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Mechanism of synchronization in switched nonlinear coupled dynamic networks

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Abstract – In this letter, by constructing a switching symmetric matrix, we derive a criterion of exponential synchronization stability in a switched nonlinear coupled dynamical network. This criterion reveals that the mechanism of the synchronization stability is the largest eigenvalue of the switching symmetric matrix and the largest switching coupling strength. We analyze the feasibility of the proposed criterion with the eigenvalue-based method and the diagonally dominant property of the matrices, respectively. And meanwhile, we give the threshold of the switching coupling strength. Unlike other existing results, the deduction processes of our main results are based on the original synchronization stability definition rather than any introduced disagreement function. Besides, the proposed synchronization stability algorithm can be used conveniently.

Introduction. – The node links in a complex network could be static or dynamic. Static node links imply that the connectivity is invariant throughout the evolution of the network, *i.e.* the coupling matrix is constant in time. Dynamic links, on the other hand, imply that the node links are switched around, *i.e.* the coupling matrix is time-varying. When the coupling matrix is constant, many important results of synchronization in complex networks are developed [1–12]. In real-world networks, some existing node links can fail and other new links are created because the node links of the network adjust for certain purposes. In terms of coupling matrix in the network, this means that certain positive values of off-diagonal elements in the coupling matrix become zero or zero values become positive values with time. Recently, a new type of small-world networks with a set of switching long-range symmetric connections has been proposed [13]. In this case, it is proven that interactions between nodes that are only sporadic and of short duration are very efficient for achieving synchronization [14]. By switching random links, it is shown that in addition to geometrical properties such as the fraction of random links in the network, dynamical information on the time dependence of these links is crucial in determining the spatiotemporal properties of complex dynamical networks [15]. Therefore, setting up some criteria of the synchronization stability in case of a dynamic complex network with switching topology is more challenging.

In this letter, along the research line in [16], we investigate the problem for exponential synchronization stability of switched nonlinear coupled dynamic network directly from the original synchronization definition. A key step in establishing the synchronization stability criteria is to construct a Lyapunov function and a symmetric constant matrix, which consists of the coupling constant matrix and its left eigenvector corresponding to eigenvalue zero of the coupling matrix. However, when the coupling matrix switches with time, its left eigenvector associated

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with eigenvalue zero of the coupling matrix will switch accordingly. Therefore, the left eigenvector can no longer be applied to construct an appropriate Lyapunov function and a symmetric constant matrix in establishing our results. Fortunately, the right eigenvector corresponding to eigenvalue zero of the switching coupling matrix is identical constant at any instant time and the set of all the coupling constant matrices at different instant time is finite, which facilitates us to construct a switching symmetric matrix and the set up one exponential synchronization stability criterion. In this criterion, we show that the mechanism can be represented by the switching symmetric matrix and the switching coupling strength. In other words, the largest eigenvalue of the switching symmetric matrix and the switching coupling strength govern the exponential synchronization stability in a dynamic network. Moreover, by using the eigenvalue-based method and the diagonally dominant property of the matrices associated with the switching symmetric matrix, respectively, we present the algorithms for judging the feasibility of the criterion and give the threshold of the switching coupling strength to guarantee that the states of the dynamic network achieve exponentially the stability of synchronization.

The network model and some preliminaries. – Consider a dynamical network consisting of m identical nodes with diffusively switching couplings, which is described by the hybrid differential equations with continuous states $x_i(t)\in R^n, i=1,\cdots,m$ and a discrete state k:

$$\dot{x}_i(t)=f(x_i(t))+c_k\sum_{j=1}^{m}a_{ijk}h(x_j(t)),\quad i=1,\cdots,m, \quad (1)$$

where $x_i(\mathrm{t})\in R^n$ represents the state vector of the i-th node, $f:R^n\to R^n$ is a smooth nonlinear vector-valued function, the function $h(x_j(t))=[h_1(x_j^1(t)),\cdots,h_n(x_j^n(t))]^T$. k which is determined by the map $s(t)=k:R\to\Theta$ is a switching signal and $\Theta\subset N$ is a finite index set. The switching coupling strength $c_k>0$ represents the coupling strength of the whole network at instant time t, the switching coupling matrix $A_k=(\mathrm{a}_{ijk})\in R^{m\times m}$ represents the coupling configuration of the network, at instant time t, if there is a connection from node j to node $i(i\neq j)$, $a_{ijk}>0$, otherwise, $a_{ijk}=0\ (i\neq j)$.

In this letter, we assume the switching matrix $A_k, k\in\Theta$ to satisfy

$$\begin{aligned} 0>a_{iik} &= -\sum_{\substack{j=1\\ j\neq i}}^{m}a_{ijk},\quad i=1,\cdots,m,\ k\in\Theta,\\ \mathrm{rank}(A_k) &= m-1,\ k\in\Theta. \end{aligned} \quad (2)$$

By condition (2), we have $A_k1_m=0_m$, where the vector $1_m=[1,\cdots,1]^T\in R^m$ is the right eigenvector corresponding to eigenvalue zero of the switching coupling matrix A_k, $0_m\in R^m$ is the vector with identical constant zero.

With the vector 1_m, we construct a symmetric constant matrix

$$W=I_m-\frac{1}{m}1_m1_m^T, \quad (3)$$

where $I_m\in R^{m\times m}$ is the identity matrix.

It is easy to prove that the matrix W has the following characteristic:

$$\begin{aligned} &W=U\Lambda U^T,\quad U=[u_1,\cdots,u_m],\\ &u_i\in R^m,\quad i=1,\cdots,m,\quad U^TU=I_m, \end{aligned} \quad (4)$$

$$\Lambda=\mathrm{diag}(\lambda_1,\cdots,\lambda_m),\quad \lambda_1=0,\quad \lambda_i=1,\quad i=2,\cdots,m, \quad (5)$$

and the vector $u_1=\frac{1}{\sqrt{m}}1_m$ is the eigenvector associated with eigenvalue zero of the matrix W. Defining the matrix $\tilde{U}=[u_2,\cdots,\ u_m]$, where $\tilde{U}^T\tilde{U}=I_{m-1}$, the column vectors of $\tilde{U}$ satisfy the homogeneous system

$$1_m^Tb=0,\quad b\in R^m. \quad (6)$$

Since the rank of $\tilde{U}$ equals $m-1$, $\tilde{U}$ forms a basis of the solution space of (6). It is easy to prove that the column vectors of the matrix W satisfy (6) and $m-1=$ rank $(W)=$ rank $(\tilde{W})$ where $\tilde{W}$ is of the form

$$\tilde{W}=\begin{bmatrix} I_{m-1}\\ 0_{m-1}^T\end{bmatrix}-\frac{1}{m}1_m1_{m-1}^T,\quad 1_{m-1}=[1,\cdots,1]^T\in R^{m-1}. \quad (7)$$

It follows that the matrix $\tilde{W}$also forms a basis of the solution space of (6). Therefore, there exists an invertible matrix $Q\in R^{(m-1)\times(m-1)}$ such that

$$\tilde{U}=\tilde{W}Q \quad (8)$$

Main results. – Introducing an average state

$$\bar{x}(t)=\frac{1}{m}\sum_{l=1}^{m}x_l(t), \quad (9)$$

by using $\dot{\bar{x}}(t)=\frac{1}{m}\sum_{i=1}^{m}f(x_i(t))+\frac{c_k}{m}\sum_{i=1}^{m}\sum_{j=1}^{m}a_{ijk}h(x_j(t))$, $k=s(t)$, $\sum_{i=1}^{m}(x_i(t)-\bar{x}(t))=0_n$ and condition (2), we obtain the variation equations near $\bar{x}(t)$,

$$\begin{aligned} \dot{x}_i(t) &= Df(\bar{x})\,x_i(t)+c_k\sum_{j=1}^{m}a_{ijk}Dh(\bar{x}(t))\,x_j(t)+J,\\ i &= 1,\cdots,m, \end{aligned} \quad (10)$$

where

$$\begin{aligned} J &= \dot{\bar{x}}(t)+f(\bar{x}(t))-Df(\bar{x}(t))\,\bar{x}(t)-\frac{c_k}{m}\\ &\quad\times\sum_{j=1}^{m}A_{jk}Dh(\bar{x}(t))\,(x_j(t)-\bar{x}(t)). \end{aligned}$$

$Df(\bar{x}(t))$, $Dh(\bar{x}(t))$ are the Jacobian matrices with respect to $\bar{x}(t)$ and $Dh(u(t))=Dh(u(t))^T$ for all $u(t)\in R^n$, $A_{jk}=\sum_{i=1}^{m}a_{ijk},\ j=1,\cdots,m$.

Assumption 1: Assume that there exists a positive constant γ such that

$$Df(\bar{x}(t))+Df(\bar{x}(t))^{\mathrm{T}}\leqslant\gamma I_n. \quad (11)$$

Using the Kronecker product, we rewrite (10) as

$$\begin{aligned}\dot{x}(t) &= [I_m\otimes Df(\bar{x}(t))+c_kA_k\otimes Dh(\bar{x}(t))]x(t)\\ &\quad +1_m\otimes J,\ k\in\Theta\end{aligned} \quad (12)$$

Based on assumption 1, we obtain the result:

Theorem 1: Let the matrix, $A_k, k=s(t)\in\Theta$, satisfy condition (2), assumption 1 hold, and there exist positive constants $\beta>0$ and $\alpha>0$ such that $Dh(u(t))=Dh(u(t))^T\geqslant\beta I_n$ for all $u(t)\in R^n$ and $\alpha>\frac{\gamma}{\beta}$, respectively. If the matrix $A_k, k\in\Theta$ and the coupling strength $c_k, k\in\Theta$ such that

$$\alpha+\max_{k\in\Theta}\{c_k\lambda_1(\bar{A}_k)\}\leqslant 0, \quad (13)$$

then, the solutions of (1) are locally exponentially stable on the synchronization manifold $x_1(t)=\cdots=x_m(t)$, where $\lambda_1(\bar{A}_k)$ is the largest eigenvalue of the matrix $\bar{A}_k$ defined in (14),

$$\bar{A}_k=\tilde{U}^T(A_k+A_k^T)\tilde{U}. \quad (14)$$

Proof: By using the symmetric matrix W in (3), we construct a Lyapunov function as

$$V(x(t))=\alpha x(t)^T(W\otimes I_n)x(t) \quad (15)$$

Differentiating (15) along the trajectory of (10), we have

$$\begin{aligned}&\dot{V}(x(t))=2x(t)^T(W\otimes Df(\bar{x}(t)))x(t)\\ &+2c_kx(t)^T(WA_k\otimes Dh(\bar{x}(t)))x(t)+2x(t)^T(W1_m\otimes J)\leqslant\\ &-\eta V(t)+x(t)^T((\alpha W+2c_kWA_k)\otimes Dh(\bar{x}(t)))x(t),\end{aligned} \quad (16)$$

where $2x(t)^T(W1_m\otimes J)=0$, $\eta=-\gamma+\alpha\beta>0$.

Introducing a transformation

$$y(t)=(U^T\otimes I_n)x(t),\quad y(t)=[y_1(t)^T,\cdots,y_m(t)^T]^T,$$

the second term in (16) with (4) and (5) is simplified as

$$\begin{aligned}&x(t)^T\left[(\alpha W+2c_kWA_k)\otimes Dh(\bar{x}(t))\right]x(t)\leqslant\\ &\left(\alpha+\max_{k\in\Theta}\{c_k\lambda_1(\bar{A}_k)\}\right)\tilde{y}(t)^T\left[I_{m-1}\otimes Dh(\bar{x}(t))\right]\tilde{y}(t),\end{aligned} \quad (17)$$

where $\bar{A}_k=\tilde{U}^T(A_k+A_k^T)\tilde{U}$, $\tilde{y}(t)=[y_2(t)^T,\cdots,y_m(t)^T]^T$, $\tilde{U}^T\tilde{U}=I_{m-1}$. Substituting (17) into (16) along with (13) gives $\dot{V}(x(t))\leqslant-\eta V(x(t))$. It follows that

$$V(x(t))\leqslant V(x(0))\exp(-\eta t), t\geqslant 0. \quad (18)$$

Because the inequality (18) holds, we have

$$\|x_i(t)-x_j(t)\|\leqslant\sqrt{2mV(x(0))}\exp\left(-\frac{\eta}{2}t\right). \quad (19)$$

The $\max_{k\in\Theta}\{c_k\lambda_1(\bar{A}_k)\}$ always exists and is achieved because Θ is a finite index set.

The proof is thus completed.

Remark 1: It is clear that to satisfy (13) we need $\bar{A}_k<0_{(m-1)\times(m-1)}\in R^{(m-1)\times(m-1)}$, which means that there exists c_k such that $c_k\geqslant-\frac{\alpha}{\lambda_1(\bar{A}_k)}$ if $\bar{A}_k<0_{(m-1)\times(m-1)}$, which is equivalent to $\alpha+\max_{k\in\Theta}\{c_k\lambda_1(\bar{A}_k)\}\leqslant 0$ if $\lambda_1(\bar{A}_k)<0$. So we can judge the feasibility of (13) by calculating the largest eigenvalues of the matrix $\bar{A}_k$ and if $\lambda_1(\bar{A}_k)<0$, and determine the threshold of the switching coupling strength $c_k, k\in\Theta$ by

$$c_k\geqslant-\frac{\alpha}{\lambda_1(\bar{A}_k)},\quad k\in\Theta. \quad (20)$$

Inequalities (20) show that the mechanism of synchronization in network (1) is the largest eigenvalues of the matrix $\bar{A}_k$, $\lambda_1(\bar{A}_k), k\in\Theta$, and the coupling strengths $c_k, k\in\Theta$.

Now, without doing an eigenvalue computation, we analyze the feasibility condition by the diagonally dominant property of matrices associated with $\bar{A}_k$. By using (8), we have $\bar{A}_k=Q^T\tilde{W}^T(A_k+A_k^T)\tilde{W}Q$. Then, $\bar{A}_k<0_{(m-1)\times(m-1)}, k\in\Theta$ is equivalent to $\tilde{W}^T(A_k+A_k^T)\tilde{W}<0_{(m-1)\times(m-1)}, k\in\Theta$.

Define

$$P_k=\tilde{W}^T\left(A_k+A_k^T\right)\tilde{W}\in R^{(m-1)\times(m-1)}. \quad (21)$$

By calculating, we have

$$P_k=\tilde{A}_k+\tilde{A}_k^T-\frac{1}{m}\left(B_k+B_k^T\right),\ k\in\Theta, \quad (22)$$

where

$$\tilde{A}_k=(a_{ijk})\in R^{(m-1)\times(m-1)},\ B_k=\begin{bmatrix}A_{1k} & A_{2k} & \cdots & A_{m-1,k}\\ A_{1k} & A_{2k} & \cdots & A_{m-1,k}\\ \vdots & \vdots & \cdots & \vdots\\ A_{1k} & A_{2k} & \cdots & A_{m-1,k}\end{bmatrix},$$

$$A_{jk}=\sum_{l=1}^{m}a_{ijk},\quad j=1,\cdots,m-1,\ k\in\Theta.$$

The diagonal elements of P_k are of form

$$p_{iik}=2\left(a_{iik}-\frac{1}{m}A_{ik}\right),\quad i=1,\cdots,\ m-1. \quad (23)$$

With condition (2), we have

$$a_{iik}-\frac{1}{m}A_{ik}\ =\ a_{iik}\left(1-\frac{1}{m}\right)-\frac{1}{m}\sum_{\substack{j=1\\ j\neq i}}^{m}a_{jik}<0,$$
$$i\ =\ 1,\cdots,\ m-1.$$

It follows that

$$p_{iik}=2\left(a_{iik}-\frac{1}{m}A_{ik}\right)<0,\quad i=1,\cdots,m-1,\ k\in\Theta. \quad (24)$$

The off-diagonal elements of P_k are

$$p_{ijk}=p_{jik}=a_{ijk}+a_{jik}-\frac{A_{ik}+A_{jk}}{m},\quad i\neq j,\quad i,j=1,\cdots,m-1. \tag{25}$$

The sum of i-th row is

$$\sum_{j=1}^{m-1}p_{ijk}=-a_{imk}-a_{mik}+\frac{A_{ik}+A_{mk}}{m},\quad i=1,\cdots,m-1. \tag{26}$$

With (26), we have

$$p_{iik}+\sum_{\substack{j=1\\ j\neq i}}^{m-1}|p_{ijk}|=\sum_{j=1}^{m-1}p_{ijk}+\sum_{\substack{j=1\\ j\neq i}}^{m-1}p_{ijk}\left[\mathrm{sgn}(p_{ijk})-1\right]=-a_{imk}-a_{mik}+\frac{A_{ik}+A_{mk}}{m}+\sum_{\substack{j=1\\ j\neq i}}^{m-1}p_{ijk}\left[\mathrm{sgn}(p_{ijk})-1\right], \tag{27}$$

where $\mathrm{sgn}(\bullet)$ is the sign function, *i.e.*

$$\mathrm{sgn}(z)=\begin{cases}1, & z>0,\\ 0, & z=0,\\ -1, & z<0.\end{cases}$$

It follows from (27) and Gerschgorin's theorem [17] that all the eigenvalues of the switching matrix $P_k, k\in\Theta$ are negative if the matrix $P_k, k\in\Theta$ is diagonally dominant. The result deduced above is summarized as follows:

Theorem 2: Let the switching matrix, $A_k, k=s(t)\in\Theta$, satisfy condition (2). Condition (13) is feasible if the following inequalities:

$$-a_{imk}-a_{mik}+\frac{A_{ik}+A_{mk}}{m}<-\sum_{\substack{j=1\\ j\neq i}}^{m-1}p_{ijk}(\mathrm{sgn}(p_{ijk})-1),\quad i=1,\cdots,m-1,\ k\in\Theta \tag{28}$$

hold.

Remark 2: Theorem 2 gives the algorithm to check the sufficient condition (13) by verifying if the switching symmetric matrix $P_k, k\in\Theta$ is strictly diagonally dominant, which is convenient to use in practical computation.

Illustrated examples. – In this section, we illustrate the results of Theorem 1 and Theorem 2 with one example, respectively. In the dynamic network (1), we take vector-valued function f as

$$f(x_i(t))=\begin{pmatrix}-p\left(x_{i1}(t)-px_{i2}(t)+g(x_{i1}(t))\right)\\ x_{i1}(t)-x_{i2}(t)+x_{i3}(t)\\ -qx_{i2}(t)\end{pmatrix},\ i=1,\cdots,m \tag{29}$$

where $g(x_{i1}(t))=m_0x_{i1}(t)+\frac{1}{2}(m_1-m_0)(|x_{i1}(t)+1|-|x_{i1}(t)-1|)$, $p=10$, $q=14.7$, $m_0=-0.68$, and $m_1=-12$.

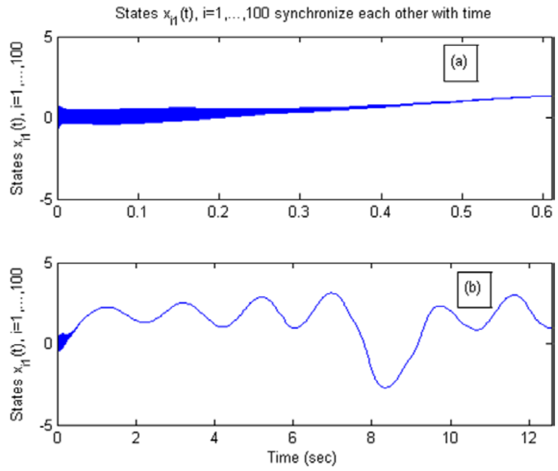

Fig. 1: (Colour on-line) Evolution process of the states $x_{i1}(t)$, $i=1,\cdots,100$ with time: (a) Transient evolution process with time $t\in[0,0.6]$. (b) The whole evolution process with time $t\in[0,4\pi]$.

The nonlinear coupled function is as follows:

$$h(x_i(t))=[h_1(x_{i1}(t)),h_2(x_{i2}(t)),h_3(x_{i3}(x_{i3}(t)))]^T,$$
$$h_l(x_{il}(t))=\exp(\tau_l x_{il}(t))-\exp(-\tau_l x_{il}(t)),\quad \tau_l>0,\quad l=1,2,3. \tag{30}$$

We consider the network (1) with $m=100$ nodes and switching signal $k=s(t)=\mathrm{round}(1+3|\sin t|)$. It is easy to obtain $k\in\{1,2,3,4\}=\Theta$. We chose the switching coupling matrix in (1) of the form (31). It is clear that the matrix, A_k, switches with time t. By computing, we have $\gamma=18.013$, which implies that Assumption 1 is satisfied. Taking $\tau_1=1.5$, $\tau_2=2$, $\tau_3=3$, we have $\beta=3$. By choosing $\alpha=6.5>\frac{\gamma}{\beta}=6.0043$, which implies that the assumption in Theorem 1 is satisfied. With (3), (14) and (31), we have $\lambda_1(\bar{A}_1)=-0.0158$, $\lambda_1(\bar{A}_2)=-1.9957$, $\lambda_1(\bar{A}_3)=-0.0474$, $\lambda_1(\bar{A}_4)=-2.0108$, which implies that (13) is feasible. By using $c_k\geqslant -6.5/\lambda_1(\bar{A}_k)$, we obtain the threshold of the switching coupling strength, $c_1=411.3924$, $c_2=3.2570$, $c_3=137.1308$, $c_4=3.2325$.

According to Theorem 1, the states of (1) with (29) and (30) are locally exponentially synchronization stable.

Without computing the eigenvalues of the switching coupling matrix (31), we have also verified from Theorem 2 that (13) can be satisfied (see appendix):

$$A_k=\begin{pmatrix}-k-\frac{1+(-1)^k}{2} & k+\frac{1+(-1)^k}{2} & 0 & \cdots & 0\\ \frac{1+(-1)^k}{2} & -k-\frac{1+(-1)^k}{2} & k & \cdots & 0\\ \vdots & & & & \\ \frac{1+(-1)^k}{2} & 0 & & \cdots & k\\ \frac{1+(-1)^k}{2}+k & 0 & \cdots & 0 & -k-\frac{1+(-1)^k}{2}\end{pmatrix},\quad k\in\{1,2,3,4\}. \tag{31}$$

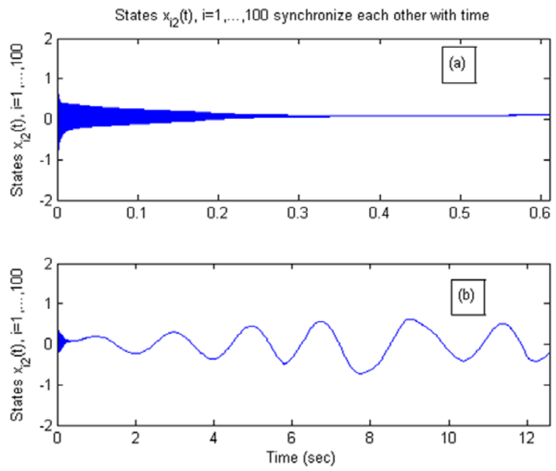

Fig. 2: (Colour on-line) Evolution process of the states $x_{i2}(t)$, $i=1,\cdots,100$ with time: (a) Transient evolution process with time $t\in[0,0.6]$. (b) The whole evolution process with time $t\in[0,4\pi]$.

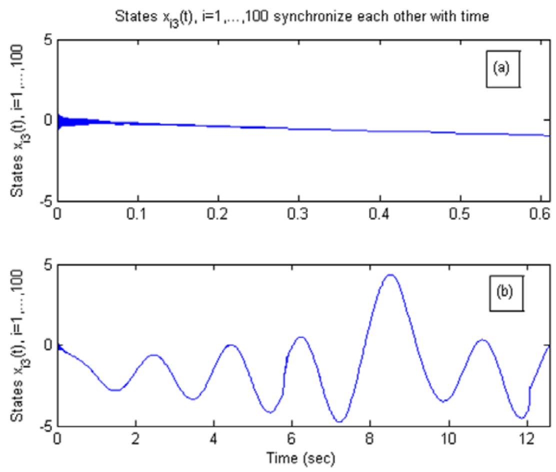

Fig. 3: (Colour on-line) Evolution process of the states $x_{i3}(t)$, $i=1,\cdots,100$ with time: (a) Transient evolution process with time $t\in[0,0.6]$. (b) The whole evolution process with time $t\in[0,4\pi]$.

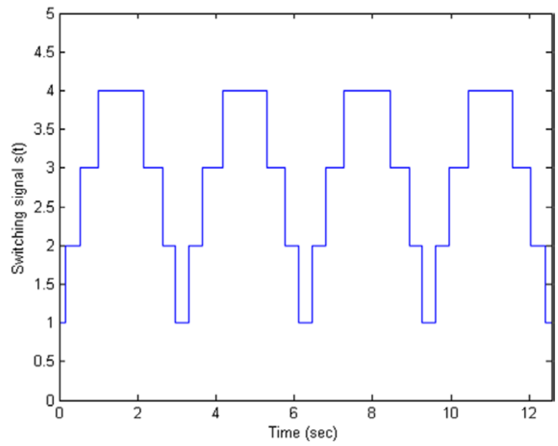

Fig. 4: (Colour on-line) Evolution of the switching signal $k=s(t)$ with time $t\in[0,4\pi]$.

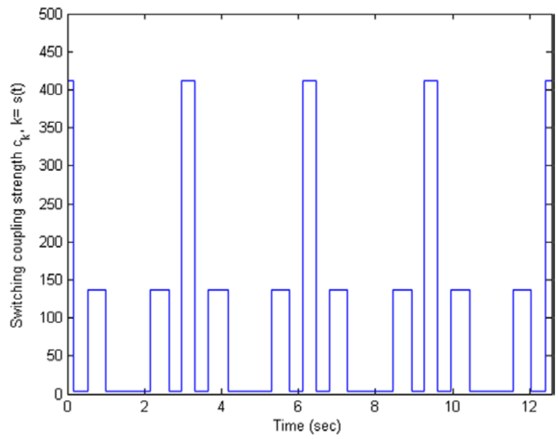

Fig. 5: (Colour on-line) Evolution of the switching coupling strength $c_k, k=s(t)$ with time $t\in[0,4\pi]$.

In the following simulations, the initial states are randomly chosen as $x_i(0)\in[-1.5,1.5]\times[-1.5,1.5]\times[-1.5,1.5]$, $i=1,2,\cdots,100$, with time $t\in[0,4\pi]$. The simulation results are shown from fig. 1 to fig. 5. In fig. 1, fig. 2 and fig. 3, we present the evolution process of the states $x_{ij}(t)$, $i=1,\cdots,100$, $j=1,2,3$ with time t in dynamical network (1). When time varies from 0 to 0.6, we present the evolution of the states in part (a) of fig. 1, fig. 2 and fig. 3, which shows transient synchronization process of the states in (1). In part (b) of fig. 1, fig. 2 and fig. 3, we present the evolution process of the states of (1) with time $t\in[0,4\pi]$, which describes the whole synchronization process of the states in network (1). Comparing part (a) and part (b) of fig. 1, fig. 2 and fig. 3, we clearly see that all states synchronize with each other when time $t\geqslant 0.6$, which shows that all states of the network have reached exponential synchronization under the synchronization algorithm in Theorem 1. We present the evolution of the switching signal and the evolution of the switching coupling strength with time $t\in[0,4\pi]$ in fig. 4 and fig. 5, respectively. From fig. 4, we can see that the switching signal $s(t)=k$ switches in a finite set $\Theta=\{1,2,3,4\}$ with time varying from 0 to 4π. From fig. 5, we can clearly see that the switching coupling strength switches with time varying from 0 to 4π, which is in agreement with our theory values. It is the switching coupling strength that should be acted on the whole network during all the states of the network (1) achieving synchronization with time varying from 0 to 4π.

In conclusion, we developed an exponential synchronization stability criterion. By this criterion, we showed that the mechanism of synchronization in a switched nonlinear coupled network is the largest eigenvalue of the switching symmetric matrix and the largest switching coupling strength. We have also given some convenient algorithms for judging the feasibility of the criterion using the eigenvalues-based method and the diagonally dominant property of symmetric matrices, respectively. The algorithms are very useful for synchronization in switched nonlinear coupled dynamical networks.

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Appendix

When $k=1$ and $k=3$, the matrices A_1 and A_3 have node balance, thus $P_1<0$ and $P_3<0$.

When $k=2$, we have

$$A_{1k}=98,\quad A_{2k}=0,\quad A_{ik}=-1,\qquad i=3,\cdots,100,$$

$$p_{12k}=4-\frac{98}{100}>0,\quad p_{1jk}=1-\frac{97}{100}>0,\qquad j=3,\cdots,99,$$

$$p_{ijk}=2+\frac{1}{100}>0,\ j=i+1,\ i=2,\cdots,98,\ p_{ijk}=\frac{1}{100}>0,$$
$$i\neq j,\qquad i=2,\cdots,99,\qquad j=4,\cdots,99,$$

$$A_{ik}+A_{mk}=98-1=97>0,\ i=1,\ A_{ik}+A_{mk}=-1, i=2,$$
$$A_{ik}+A_{mk}=-2,\quad i=3,\cdots,99,$$

$$-a_{imk}-a_{mik}+\frac{A_{ik}+A_{mk}}{m}=-\frac{203}{m}<0,\ i=1,\quad k\in\Theta$$

and

$$\frac{A_{ik}+A_{mk}}{m}<0, i=2,\cdots,99,$$

which implies that

$$-a_{imk}-a_{mik}+\frac{A_{ik}+A_{mk}}{m}<0,\quad i=2,\cdots,99.$$

It follows that $P_2<0$.

When $k=4$,

$$A_{1k}=98,\ A_{2k}=0,\ A_{ik}=-1,\quad i=3,\cdots,100,$$

$$p_{12k}=6-\frac{98}{100}>0,\ p_{1j2}=1-\frac{97}{100}>0,\quad j=3,\cdots,99,$$

$$p_{23k}=4+\frac{1}{100}>0,\ j=i+1,\ i=2,\cdots,98,\ p_{ijk}=\frac{1}{100}>0,$$
$$i\neq j,\qquad i=2,\cdots,99,\qquad j=4,\cdots,99,$$

$$A_{ik}+A_{mk}=98-1=97>0,\quad i=1,$$
$$A_{ik}+A_{mk}=-1<0,\quad i=2,$$
$$A_{ik}+A_{mk}=-2<0,\quad i=3,\cdots,99$$

and

$$-a_{imk}-a_{mik}+\frac{A_{ik}+A_{mk}}{m}=-\frac{403}{m}<0,\quad i=1,\ k\in\Theta,$$
$$\frac{A_{ik}+A_{mk}}{m}<0,\quad i=2,\cdots,99,$$

which implies

$$-a_{imk}-a_{mik}+\frac{A_{ik}+A_{mk}}{m}<0,\quad i=2,\cdots,99.$$

It follows that $P_4<0$.

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