3 & 0 & 0 & -x_4y_4 & 0 & 0 & -x_5y_5 & 0 & 0 & -x_6y_6 & 0 & 0 & -x_7y_7 \\ 0 & -x_1z_1 & 0 & 0 & x_2z_2 & 0 & 0 & x_3z_3 & 0 & 0 & -x_4z_4 & 0 & 0 & x_5z_5 & 0 & 0 & -x_6z_6 & 0 & 0 & x_7z_7 & 0 \\ z_1^2 & 0 & x_1^2 & z_2^2 & 0 & x_2^2 & z_3^2 & 0 & x_3^2 & z_4^2 & 0 & x_4^2 & z_5^2 & 0 & x_5^2 & z_6^2 & 0 & x_6^2 & z_7^2 & 0 & x_7^2 \\ y_1z_1 & 0 & 0 & -y_2z_2 & 0 & 0 & y_3z_3 & 0 & 0 & -y_4z_4 & 0 & 0 & y_5z_5 & 0 & 0 & -y_6z_6 & 0 & 0 & y_7z_7 & 0 & 0 \\ y_1^2 & x_1^2 & 0 & y_2^2 & x_2^2 & 0 & y_3^2 & x_3^2 & 0 & y_4^2 & x_4^2 & 0 & y_5^2 & x_5^2 & 0 & y_6^2 & x_6^2 & 0 & y_7^2 & x_7^2 & 0 \end{bmatrix} \begin{bmatrix} {}^1\mathbf{K}_{fxx} \\ {}^1\mathbf{K}_{fyy} \\ {}^1\mathbf{K}_{fzz} \\ {}^2\mathbf{K}_{fxx} \\ {}^2\mathbf{K}_{fyy} \\ {}^2\mathbf{K}_{fzz} \\ {}^3\mathbf{K}_{fxx} \\ {}^3\mathbf{K}_{fyy} \\ {}^3\mathbf{K}_{fzz} \\ {}^4\mathbf{K}_{fxx} \\ {}^4\mathbf{K}_{fyy} \\ {}^4\mathbf{K}_{fzz} \\ {}^5\mathbf{K}_{fxx} \\ {}^5\mathbf{K}_{fyy} \\ {}^5\mathbf{K}_{fzz} \\ {}^6\mathbf{K}_{fxx} \\ {}^6\mathbf{K}_{fyy} \\ {}^6\mathbf{K}_{fzz} \\ {}^7\mathbf{K}_{fxx} \\ {}^7\mathbf{K}_{fyy} \\ {}^7\mathbf{K}_{fzz} \end{bmatrix}, \quad (17)$$

where x_i, y_i, and z_i denote the elements of position vectors directing from the ith finger contact position to the task position, and they are given all positive. ${}^i\mathbf{K}_{fxx}$, ${}^i\mathbf{K}_{fyy}$, and ${}^i\mathbf{K}_{fzz}$ represent the x-, y-, and z-directional stiffness elements in the fingertip space of the ith finger, respectively.

Note that the elements of the second, third, fourth, eighth, tenth, and fifteenth rows in (17) are shown zero. This is because we excluded the coupling terms [${}^i\mathbf{K}_{fxy}$, ${}^i\mathbf{K}_{fxz}$, and ${}^i\mathbf{K}_{fyz}$ $(i=1,2,...,7)$] in the fingertip space so as to facilitate the fingertip's control. Thus, we have zero $\mathbf{K}_{oxy}$, $\mathbf{K}_{oxz}$, $\mathbf{K}_{ox\gamma}$, $\mathbf{K}_{oyz}$, $\mathbf{K}_{oy\beta}$, and $\mathbf{K}_{oz\alpha}$, which, in fact, is a linear combination of ${}^i\mathbf{K}_{fxy}$, ${}^i\mathbf{K}_{fxz}$, and ${}^i\mathbf{K}_{fyz}$ for $i=1,2,...,7$. Also, note that the fifth row of $[\mathbf{B}_f^o]$ corresponds to the modulation of $\mathbf{K}_{ox\beta}$ and the ninth row of $[\mathbf{B}_f^o]$ corresponds to the modulation of $\mathbf{K}_{oy\gamma}$. However, we can easily notice that zero $\mathbf{K}_{ox\beta}$ and $\mathbf{K}_{ox\gamma}$ cannot be achieved by all positive stiffness components K_{ff} defined in the fingertip space and positive influence coefficients [i.e., z_i $(i=1,2,...,7)$] in this grasp configuration.

Consequently, the stiffness matrix in the operational space can be simply specified as follows:

$$\begin{bmatrix} \mathbf{K}_{oxx} & \mathbf{K}_{oxy} & \mathbf{K}_{oxz} & \mathbf{K}_{ox\gamma} & \mathbf{K}_{ox\beta} & \mathbf{K}_{ox\alpha} \\ \mathbf{K}_{oyx} & \mathbf{K}_{oyy} & \mathbf{K}_{oyz} & \mathbf{K}_{oy\gamma} & \mathbf{K}_{oy\beta} & \mathbf{K}_{oy\alpha} \\ \mathbf{K}_{ozx} & \mathbf{K}_{ozy} & \mathbf{K}_{ozz} & \mathbf{K}_{oz\gamma} & \mathbf{K}_{oz\beta} & \mathbf{K}_{oz\alpha} \\ \mathbf{K}_{o\gamma x} & \mathbf{K}_{o\gamma y} & \mathbf{K}_{o\gamma z} & \mathbf{K}_{o\gamma\gamma} & \mathbf{K}_{o\gamma\beta} & \mathbf{K}_{o\gamma\alpha} \\ \mathbf{K}_{o\beta x} & \mathbf{K}_{o\beta y} & \mathbf{K}_{o\beta z} & \mathbf{K}_{o\beta\gamma} & \mathbf{K}_{o\beta\beta} & \mathbf{K}_{o\beta\alpha} \\ \mathbf{K}_{o\alpha x} & \mathbf{K}_{o\alpha y} & \mathbf{K}_{o\alpha z} & \mathbf{K}_{o\alpha\gamma} & \mathbf{K}_{o\alpha\beta} & \mathbf{K}_{o\alpha\alpha} \end{bmatrix} = \begin{bmatrix} k_1 & 0 & 0 & 0 & -\psi_1 & 0,\pm\psi_2 \\ 0 & k_2 & 0 & +\psi_3 & 0 & 0,\pm\psi_4 \\ 0 & 0 & k_3 & 0,\pm\psi_5 & 0,\pm\psi_6 & 0 \\ 0 & +\psi_3 & 0,\pm\psi_5 & k_4 & 0,\pm\psi_7 & 0,\pm\psi_8 \\ -\psi_1 & 0 & 0,\pm\psi_6 & 0,\pm\psi_7 & k_5 & 0,\pm\psi_9 \\ 0,\pm\psi_2 & 0,\pm\psi_4 & 0 & 0,\pm\psi_8 & 0,\pm\psi_9 & k_6 \end{bmatrix}, \tag{18}$$

where k_i $(i=1,2,...,6)$ and ψ_i $(i=1,2,...,9)$ are all positive parameters. Particularly, the diagonal elements can be determined by considering the task characteristic for each direction.[24–26]

In (18), ψ_2 and ψ_i $(i=4,...,9)$ can be arbitrarily determined and also those parameters can be suitably adjusted by considering additional control performance (e.g., to avoid the jamming effect of assembly tasks). On the other hand, ψ_1 and ψ_3 corresponding to $\mathbf{K}_{ox\beta}$ and $\mathbf{K}_{oy\gamma}$, respectively, cannot be arbitrarily planned according to the previous analysis.

In typical contact tasks, RCC point exists at the distal position of the grasped object as shown in Figure 5. However, if the RCC point lies inside the polygon formed by the finger contact point, the coupling stiffness elements (i.e., $\mathbf{K}_{ox\beta}$ and $\mathbf{K}_{oy\gamma}$) can be made zero since some of the kinematic influence coefficients of the fifth and ninth, rows of (17) change their signs. Thus, it is pointed out that the location of the compliance center plays an important role for successful compliant robotic tasks. For example, in assembling and/or character writing tasks, it is confirmed that a proper selection of the location of the compliance center can improve the given performance.[34,35] Assuming a point contact with friction, a multi-fingered hand should have at least seven fingers to modulate 6×6 operational stiffness characteristic in three-dimensional space,[21] since the operational stiffness matrix has 21 independent stiffness elements. However, as mentioned above, the independent compliance control of a multi-fingered hand provides us with partially decoupled compliance characteristics in the operational space. According to (17), the number of the independent compliance elements is 15. Thus, we can find that the six-degree-of-freedom compliance control can be achievable with a five-fingered hand.

Specifically, when the RCC point lies outside the polygon formed by the grasped fingers, the procedure to determine the stiffness elements at the fingertip space is as follows. Let $[\mathbf{D}_f^o]$ be the matrix excluding the second, third, fourth, fifth, eighth, ninth, tenth, and fifteenth rows of (17) and K_{oo}^{*} be the vector excluding $\mathbf{K}_{oxy}$, $\mathbf{K}_{oxz}$, $\mathbf{K}_{ox\gamma}$, $\mathbf{K}_{ox\beta}$, $\mathbf{K}_{oyz}$, $\mathbf{K}_{oy\gamma}$, $\mathbf{K}_{oy\beta}$, and $\mathbf{K}_{oz\alpha}$ of K_{oo}. Then, $K_{ff}(\geqslant 0)$ can be obtained by solving the following linear matrix equation:

$$[K_{oo}^{*}]_{(13\times 1)} = [\mathbf{D}_f^o]_{(13\times 21)}[K_{ff}]_{(21\times 1)}. \tag{19}$$

Then, the coupling stiffness element ψ_1 and ψ_3 can be determined by

$$\psi_1 = -[\mathbf{B}_f^o]_5 K_{ff}, \tag{20}$$

$$\psi_3 = [\mathbf{B}_f^o]_9 K_{ff}, \tag{21}$$

where $[\mathbf{B}_f^o]_i$ denotes the ith row of $[\mathbf{B}_f^o]$.

On the other hand, it is well known that a soft finger has four intermediate joints.[36] Let us consider the necessary finger condition to implement the soft finger contact from Table II. In particular, in the case of a five soft-fingered hand, it is observed that the number of independent elements of $[\mathbf{K}_o]_{(6\times 6)}$ is 21, the number of independent elements of $[\mathbf{K}_f]_{(20\times 20)}$ is 210, and the number of finger coupling elements of $[\mathbf{K}_f]_{(20\times 20)}$ is 190, respectively. And, thus, it is concluded that a five soft-fingered hand is insufficient to implement 6×6 operational compliance matrix. Similar to (17), a stiffness mapping for the soft contact is derived as

$$[K_{oo}]_{(21\times 1)} = [\mathbf{B}_f^o]_{(21\times 20)} [K_{ff}]_{(20\times 1)} . \tag{22}$$

Noting that six operational stiffness elements are zero as discussed in (17), we can have five surplus degrees of freedom when we use a five soft-fingered hand [that is, 210–190–(21–6)=5]. Moreover, it is possible to implement the compliant task by a four soft-fingered hand because the hand has one surplus degree of freedom [136–120–(21–6)=1].

As a result, we can conclude that only four soft fingers are necessary to achieve a 6×6 operational stiffness characteristic. A compliance control approach based on the analysis of this work was implemented for a compliant contact task in planar space.[32]

5. CONCLUDING REMARKS

A guideline for specifying compliance characteristics in the three-dimensional operational space of multi-fingered hands was analyzed in this paper. Through analyzing the stiffness relation between the operational space and the fingertip space of multi-fingered hands, it is shown that some of the coupling stiffness elements cannot be planned arbitrarily. Thus, it is desired to consider the objective of the given task as well as the grasp geometry of multi-fingered hands in specifying the operational stiffness matrix for successful grasping and manipulation tasks. In this paper, we showed that in the case of the point contact, a five-fingered hand is necessary to implement a three-dimensional operational stiffness matrix through an example dealing with allocation of RCC point. It is also pointed out that a three-dimensional compliant contact task can be achieved with a four soft-fingered hand. As a result, we can conclude that a five-fingered hand is sufficient for the implementation of proper three-dimensional compliance characteristics by applying our approach. The fundamental analysis on compliance characteristics performed in this study can be extended to general robotic manipulating tasks.

6. APPENDIX: INDEPENDENT STIFFNESS RELATION BETWEEN OPERATIONAL SPACE AND FINGERTIP SPACE

The stiffness relation between the operational space and the fingertip space given in (4) can be generally expressed by

$$\begin{bmatrix} \mathbf{K}_{oxx} & \mathbf{K}_{oxy} & \mathbf{K}_{oxz} & \mathbf{K}_{ox\gamma} & \mathbf{K}_{ox\beta} & \mathbf{K}_{ox\alpha} \\ \mathbf{K}_{oyx} & \mathbf{K}_{oyy} & \mathbf{K}_{oyz} & \mathbf{K}_{oy\gamma} & \mathbf{K}_{oy\beta} & \mathbf{K}_{oy\alpha} \\ \mathbf{K}_{ozx} & \mathbf{K}_{ozy} & \mathbf{K}_{ozz} & \mathbf{K}_{oz\gamma} & \mathbf{K}_{oz\beta} & \mathbf{K}_{oz\alpha} \\ \mathbf{K}_{o\gamma x} & \mathbf{K}_{o\gamma y} & \mathbf{K}_{o\gamma z} & \mathbf{K}_{o\gamma\gamma} & \mathbf{K}_{o\gamma\beta} & \mathbf{K}_{o\gamma\alpha} \\ \mathbf{K}_{o\beta x} & \mathbf{K}_{o\beta y} & \mathbf{K}_{o\beta z} & \mathbf{K}_{o\beta\gamma} & \mathbf{K}_{o\beta\beta} & \mathbf{K}_{o\beta\alpha} \\ \mathbf{K}_{o\alpha x} & \mathbf{K}_{o\alpha y} & \mathbf{K}_{o\alpha z} & \mathbf{K}_{o\alpha\gamma} & \mathbf{K}_{o\alpha\beta} & \mathbf{K}_{o\alpha\alpha} \end{bmatrix}
= \begin{bmatrix} g_{11} & g_{1j} & \cdots & g_{16} \\ g_{k1} & g_{kj} & \cdots & g_{k6} \\ \vdots & \vdots & \ddots & \vdots \\ g_{m1} & g_{mj} & \cdots & g_{m6} \end{bmatrix}^T
\times \begin{bmatrix} {}^{1}\mathbf{K}_f & {}^{12}\mathbf{K}_f & \cdots & {}^{1n_f}\mathbf{K}_f \\ {}^{21}\mathbf{K}_f & {}^{2}\mathbf{K}_f & \cdots & {}^{2n_f}\mathbf{K}_f \\ \vdots & \vdots & \ddots & \vdots \\ {}^{n_f1}\mathbf{K}_f & {}^{n_f2}\mathbf{K}_f & \cdots & {}^{n_f}\mathbf{K}_f \end{bmatrix}
\times \begin{bmatrix} g_{11} & g_{1j} & \cdots & g_{16} \\ g_{k1} & g_{kj} & \cdots & g_{k6} \\ \vdots & \vdots & \ddots & \vdots \\ g_{m1} & g_{mj} & \cdots & g_{m6} \end{bmatrix}, \tag{A1}$$

where subscripts γ, β, and α imply the rotational angles about x-, y-, and z-axis, respectively. The element $g_{kj}\,(j=2,\ldots,5, k=2,\ldots,m-1)$ denotes the kj-element of $[\mathbf{G}_o^f]$ presented in (1) and thus it depends on the grasp geometry.

If all element except the diagonal elements in (A1) are zero, an independent finger-based control can be implemented. Thus, the independent stiffness relation between the operational space and the fingertip space given in (13) can be described as follows:

$$K_{oo}=\begin{bmatrix} g_{11}^2 & g_{l1}^2 & \cdots & g_{m1}^2 \\ g_{11}g_{1j} & g_{l1}g_{lj} & \cdots & g_{m1}g_{mj} \\ \vdots & \vdots & \ddots & \vdots \\ g_{11}g_{16} & g_{l1}g_{l6} & \cdots & g_{m1}g_{m6} \\ g_{12}^2 & g_{l2}^2 & \cdots & g_{m2}^2 \\ g_{12}g_{1k} & g_{l2}g_{lk} & \cdots & g_{m2}g_{mk} \\ \vdots & \vdots & \ddots & \vdots \\ g_{12}g_{16} & g_{l2}g_{l6} & \cdots & g_{m2}g_{m6} \\ \vdots & \vdots & \ddots & \vdots \\ g_{15}^2 & g_{l5}^2 & \cdots & g_{m5}^2 \\ g_{15}g_{16} & g_{l5}g_{l6} & \cdots & g_{m5}g_{m6} \\ g_{16}^2 & g_{l6}^2 & \cdots & g_{m6}^2 \end{bmatrix} K_{ff}, \quad \text{(A2)}$$

where $j=2,...,5$, $k=3,...,5$, $l=2,...,m-1$, and the independent vectors rearranged in the operational space and the fingertip space, K_{oo} and K_{ff}, are given by

$$K_{oo}=[\mathbf{K}_{oxx}\ \ \mathbf{K}_{oxy}\ \ \mathbf{K}_{oxz}\ \ \mathbf{K}_{ox\gamma}\ \ \mathbf{K}_{ox\beta}\ \ \mathbf{K}_{ox\alpha}\ \ \mathbf{K}_{oyy}\cdots\mathbf{K}_{o\beta\beta}\ \ \mathbf{K}_{o\beta\alpha}\ \ \mathbf{K}_{o\alpha\alpha}]^T$$

and

$$K_{ff}=[{}^1\mathbf{K}_{fxx}\ \ {}^1\mathbf{K}_{fxy}\ \ {}^1\mathbf{K}_{fxz}\ \ {}^1\mathbf{K}_{fyy}\ \ {}^1\mathbf{K}_{fyz}\ \ {}^1\mathbf{K}_{fzz}\ \ {}^2\mathbf{K}_{fxx}\cdots{}^{n_f}\mathbf{K}_{fzz}]^T.$$

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